Writing Variable Expressions

GUIDED SKILL SUPPORT

Overview

A variable is a symbol used to represent one or more numbers. It is common to use letters of the alphabet for variables. An algebraic expression, also known as a "variable expression," is a combination of numbers and variables, connected by mathematical operations. When one variable is used multiple times in one expression, its value must be the same each time. A variable can be used to represent an unknown quantity in a situation.

For additional information, see the Methods & Meanings box in Lesson 4.2.1 of the *Core Connections Course 1* text.

Examples

1. Write a variable expression for each statement.

a. Seven less than x.

$x - 7$

b. Half of a number.

$\frac{1}{2}x$

c. Three more than twice m.

$2m + 3$

d. Nine less than six times y.

$6y - 9$

e. The product of five plus a number and ten.

$(5 + x) \cdot 10$

2. If the unknown distance of Croakie's hop is represented by the variable h, then write an expression for each of the following.

a. Three equal hops

$h + h + h$ or $3h$

b. Five equal hops

$h + h + h + h + h$ or $5h$

c. Two equal hops and a hop of 3 feet

$h + h + 3$ or $2h + 3$

3. If b represents the cost of a banana and a represents the cost of an apple, write an expression to represent the cost of each of the following.

a. Three bananas and two apples

$b + b + b + a + a$ or $3b + 2a$

b. One banana and 3 apples

$b + a + a + a$ or $b + 3a$

c. One banana, one apple, a \$2 item, and a \$3 item

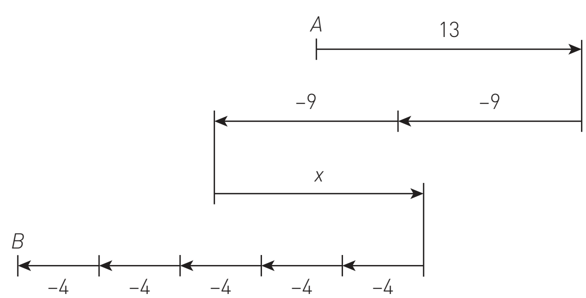
$b + a + 2 + 3$ or $b + a + 5$

4. In 2024, a frog named Eddie Ribbit competed in the Calaveras County Fair frog jumping contest. A reporter was recording the distance of his jumps and wrote the following:

> Eddie starts at point A and hops 13 feet to the right, does two quick hops of 9 feet each to the left, twirls once to the right, does five slow 4-foot hops left, and ends at point B.

a. Draw a diagram to show Eddie Ribbit's entire routine as recorded.

One possible diagram is shown. The twirl is represented using a variable, since the length is unknown, and hops to the left are represented using negative numbers.



b. Write an expression that represents the distance from point A to point B based on Eddie Ribbit's moves.

$13 + 2(-9) + x + 5(-4)$

Practice

1. Write a variable expression for each statement.

a. g divided by ten

b. One-third of k

c. Eight increased by p

d. Four more than three times d

2. If the unknown distance of Croakie's jump is represented by j, write an expression for:

a. Three jumps

b. Six jumps

c. Four jumps and a hop of 5 feet

d. A hop of 3 feet, two jumps, then a hop of 2 feet

3. If the unknown distance of Croakie's jump is represented by j, and the unknown distance of Croakie's hop is represented by h, write an expression for:

a. Two jumps and two hops

b. One jump, three hops, and two jumps

c. One jump, three hops, and a hop of 7 feet

d. A hop of 6 feet, three hops, and two jumps

4. If the unknown cost of a taco is t, and the unknown cost of a carton of milk is m, write an expression for the cost of:

a. Three tacos and two milks

b. One taco and four milks

c. One taco, one milk, two tacos, and one milk

d. Two tacos, one milk, and a $2 item

Answers

1. a. $g \div 10$ or $\frac{g}{10}$

b. $\frac{1}{3}k$ or $\frac{k}{3}$

c. $8 + p$

d. $4 + 3d$

2. a. $j + j + j = 3j$

b. $j + j + j + j + j + j = 6j$

c. $j + j + j + j + 5 = 4j + 5$

d. $3 + j + j + 2 = 2j + 5$

3. a. $j + j + h + h = 2j + 2h$

b. $j + h + h + h + j + j = 3j + 3h$

c. $j + h + h + h + 7 = j + 3h + 7$

d. $6 + h + h + h + j + j = 3h + 2j + 6$

4. a. $3t + 2m$

b. $t + 4m$

c. $t + m + 2t + m = 3t + 2m$

d. $2t + m + 2$

Using Variable Expressions

GUIDED SKILL SUPPORT

Overview

Variables are letters or symbols used to represent one or more numbers. They can be used to represent an unknown value or values in a situation or they can be used to generalize patterns to include all possible numbers. Algebraic expressions contain variables, numbers, and operations (e.g., addition, division, exponents, etc.). They can be evaluated for specific values of the variable(s). Replace the variable with the given value and evaluate the expression, following the Order of Operations.

For additional information, see the Methods & Meanings box in Lesson 4.2.1 of the *Core Connections Course 1* text.

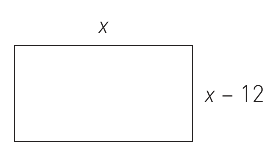
Examples

1. The local playground is rectangular. As a safety precaution, the town plans to put a fence around the entire playground. The width of the playground is 12 feet shorter than its length.

a. If the length of the playground is 45 feet, how many feet of fencing is needed to fence it in completely?

> If the length is 45 feet, then the width is 45 − 12, or 33 feet. The total fencing needed is 45 + 45 + 33 + 33 = 156 feet.

b. The mayor of the town wants an easy way to calculate the amount of fencing needed to border any rectangular playground with a width that is 12 feet shorter than its length. Write at least two algebraic expressions that can be used to calculate the amount of fencing for any playground with this configuration.



x = length of playground

Length of fencing $= x + x + x - 12 + x - 12$
$= 2x + 2(x - 12)$

c. Using one of your algebraic expressions, determine the amount of fencing needed for a playground with a length of 33 feet.

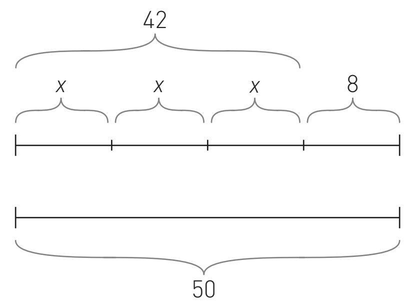
> Length of fencing $= x + x + x - 12 + x - 12$
> $= 33 + 33 + 33 - 12 + 33 - 12$
> $= 108$ feet

2. Saul, Seth, and Sammy are cousins. Seth is 3 years older than Saul and Sammy is 5 years older than Saul.

a. Write a variable expression to represent the sum of the ages of the three cousins.

If x = Saul's age, then Seth's age = $x + 3$, Sammy's age = $x + 5$, and the sum of their ages could be represented by $x + x + 3 + x + 5$ or $3x + 8$.

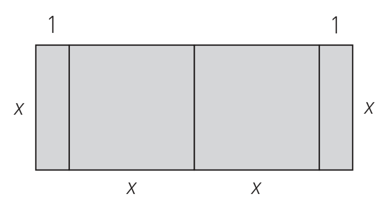
b. How old will Saul be when the sum of the ages of the cousins is 50 years?



$42 \div 3 = 14$; Saul will be 14.

Practice

1. Use the following diagram to complete parts (a) and (b). All shapes that appear to be rectangles are rectangles.



a. Write an expression to represent the perimeter of the large rectangular shape.

b. Suppose the perimeter of the large rectangular shape is 130 units. Is it possible for 18 to be the value of x? Justify your reasoning.

2. Bonnie has designed a rectangular picture frame with a length that is 8 inches longer than the width.

a. If the width of the frame is 10 inches long, what is the perimeter of the entire frame?

b. Write an algebraic expression to represent the perimeter for a frame with any width.

c. Using one of your algebraic expressions, determine the number of tiles needed for a rectangular frame if the width is 52 inches.

3. The ingredients for a shortbread cookie recipe are flour, butter, and sugar. The number of cups of flour for any number of servings is twice as much as the number of cups of butter and four times as much as the number of cups of sugar.

a. If f represents the number of cups of flour in any number of servings, what expressions could represent the number of cups of butter and sugar?

b. 12 servings of cookies use 2 cups of flour. How many cups of butter and sugar are needed?

4. You are given 108 square tiles to create a square frame. What is the largest frame size (on each side) that you can make with these tiles?

Answers

1. a. Expressions may vary but should be equivalent to $6x + 4$.

b. No, substituting 18 for x yields $6(18) + 4 = 108 + 4 = 112$, which is not equal to 130.

2. a. $10 + 18 + 10 + 18 = 56$ inches

b. Let x = width, then the length is $x + 8$; perimeter = $4x + 16$.

c. $4(52) + 16 = 224$ inches

3. a. butter = $\frac{1}{2}f$, sugar = $\frac{1}{4}f$

b. 1 cup butter, $\frac{1}{2}$ cup sugar

4. Methods may vary. $x + x + (x - 2) + (x - 2)$; $108 - 4 = 104$ then $104 \div 4 = 26$

Adding and Subtracting With Mixed Numbers

GUIDED SKILL SUPPORT

Overview

A mixed number is a combination of a whole number and a fraction that is less than 1. When adding or subtracting mixed numbers and fractions, it is important to pay attention to the denominators.

METHOD 1

1. Convert the mixed numbers into fractions greater than 1.
2. Use a Giant One to rewrite each fraction, as needed, so all fractions have a common denominator.
3. Add or subtract in the same way you would if they were fractions between 0 and 1.
4. Rewrite as a fraction in lowest terms or as a mixed number, as necessary.

Other strategies are also possible, such as subtracting parts (whole number parts and fraction parts).

METHOD 2

1. Separate the mixed numbers into whole number parts and fraction parts.
2. Use a Giant One to rewrite each fraction part, as needed, so all fractions have a common denominator.
3. Add or subtract the whole number parts and the fraction parts. If the larger fraction part is being subtracted from the smaller fraction part, then regroup the smaller fraction using a Giant One.
4. Rewrite as a fraction in lowest terms or a mixed number, as necessary.

For additional information, see the Adding and Subtracting Fractions topic in the Chapter 3 Guided Skill Support, and the Methods & Meanings box in Lesson 4.1.4 of the *Core Connections Course 1* text. For additional examples and practice, see the *Core Connections Course 1* Checkpoint 4 materials.

Examples

1. Subtract: $3\frac{1}{5} - 1\frac{2}{3}$

METHOD 1

$$3\frac{1}{5} - 1\frac{2}{3}$$
$$= \frac{16}{5} - \frac{5}{3}$$
$$= \frac{16}{5} \cdot \frac{3}{3} - \frac{5}{3} \cdot \frac{5}{5}$$
$$= \frac{48}{15} - \frac{25}{15}$$
$$= \frac{23}{15}$$
$$= 1\frac{8}{15}$$

METHOD 2

$$3\frac{1}{5} - 1\frac{2}{3}$$
$$= 3 + \frac{1}{5} - \left(1 + \frac{2}{3}\right)$$
$$= 3 + \frac{1}{5} \cdot \frac{3}{3} - \left(1 + \frac{2}{3} \cdot \frac{5}{5}\right)$$
$$= 3 + \frac{3}{15} - \left(1 + \frac{10}{15}\right)$$
$$= 3 - 1 + \frac{3}{15} + \frac{15}{15} - \left(1 + \frac{10}{15}\right)$$
$$= 2 + \frac{18}{15} - \left(1 + \frac{10}{15}\right)$$
$$= (2 - 1) + \left(\frac{18}{15} - \frac{10}{15}\right)$$
$$= 1\frac{8}{15}$$

2. Add: $8\frac{3}{4} + 4\frac{2}{5}$

METHOD 1

$$8\frac{3}{4} + 4\frac{2}{5}$$
$$= \frac{35}{4} + \frac{22}{5}$$
$$= \frac{35}{4} \cdot \frac{5}{5} + \frac{22}{5} \cdot \frac{4}{4}$$
$$= \frac{175}{20} + \frac{88}{20}$$
$$= \frac{263}{20}$$
$$= 13\frac{3}{20}$$

METHOD 2

$$8\frac{3}{4} + 4\frac{2}{5}$$
$$= 8 + \frac{3}{4} + 4 + \frac{2}{5}$$
$$= 8 + \frac{3}{4} \cdot \frac{5}{5} + 4 + \frac{2}{5} \cdot \frac{4}{4}$$
$$= 8 + 4 + \frac{15}{20} + \frac{8}{20}$$
$$= 12\frac{23}{20}$$
$$= 13\frac{3}{20}$$

Practice

Add or subtract:

1. $2\frac{1}{2} - 1\frac{3}{4}$

2. $4\frac{1}{3} - 3\frac{5}{6}$

3. $1\frac{1}{6} - \frac{3}{4}$

4. $5\frac{2}{5} - 3\frac{2}{3}$

5. $7 - 1\frac{2}{3}$

6. $5\frac{3}{8} - 2\frac{2}{3}$

7. $2\frac{1}{2} + 1\frac{3}{4}$

8. $4\frac{1}{3} + 3\frac{5}{6}$

9. $1\frac{1}{6} + \frac{3}{4}$

10. $5\frac{2}{5} + 3\frac{2}{3}$

11. $7 + 1\frac{2}{3}$

12. $5\frac{3}{8} + 2\frac{2}{3}$

Answers

1. $\frac{5}{2} - \frac{7}{4} = \frac{10}{4} - \frac{7}{4} = \frac{3}{4}$

2. $\frac{13}{3} - \frac{23}{6} = \frac{26}{6} - \frac{23}{6} = \frac{3}{6}$ or $\frac{1}{2}$

3. $\frac{7}{6} - \frac{3}{4} = \frac{14}{12} - \frac{9}{12} = \frac{5}{12}$

4. $\frac{27}{5} - \frac{11}{3} = \frac{81}{15} - \frac{55}{15} = \frac{26}{15}$ or $1\frac{11}{15}$

5. $\frac{7}{1} - \frac{5}{3} = \frac{21}{3} - \frac{5}{3} = \frac{16}{3}$ or $5\frac{1}{3}$

6. $\frac{43}{8} - \frac{8}{3} = \frac{129}{24} - \frac{64}{24} = \frac{65}{24}$ or $2\frac{17}{24}$

7. $\frac{17}{4}$ or $4\frac{1}{4}$

8. $\frac{49}{6}$ or $8\frac{1}{6}$

9. $\frac{23}{12}$ or $1\frac{11}{12}$

10. $\frac{136}{15}$ or $9\frac{1}{15}$

11. $\frac{26}{3}$ or $8\frac{2}{3}$

12. $\frac{193}{24}$ or $8\frac{1}{24}$

Ratios

GUIDED SKILL SUPPORT

Overview

A ratio is a comparison of two quantities. A ratio can be written using the word "to," using a colon, or as a fraction.

Equivalent ratios can be determined using a variety of methods, including ratio tables, double number lines, tape diagrams, and Giant Ones. Giant Ones are fractions that are equivalent to 1. They are used in this course as an application of the Multiplicative Identity, that is, multiplying a number by a form of the number 1, such as $\frac{2}{2}$ or $\frac{5}{5}$, so the value of the number does not change.

For additional information, see the Methods & Meanings box in Lesson 4.2.3 of the *Core Connections Course 1* text

Examples

1. A bag contains the following marbles: 7 yellow, 8 red, and 5 blue. Name the following ratios.

a. Red to yellow marbles

8 to 7, 8:7, or $\frac{8}{7}$

b. Red to blue marbles

8 to 5, 8:5, or $\frac{8}{5}$

c. Blue to red marbles

5 to 8, 5:8, or $\frac{5}{8}$

d. Blue to total number of marbles

5 to 20, 5:20, or $\frac{5}{20}$

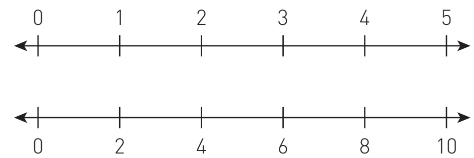
2. Write ratios equivalent to $\frac{1}{2}$ using the following methods.

a. Ratio table

1	2	4	5
2	4	8	10

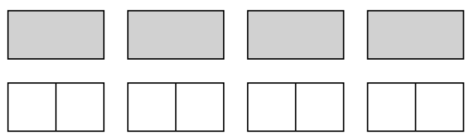
2 to 4, 4 to 8, 5 to 10

b. Double number line



2:4, 3:6, 4:8, 5:10

c. Tape diagram



2 to 4, 3 to 6, 4 to 8

d. Giant One

$\frac{1}{2}\cdot\frac{2}{2}=\frac{2}{4}$ $\frac{1}{2}\cdot\frac{3}{3}=\frac{3}{6}$ $\frac{1}{2}\cdot\frac{4}{4}=\frac{4}{8}$

Practice

Molly's favorite juice drink is made by mixing 3 cups of apple juice, 5 cups of cranberry juice, and 2 cups of ginger ale. Use this information to write the following ratios.

1. 5 cups cranberry juice to _____ cups apple juice

2. $\frac{\text{5 cups cranberry juice}}{\square \text{ cups apple juice}}$

3. 5 cups cranberry juice:_____ cups apple juice

4. 15 cups cranberry juice:_____ cups apple juice

5. 15 cups cranberry juice:_____ cups ginger ale

6. 15 cups apple juice:_____ cups ginger ale

7. 15 cups apple juice to _____ cups ginger ale

8. $\frac{\text{15 cups apple juice}}{\square \text{ cups ginger ale}}$

9. _____ cups finished juice drink to 2 cups ginger ale

10. 30 cups finished juice drink to ______ cups ginger ale

11. 25 cups finished juice drink to ______ cups ginger ale

12. Using at least two different methods (see Example 2), write an equivalent ratio.

a. $\frac{3}{5} = \frac{\square}{30}$ b. $\frac{\square}{7} = \frac{36}{63}$

13. Using any method, write an equivalent ratio.

a. $\frac{3}{7} = \frac{27}{\square}$ b. $\frac{\square}{7} = \frac{36}{63}$ c. $\frac{4}{7} = \frac{48}{\square}$

14. Using any method, write an equivalent ratio.

a. $\frac{4}{12} = \frac{48}{\square}$ b. $\frac{4}{12} = \frac{\square}{3}$ c. $\frac{4}{12} = \frac{84}{\square}$

Answers

1. 5 cups cranberry juice to 3 cups apple juice

2. $\frac{\text{5 cups cranberry juice}}{\text{3 cups apple juice}}$

3. 5 cups cranberry juice:3 cups apple juice

4. 15 cups cranberry juice:9 cups apple juice

5. 15 cups cranberry juice:6 cups ginger ale

6. 15 cups apple juice:10 cups ginger ale

7. 15 cups apple juice to 10 cups ginger ale

8. $\frac{\text{15 cups apple juice}}{\text{10 cups ginger ale}}$

9. 10 cups finished juice drink to 2 cups ginger ale

10. 30 cups finished juice drink to 6 cups ginger ale

11. 25 cups finished juice drink to 5 cups ginger ale

12. a. $\frac{3}{5} = \frac{18}{30}$ b. $\frac{3}{5} = \frac{27}{45}$

13. a. $\frac{3}{7} = \frac{27}{63}$ b. $\frac{4}{7} = \frac{36}{63}$ c. $\frac{4}{7} = \frac{48}{84}$

14. a. $\frac{4}{12} = \frac{48}{144}$ b. $\frac{4}{12} = \frac{1}{3}$ c. $\frac{4}{12} = \frac{84}{252}$