Equivalent Fractions

GUIDED SKILL SUPPORT

Overview

Fractions that name the same value are called equivalent fractions, such as $\frac{2}{3} = \frac{6}{9}$.

One method for writing equivalent fractions is to use the Multiplicative Identity, that is, multiplying the given fraction by a form of the number 1, such as $\frac{2}{2}$, $\frac{5}{5}$, etc. In this course, this type of fraction is called a "Giant One." Multiplying by 1 does not change the value of a number; this is stated in the Identity Property of Multiplication.

For additional information, see the Methods & Meanings box in Lesson 3.1.1 of the *Core Connections Course 1* text.

Examples

1. Write three fractions that are equivalent to $\frac{1}{2}$.

$\frac{1}{2} \cdot \frac{2}{2} = \frac{2}{4}$ $\quad$ $\frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6}$ $\quad$ $\frac{1}{2} \cdot \frac{4}{4} = \frac{4}{8}$

2. Use a Giant One to write an equivalent fraction to $\frac{7}{12}$ that has a denominator of 96.

$$\frac{7}{12} \cdot \frac{}{} = \frac{?}{96}$$

Since $\frac{96}{12} = 8$, the Giant One is $\frac{8}{8}$: $\frac{7}{12} \cdot \frac{8}{8} = \frac{56}{96}$.

Practice

1. $\frac{4}{3} \cdot \frac{}{} = \frac{}{15}$

2. $\frac{5}{9} \cdot \frac{}{} = \frac{}{36}$

3. $\frac{9}{2} \cdot \frac{}{} = \frac{}{38}$

4. $\frac{3}{7} \cdot \frac{}{} = \frac{}{28}$

5. $\frac{5}{3} \cdot \frac{}{} = \frac{}{18}$

6. $\frac{6}{5} \cdot \frac{}{} = \frac{}{15}$

Answers

1. $\frac{4}{3} \cdot \frac{5}{5} = \frac{20}{15}$

2. $\frac{5}{9} \cdot \frac{4}{4} = \frac{20}{36}$

3. $\frac{9}{2} \cdot \frac{19}{19} = \frac{171}{38}$

4. $\frac{3}{7} \cdot \frac{4}{4} = \frac{12}{28}$

5. $\frac{5}{3} \cdot \frac{6}{6} = \frac{30}{18}$

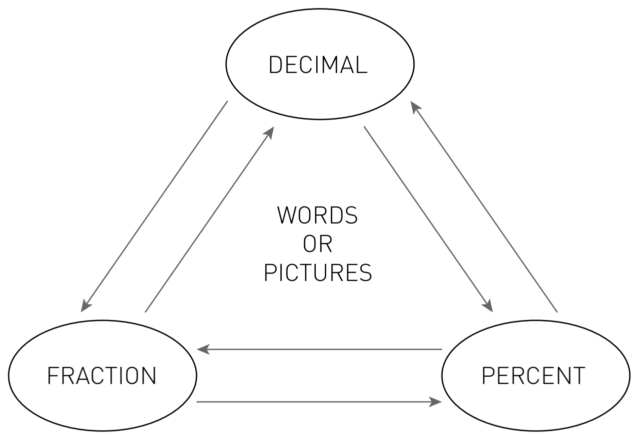
6. $\frac{6}{5} \cdot \frac{3}{3} = \frac{18}{15}$

Fraction–Decimal–Percent Equivalents

GUIDED SKILL SUPPORT

Overview

Fractions, decimals, and percents are different ways to represent the same portion or number.



For additional information, see the Methods & Meanings box in Lesson 3.1.6 of the *Core Connections Course 1* text. For additional examples and practice, see the *Core Connections Course 1* Checkpoint 5 materials.

Examples

1. Decimal to percent: 0.81

Multiply the decimal by 100.

$$(0.81)(100) = 81\%$$

2. Percent to decimal: 43%

Divide the percent by 100.

$$43\% \div 100 = 0.43$$

3. Fraction to percent: $\frac{4}{5}$

Set up an equivalent fraction with 100 as the denominator. The numerator is the percent.

$$\frac{4}{5} \cdot \boxed{\frac{20}{20}} = \frac{80}{100} = 80\%$$

4. Percent to fraction: 22%

Use 100 as the denominator. Use the digits in the percent as the numerator.

$$22\% = \frac{22}{100} \cdot \boxed{\frac{\frac{1}{2}}{\frac{1}{2}}} = \frac{11}{50}$$

5. Decimal to fraction:

a. 0.2

Use the digits in the decimal as the numerator. Use the decimal place value name as the denominator.

$$0.2 = \frac{2}{10} = \frac{1}{5}$$

b. 0.17

$$0.17 = \frac{17}{100}$$

6. Fraction to decimal:

a. $\frac{13}{25}$

Use a Giant One to write the fraction with a denominator that is a multiple of 10. Divide the numerator by the denominator.

$$\frac{13}{25} = \frac{13}{25} \cdot \frac{4}{4} = \frac{52}{100} = 0.52$$

b. $\frac{3}{8}$

$$\frac{3}{8} = \frac{3}{8} \cdot \frac{12.5}{12.5} = \frac{37.5}{100} \cdot \frac{10}{10} = \frac{375}{1{,}000} = 0.375$$

c. $\frac{7}{40}$

$$\frac{7}{40} = \frac{7}{40} \cdot \frac{2.5}{2.5} = \frac{17.5}{100} \cdot \frac{10}{10} = \frac{175}{1{,}000} = 0.175$$

Practice

Convert the fraction, decimal, or percent as indicated.

1. $\frac{1}{4}$ to a decimal.

2. 50% to a fraction in lowest terms.

3. 0.75 to a fraction in lowest terms.

4. 75% to a decimal.

5. 0.38 to a percent.

6. $\frac{1}{5}$ to a percent.

7. 0.3 to a fraction.

8. $\frac{1}{8}$ to a decimal.

9. $\frac{11}{20}$ to a decimal.

10. 0.08 to a percent.

11. 87% to a decimal.

12. $\frac{3}{5}$ to a percent.

13. 0.4 to a fraction in lowest terms.

14. 65% to a fraction in lowest terms.

15. $\frac{10}{25}$ to a decimal.

16. 125% to a fraction.

17. $\frac{8}{5}$ to a decimal.

18. 3.25 to a percent.

19. $\frac{1}{16}$ to a decimal. Then convert the decimal to a percent.

20. $\frac{3}{20}$ to a decimal.

21. 43% to a fraction. Then convert the fraction to a decimal.

22. 0.375 to a percent. Then convert the percent to a fraction.

23. $\frac{7}{8}$ to a decimal. Then convert the decimal to a percent.

24. 0.12 to a fraction.

Answers

1.	0.25	**2.**	$\frac{1}{2}$	**3.**	$\frac{3}{4}$	**4.**	0.75
5.	38%	**6.**	20%	**7.**	$\frac{3}{10}$	**8.**	0.125
9.	0.55	**10.**	8%	**11.**	0.87	**12.**	60%
13.	$\frac{2}{5}$	**14.**	$\frac{13}{20}$	**15.**	0.4	**16.**	$\frac{5}{4}$ or $1\frac{1}{4}$
17.	1.6	**18.**	325%	**19.**	0.0625; 6.25%	**20.**	0.15
21.	$\frac{43}{100}$; 0.43	**22.**	$37\frac{1}{2}$%; $\frac{3}{8}$	**23.**	0.875; 87.5%	**24.**	$\frac{12}{100} = \frac{3}{25}$

Adding and Subtracting Fractions

GUIDED SKILL SUPPORT

Overview

To add or subtract two fractions that are written with the same denominator (the number on the bottom), simply add or subtract the numerators (the numbers on the top) and keep the denominator the same. For example, $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$.

If the fractions have different denominators, rewrite them first as fractions with the same denominator. Two methods for adding and subtracting two fractions with different denominators are using an area model and using the Multiplicative Identity (Giant One).

AREA MODEL METHOD

Fractions can be modeled as a shaded portion of a rectangle. The area model method involves creating common denominators by using rectangles to represent each fraction by dividing and shading the rectangles into equal portions.

The steps of the area model method are as follows:

1. Model each fraction using a shaded rectangle. Cut one rectangle vertically into an equal number of pieces based on the first denominator (bottom number). Cut the other rectangle horizontally, using the second denominator. Shade a number of pieces in each rectangle to represent the numerator (top number). Label each rectangle with the fraction it represents.
2. Superimpose the lines from each rectangle onto the other rectangle, as if one rectangle is placed on top of the other one. Rewrite the fractions by changing the numerators to match the number of shaded portions in each figure and changing the denominators to match the new total number of equal parts.
3. Add or subtract the equivalent parts to determine the sum or difference.

IDENTITY PROPERTY OF MULTIPLICATION (GIANT ONE) METHOD

The Giant One, a representation of the number 1, is a representation of the mathematics of the Multiplicative Identity. It is a fraction with the same numerator and denominator ($\frac{3}{3}$, for example). Multiplying by a Giant One can create common denominators.

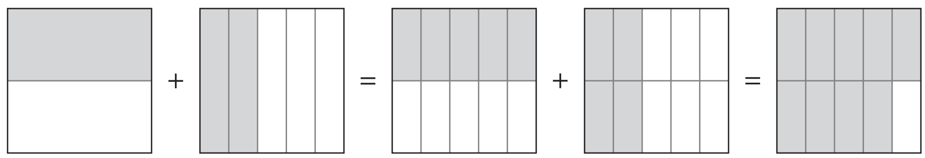
The steps of the Giant One method are as follows:

1. Rewrite the fractions as equivalent fractions that have a common denominator.
2. Add or subtract only the numerators, keeping the common denominator.
3. Rewrite the sum or difference in the lowest terms possible.

Examples

1. Use an area model to calculate the sum.

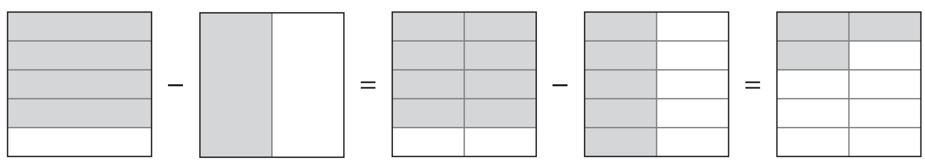
$\frac{1}{2} + \frac{2}{5}$



$$\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{9}{10}$$

2. Use an area model to calculate the difference.

$\frac{4}{5} - \frac{1}{2}$



$$\frac{4}{5} - \frac{1}{2} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$$

3. Use the Giant One method to calculate the sum.

$\frac{1}{3} + \frac{1}{4}$

Multiply both $\frac{1}{3}$ and $\frac{1}{4}$ by Giant Ones to get a common denominator.

$$\frac{1}{3} \cdot \frac{4}{4} + \frac{1}{4} \cdot \frac{3}{3} = \frac{4}{12} + \frac{3}{12}$$

Add the numerators of both fractions to get the sum.

$$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

4. Use the Giant One method to calculate the difference.

$\frac{5}{6} - \frac{1}{4}$

Multiply both $\frac{5}{6}$ and $\frac{1}{4}$ by Giant Ones to get a common denominator.

$$\frac{5}{6} \cdot \frac{2}{2} - \frac{1}{4} \cdot \frac{3}{3} = \frac{10}{12} - \frac{3}{12}$$

Subtract the numerators of both fractions to get the answer.

$$\frac{10}{12} - \frac{3}{12} = \frac{7}{12}$$

Practice

Use the area model method to add or subtract the following fractions.

1. $\frac{3}{4}+\frac{1}{5}$ **2.** $\frac{1}{3}+\frac{2}{7}$ **3.** $\frac{3}{4}-\frac{2}{3}$

Use the Giant One method to add or subtract the following fractions.

4. $\frac{1}{3}+\frac{2}{5}$ **5.** $\frac{4}{5}-\frac{1}{3}$ **6.** $\frac{3}{4}-\frac{1}{5}$

Determine each sum or difference. Use the method of your choice.

7. $\frac{1}{6}+\frac{2}{3}$ **8.** $\frac{3}{8}+\frac{2}{5}$ **9.** $\frac{1}{4}+\frac{3}{7}$ **10.** $\frac{2}{9}+\frac{3}{4}$

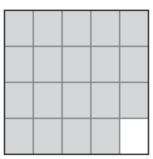
11. $\frac{5}{12}+\frac{1}{3}$ **12.** $\frac{7}{9}-\frac{2}{3}$ **13.** $\frac{3}{4}+\frac{1}{3}$ **14.** $\frac{5}{6}+\frac{2}{3}$

15. $\frac{7}{8}+\frac{1}{4}$ **16.** $\frac{6}{7}-\frac{2}{3}$ **17.** $\frac{1}{3}-\frac{1}{4}$ **18.** $\frac{3}{5}+\frac{3}{4}$

19. $\frac{5}{5}-\frac{3}{4}$ **20.** $\frac{3}{4}-\frac{1}{3}$ **21.** $\frac{2}{5}+\frac{9}{15}$ **22.** $\frac{2}{3}-\frac{3}{5}$

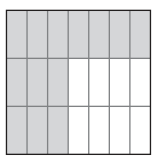
Answers

1.



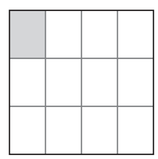
$\frac{19}{20}$

2.



$\frac{13}{21}$

3.



$\frac{1}{12}$

4. $\frac{11}{15}$ **5.** $\frac{7}{15}$ **6.** $\frac{11}{20}$ **7.** $\frac{5}{6}$

8. $\frac{31}{40}$ **9.** $\frac{19}{28}$ **10.** $\frac{35}{36}$ **11.** $\frac{3}{4}$

12. $\frac{1}{9}$ **13.** $\frac{13}{12}=1\frac{1}{12}$ **14.** $\frac{9}{6}=\frac{3}{2}=1\frac{1}{2}$ **15.** $\frac{9}{8}=1\frac{1}{8}$

16. $\frac{4}{21}$ **17.** $\frac{1}{12}$ **18.** $\frac{27}{20}=1\frac{7}{20}$ **19.** $\frac{1}{4}$

20. $\frac{5}{12}$ **21.** 1 **22.** $\frac{1}{15}$

Locating Integers on a Number Line

GUIDED SKILL SUPPORT

Overview

Integers are positive and negative whole numbers, or zero. On a number line, think of integers as "whole steps or no steps" in either direction from 0. On a horizontal number line, positive integers are located to the right of zero and negative integers are to the left of zero. On a vertical number line, positive integers are located above zero and negative integers are below zero.

Examples

1. A rabbit is sitting at −7 on a number line. It makes 4 hops of 10 units each to the right. Where does the rabbit land?

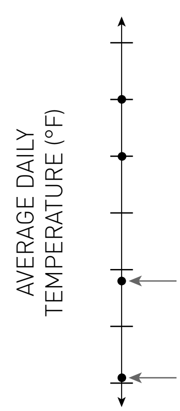
The rabbit lands at 33 on the number line.
−7 + 4(10)

2. A frog is sitting at 3 on a number line. It hops 5 units to the left, 2 units to the right, and then 6 more units to the left. Where does the frog land?

The frog lands at −6 on the number line.
3 + (−5) + 2 + (−6)

3. The average daily temperature in degrees Fahrenheit for four days in January in Green Bay, Wisconsin are as follows: −9, 10, −2, and 15. What are the highest and lowest temperatures shown? Show your solution with a number line.

−9 °F is the lowest temperature and 15 °F is the highest.



Practice

1. A rabbit is sitting at –5 on a number line. It makes 3 equal hops of 7 units each to the left. Where does the rabbit land?

2. A rabbit is sitting at 11 on a number line. It makes 4 equal hops of 2.5 units each to the left. Where does the rabbit land?

3. A grasshopper is sitting at –3 on a number line. It jumps 5 units to the right, 2 units to the left, and then 9 more units to the left. Where does the grasshopper land?

4. A grasshopper is sitting at 2.5 on a number line. It jumps 3 units to the left, $7\frac{1}{4}$ units to the right, and then 5 units to the left. Where does the grasshopper land?

5. Jeremy starts hiking at sea level, which has an elevation of 0 feet. He then hikes 250 feet up to the first summit and then 400 feet down to the valley. What is the elevation of the valley?

6. Kira created a new dance that involves only hops of 2 feet in length. The first set of moves she created was left, left, left, right. After completing this set of moves, how far did Kira end up from where she began the dance? If her starting point is at 0, what integer could represent this location?

7. In an American football game, a team has to travel 10 yards in four plays (downs) to get a new set of downs (a first down). A team completes the following plays: 2-yard loss, 5-yard gain, 1-yard loss, 8-yard gain. Did they get a first down (that is, did they travel at least 10 yards)? Show how you know.

8. Maya is keeping track of her savings account. Over the first three weeks, she saved $8, spent $5, and then saved $15. What integers could she use to record each week?

Answers

1. –26

2. 1

3. –9

4. $1\frac{3}{4}$

5. 150 feet below sea level or –150 feet

6. 4 feet left of the starting point; –4

7. yes; The team ended up 10 yards from where they started.

8. 8, –5, 15

Rational Numbers and Absolute Value

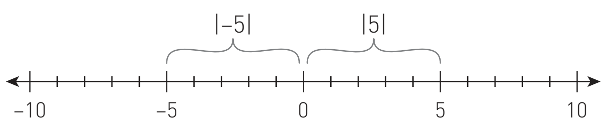
GUIDED SKILL SUPPORT

Overview

Rational numbers are numbers that can be written as fractions, where both the numerator and the denominator are integers (as long as the denominator does not equal 0).

The opposite of a number is the same number but with the opposite sign (+ or –). A number and its opposite are both the same distance from 0 on the number line.

The absolute value of a number is the distance of the number from zero. Absolute value is always non-negative since it represents a distance, without regard to direction. The symbol for absolute value is | |. On the number line shown, both 5 and –5 are 5 units from zero. This distance can be displayed as |–5| = 5, which is read, "The absolute value of negative five equals five."



Similarly, |5| = 5 means, "The absolute value of five is five." Numbers that are opposites have the same absolute value.

Number lines can be used to organize and compare rational numbers. Use inequality symbols to write comparison statements.

Symbol	Meaning in Words
$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to

For additional information, see the Methods & Meanings boxes in Lessons 3.2.1 and 3.2.5 of the *Core Connections Course 1* text.

Examples

1. What is the absolute value of $|-6|$?

$|-6| = 6$

2. What is the absolute value of $|7|$?

$|7| = 7$

3. Compare $|-5|$ and 5.

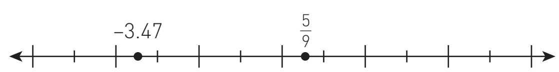
$-5 = 5$, so the two numbers are equal.

4. Compare −13 and −25.

$|-13| = 13$ and $|-25| = 25$ which means $|-25|$ is farther from zero. Therefore, $|-13| < |-25|$.

5. Compare −3.47 and $\frac{5}{9}$.

On a number line, −3.47 is to the left of $\frac{5}{9}$. Therefore, $-3.47 < \frac{5}{9}$.



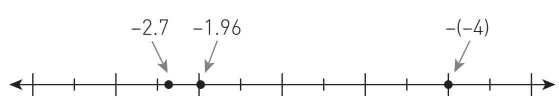
6. Order the following numbers from least to greatest using a number line.

$$-1.96, -(-4), -2.7$$

−(−4) is the opposite of negative 4, which is positive 4.

−2.7 is a larger distance from zero than −1.96.

Therefore, the order from least to greatest is −2.7, −1.96, −(−4).



Practice

Determine the absolute value

1. $|-11|$

2. $|12|$

3. $|-9|$

4. $-|7|$

5. $|-7|$

Compare each pair of values.

6. Compare $5\frac{3}{5}$ and $|5.60|$.

7. Compare $\left|\frac{3}{5}\right|$ and $\frac{1}{5}$.

8. Compare $-\frac{3}{4}$ and $-\frac{3}{8}$.

9. Compare $\left|-\frac{7}{4}\right|$ and -7.2.

10. $\left|-9\frac{8}{11}\right|$, -7.2 , -8.79

11. $-\frac{6}{1{,}000}$, $\frac{81}{10}$, -0.812

12. $\left|-\frac{6}{1{,}000}\right|$, 1%, -0.812

Answers

1. 11

2. 12

3. 9

4. -7

5. 7

6. $5\frac{3}{5} = |5.60|$

7. $\left|\frac{3}{5}\right| > \frac{1}{5}$

8. $-\frac{3}{4} < -\frac{3}{8}$

9. $\left|-\frac{7}{4}\right| > -7.2$

10. $-8.79 < -7.2 < \left|-9\frac{8}{11}\right|$

11. $-0.812 < -\frac{6}{1{,}000} < \frac{81}{10}$

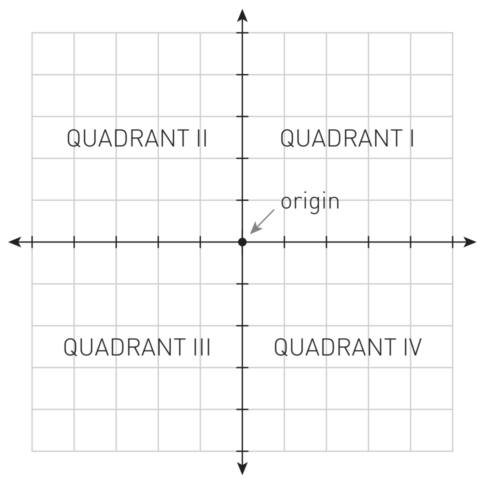
12. $-0.812 < \left|-\frac{6}{1{,}000}\right| < 1\%$

Graphing in Four Quadrants

GUIDED SKILL SUPPORT

Overview

The coordinate plane is made of two axes: a horizontal axis called the x-axis and a vertical axis called the y-axis. The axes intersect at the point (0, 0), called the origin. The axes divide the plane into four quadrants, labeled I, II, III, and IV.



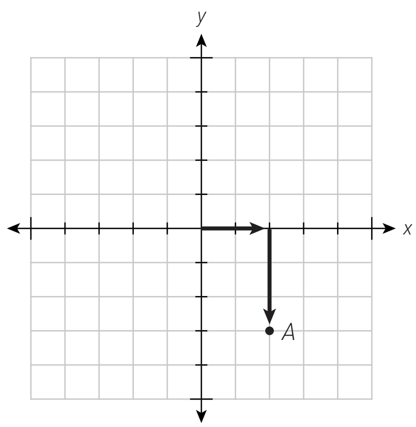
Points on a coordinate graph are identified by two numbers in an ordered pair written as (x, y). The first number is the x-coordinate of the point, and the second number is the y-coordinate. Taken together, the two coordinates name exactly one point on the graph.

The graphing you studied in earlier grades is now extended to include negative values. For additional information, see the Methods & Meanings box in Lesson 3.2.3 of in the *Core Connections Course 1* text.

Examples

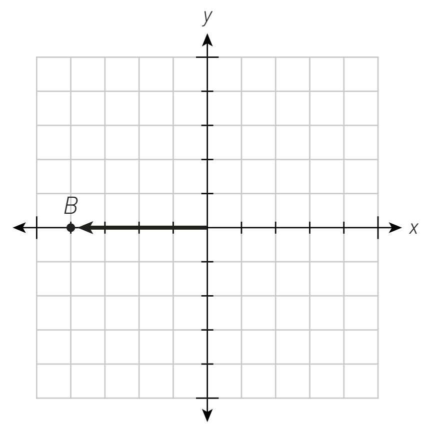
1. Graph point A at $(2, -3)$.

Go right 2 units from the origin $(0, 0)$, then go down 3 units. Mark and label the point.

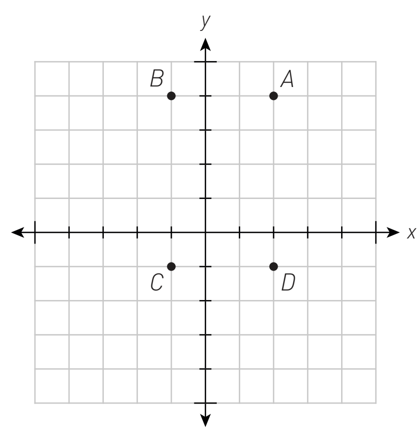


2. Graph point B at $(-4, 0)$.

Go to the left from the origin 4 units, but do not go up or down. Mark and label the point.



3. Consider the graph shown for parts (a) and (b).



a. What are the coordinates of points A, B, C, and D?

$A = (2, 4)$, $B = (-1, 4)$, $C = (-1, -1)$, $D = (2, -1)$

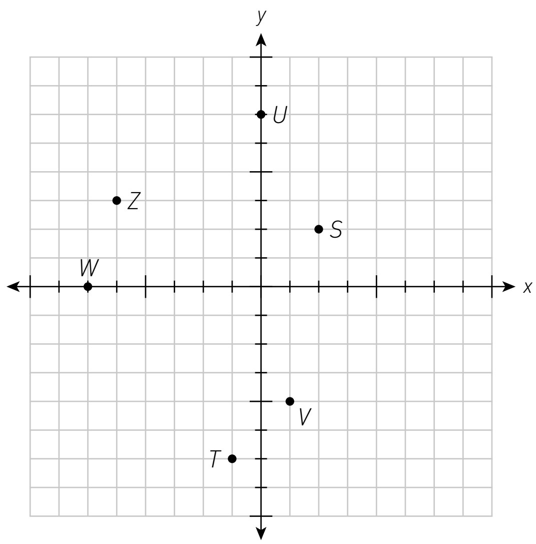
b. What is the distance between point C and point D?

Since point C, $(-1, -1)$, and point D, $(2, -1)$, have the same y-coordinate, (-1), the distance between them can be represented as a horizontal line, or the distance between their x-coordinates. The distance can be counted or calculated as the absolute value of the difference of the x-coordinates.

$|-1 - 2| = |-3| = 3$

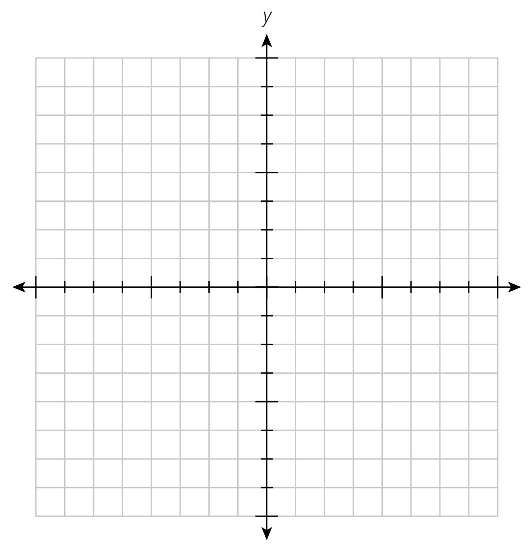
Practice

1. Name the coordinate pair for each point shown on the grid below.



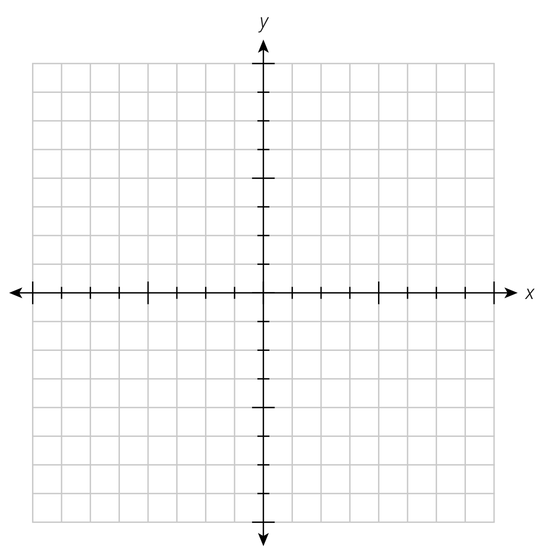
2. Use the ordered pair to locate each point on a coordinate grid. Place a dot at each point and label it with its name.

J(3, 4), *K*(0, −4), *L*(−5, 0), *M*(−2, −3), *N*(−2, 3), *O*(2, −3), *P*(−4, −6), *Q*(4, −5), *R*(−5, −4)



3. Graph the following points and determine the perimeter of the polygon they form.

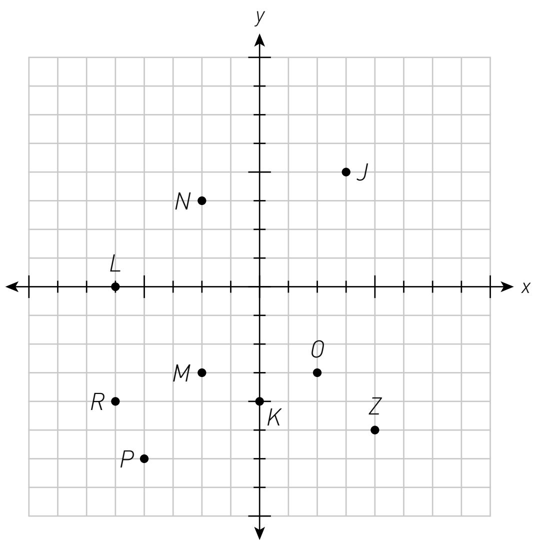
A(2, 2), *B*(−3, 2), *C*(−3, −4), *D*(2, −4)



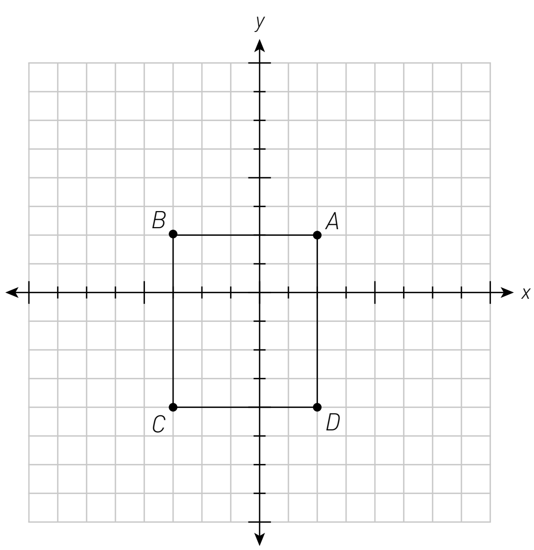
Answers

1. $S(2, 2)$, $T(-1, -6)$, $U(0, 6)$, $V(1, -4)$, $W(-6, 0)$, $Z(-5, 3)$

2.



3.



The perimeter is 2(5) + 2(6) = 22 units.

