Multiplying Fractions

GUIDED SKILL SUPPORT

Overview

Multiplication of fractions can be shown using a rectangular area model. Divide a unit rectangle into vertical sections to represent the denominator of the dividend, the second fraction. Then shade the number of parts represented by the numerator of that fraction. Next, divide the rectangle horizontally using the denominator of the divisor, the first fraction. The numerator of this fraction again indicates the number of parts to shade. The double-shaded parts, or overlap of the two fractions, represent the product.

The rule for multiplying fractions derived from the area models is to multiply the numerators, then multiply the denominators, as shown.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

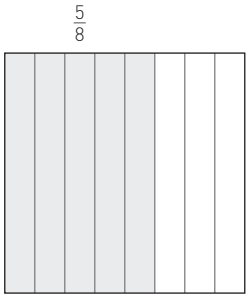
Write the answer, or product, in the lowest terms possible.

For additional information, see the Methods & Meanings box in Lesson 5.1.3 of the *Core Connections Course 1* text. For additional examples and practice, see the *Core Connections Course 1* Checkpoint 7A materials.

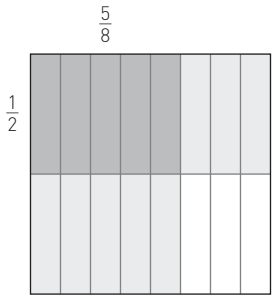
Examples

1. $\frac{1}{2} \cdot \frac{5}{8}$ (that is, $\frac{1}{2}$ of $\frac{5}{8}$)

Draw a unit rectangle and divide it into 8 pieces vertically to represent the denominator of $\frac{5}{8}$. Lightly shade 5 of those pieces to represent the numerator. Label it $\frac{5}{8}$.



Divide the rectangle in half horizontally. Shade one half and label it $\frac{1}{2}$.

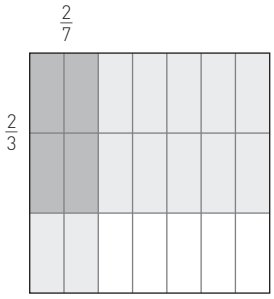


The double-shaded region represents $\frac{1}{2}$ of $\frac{5}{8}$.

Write an equation to represent this.

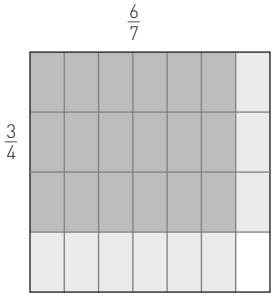
$$\frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16}$$

2. $\frac{2}{3} \cdot \frac{2}{7}$



$$\frac{2}{3} \cdot \frac{2}{7} = \frac{2 \cdot 2}{3 \cdot 7} = \frac{4}{21}$$

3. $\frac{3}{4} \cdot \frac{6}{7}$



$$\frac{3}{4} \cdot \frac{6}{7} = \frac{3 \cdot 6}{4 \cdot 7} = \frac{18}{28} = \frac{9}{14}$$

Practice

Draw an area model for each of the following problems and write the answer.

1. $\frac{1}{3} \cdot \frac{1}{6}$ **2.** $\frac{1}{4} \cdot \frac{3}{5}$ **3.** $\frac{2}{3} \cdot \frac{5}{9}$

Determine the following products using any method. Write your answer in the lowest terms.

4. $\frac{1}{3} \cdot \frac{2}{5}$ **5.** $\frac{2}{3} \cdot \frac{2}{7}$ **6.** $\frac{3}{4} \cdot \frac{1}{5}$ **7.** $\frac{2}{5} \cdot \frac{2}{3}$ **8.** $\frac{2}{3} \cdot \frac{1}{4}$

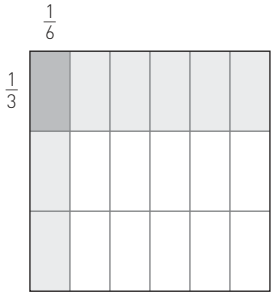
9. $\frac{5}{6} \cdot \frac{2}{3}$ **10.** $\frac{4}{5} \cdot \frac{3}{4}$ **11.** $\frac{2}{15} \cdot \frac{1}{2}$ **12.** $\frac{3}{7} \cdot \frac{1}{2}$ **13.** $\frac{3}{8} \cdot \frac{4}{5}$

14. $\frac{2}{9} \cdot \frac{3}{5}$ **15.** $\frac{3}{10} \cdot \frac{5}{7}$ **16.** $\frac{5}{11} \cdot \frac{6}{7}$ **17.** $\frac{5}{6} \cdot \frac{3}{10}$ **18.** $\frac{10}{11} \cdot \frac{3}{5}$

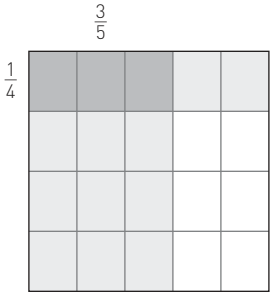
19. $\frac{5}{12} \cdot \frac{3}{5}$ **20.** $\frac{7}{9} \cdot \frac{5}{14}$

Answers

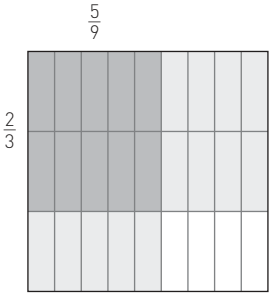
1. $\frac{1}{18}$



2. $\frac{3}{20}$



3. $\frac{10}{27}$



4. $\frac{2}{15}$ **5.** $\frac{4}{21}$ **6.** $\frac{3}{20}$ **7.** $\frac{4}{15}$ **8.** $\frac{2}{12} = \frac{1}{6}$

9. $\frac{10}{18} = \frac{5}{9}$ **10.** $\frac{12}{20} = \frac{3}{5}$ **11.** $\frac{2}{30} = \frac{1}{15}$ **12.** $\frac{3}{14}$ **13.** $\frac{12}{40} = \frac{3}{10}$

14. $\frac{6}{45} = \frac{2}{15}$ **15.** $\frac{15}{70} = \frac{3}{14}$ **16.** $\frac{30}{77}$ **17.** $\frac{15}{60} = \frac{1}{4}$ **18.** $\frac{30}{55} = \frac{6}{11}$

19. $\frac{15}{60} = \frac{1}{4}$ **20.** $\frac{35}{126} = \frac{5}{18}$

Dividing Fractions

GUIDED SKILL SUPPORT

Overview

Division such as 8 ÷ 2 can be thought of as 8 divided into groups of 2. Similarly, $\frac{1}{2} \div \frac{1}{4}$ can mean $\frac{1}{2}$ divided into groups of $\frac{1}{4}$, or as asking how many one-fourths are in one-half.

Three methods for dividing fractions are described. You might choose any of these methods, depending on the problem, but sometimes one method (particularly the area model) is not always the most efficient or appropriate.

DIAGRAMS

To divide by a fraction using a diagram, you can create a model of the situation using rectangles, a linear model, or another visual representation.

COMMON DENOMINATOR

Express both fractions with a common denominator, then divide the first numerator by the second.

GIANT ONE

Write the division problem in fraction form, with a fraction in both the numerator and the denominator. Use the reciprocal of the denominator for the numerator and denominator in the Giant One. Multiply the fractions as usual, and rewrite the resulting fraction in an equivalent form if appropriate.

For more information, see the Methods & Meanings boxes in Lessons 7.2.2 and 7.2.5 of the *Core Connections Course 1* text. For additional examples and practice, see the *Core Connections Course 1* Checkpoint 8B materials.

Examples

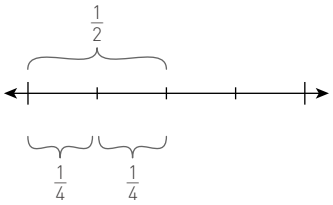
1. $\frac{1}{2} \div \frac{1}{4}$

Three methods are shown. Each method shows that $\frac{1}{2} \div \frac{1}{4} = 2$.

LINEAR MODEL

Divide a number line into two equal pieces and label one of them to represent $\frac{1}{2}$.

Then, divide the number line further into four equal pieces that each represent $\frac{1}{4}$. Label the number of $\frac{1}{4}$ pieces in $\frac{1}{2}$. There are 2 of these $\frac{1}{4}$ pieces.



Therefore, $\frac{1}{2} \div \frac{1}{4} = 2$.

COMMON DENOMINATOR

$$\frac{1}{2} \div \frac{1}{4}$$
$$= \frac{2}{4} \div \frac{1}{4}$$
$$= \frac{2}{1}$$
$$= 2$$

GIANT ONE

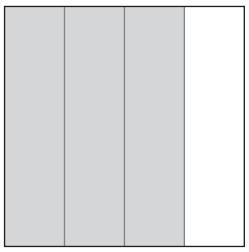
$$\frac{1}{2} \div \frac{1}{4} = \frac{\frac{1}{2}}{\frac{1}{4}} \cdot \boxed{\frac{\frac{4}{1}}{\frac{4}{1}}} = \frac{\frac{4}{2}}{1} = \frac{4}{2} = 2$$

2. $\frac{3}{4} \div \frac{1}{6}$

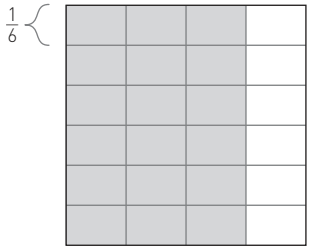
Three methods are shown. Each method shows that $\frac{3}{4} \div \frac{1}{6} = 4\frac{1}{2}$.

AREA MODEL

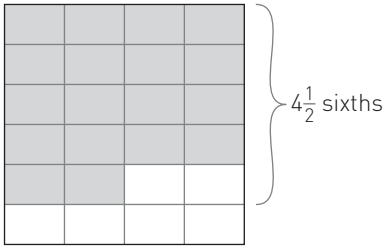
Represent $\frac{3}{4}$.



Then, divide the rectangle into $\frac{1}{6}$s.



Rearrange the shaded rectangles.



In the shaded section, $\frac{3}{4}$, there are $4\frac{1}{2}$ sixths.

Therefore, $\frac{3}{4} \div \frac{1}{6} = 4\frac{1}{2}$.

COMMON DENOMINATOR

$$\frac{3}{4} \div \frac{1}{6}$$
$$= \frac{9}{12} \div \frac{2}{12}$$
$$= \frac{9}{2}$$
$$= 4\frac{1}{2}$$

GIANT ONE

$$\frac{3}{4} \div \frac{1}{6} = \frac{\frac{3}{4}}{\frac{1}{6}} \cdot \boxed{\frac{\frac{6}{1}}{\frac{6}{1}}} = \frac{\frac{18}{4}}{1} = \frac{9}{2} = 4\frac{1}{2}$$

3. $\frac{2}{3} \div \frac{3}{5}$

Three methods are shown. The rectangular model is less appropriate for this problem. Each method shows that $\frac{2}{3} \div \frac{3}{5} = 1\frac{1}{9}$.

COMMON DENOMINATOR

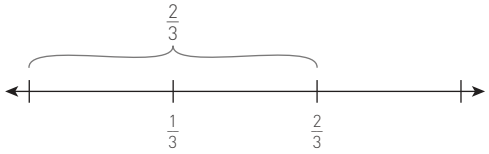
$$\frac{2}{3} \div \frac{3}{5}$$
$$= \frac{10}{15} \div \frac{9}{15}$$
$$= \frac{10}{9}$$
$$= 1\frac{1}{9}$$

GIANT ONE

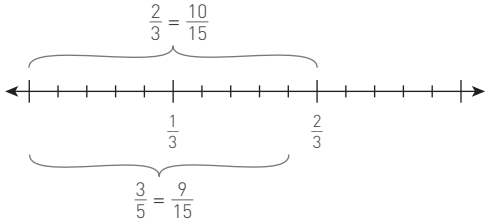
$$\frac{2}{3} \div \frac{3}{5} = \frac{\frac{2}{3}}{\frac{3}{5}} \cdot \boxed{\frac{\frac{5}{3}}{\frac{5}{3}}} = \frac{\frac{10}{9}}{1} = \frac{10}{9} = 1\frac{1}{9}$$

LINEAR MODEL

Divide a number line into three equal pieces and label two of them to represent $\frac{2}{3}$.



Divide the number line further into fifteenths and label $\frac{3}{5}$, which is equivalent to $\frac{9}{15}$.



Since $\frac{3}{5}$ divides $\frac{2}{3}$ one time and the remaining piece represents $\frac{1}{9}$ of $\frac{3}{5}$, $\frac{2}{3} \div \frac{3}{5} = 1\frac{1}{9}$.

4. $1\frac{1}{3} \div 1\frac{1}{2}$

Three methods are shown. Each method shows that $1\frac{1}{3} \div 1\frac{1}{2} = \frac{8}{9}$.

COMMON DENOMINATOR

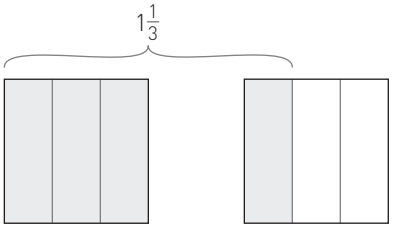
$$1\frac{1}{3} \div 1\frac{1}{2}$$
$$= \frac{4}{3} \div \frac{3}{2}$$
$$= \frac{8}{6} \div \frac{9}{6}$$
$$= \frac{8}{9}$$

GIANT ONE

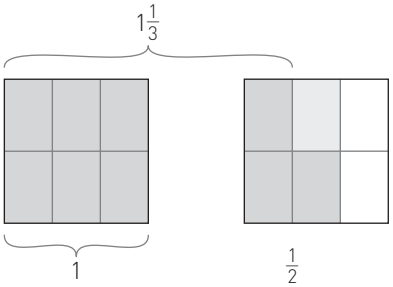
$$1\frac{1}{3} \div 1\frac{1}{2} = \frac{1\frac{1}{3}}{1\frac{1}{2}} = \frac{\frac{4}{3}}{\frac{3}{2}} \cdot \boxed{\frac{\frac{2}{3}}{\frac{2}{3}}} = \frac{\frac{8}{9}}{1} = \frac{8}{9}$$

AREA MODEL

Represent $1\frac{1}{3}$.



Then divide the rectangles into sixths and shade to show $1\frac{1}{2}$.



$\frac{9}{6}$ are shaded to represent $1\frac{1}{2}$, but only 8 of the 9 sixths fit in the $1\frac{1}{3}$ portion, therefore $1\frac{1}{3} \div 1\frac{1}{2} = \frac{8}{9}$.

Practice

Complete the division problems. Use any method.

1. $1 \div \frac{1}{4}$

2. $5 \div \frac{1}{3}$

3. $\frac{3}{7} \div \frac{1}{8}$

4. $\frac{5}{8} \div \frac{4}{5}$

5. $\frac{3}{2} \div \frac{3}{4}$

6. $\frac{7}{6} \div \frac{2}{3}$

7. $\frac{4}{7} \div \frac{1}{3}$

8. $1\frac{3}{7} \div \frac{1}{2}$

9. $1\frac{1}{4} \div \frac{3}{5}$

10. $2\frac{2}{3} \div \frac{1}{9}$

11. $3\frac{1}{3} \div \frac{5}{6}$

12. $\frac{5}{8} \div 1\frac{1}{4}$

13. $2\frac{1}{2} \div 6\frac{4}{7}$

14. $1\frac{9}{10} \div 3\frac{3}{4}$

15. $1\frac{9}{10} \div 4\frac{3}{4}$

Answers

1. 4

2. 15

3. $\frac{24}{7} = 3\frac{3}{7}$

4. $\frac{25}{32}$

5. 2

6. $\frac{7}{4}$

7. $\frac{12}{7} = 1\frac{5}{7}$

8. $\frac{20}{7} = 2\frac{6}{7}$

9. $\frac{25}{12} = 2\frac{1}{12}$

10. 24

11. 4

12. $\frac{1}{2}$

13. $\frac{35}{92}$

14. $\frac{76}{150} = \frac{38}{75}$

15. $\frac{76}{190} = \frac{2}{5}$

Multiplying Decimals and Percents

GUIDED SKILL SUPPORT

Overview

When multiplying multi-digit decimals, it is important to pay attention to place value. The process is similar to multiplying whole numbers. In the product, the number of decimal places is equal to the total number of decimal places in the factors (numbers you multiplied). Sometimes zeros need to be added to place the decimal point.

For calculations that involve determining a percent of a number, the percent can be converted to a decimal before multiplying. Multiple methods for multiplying decimals are shown in the examples.

For additional information, see the Methods & Meanings box in Lesson 5.2.2 of the *Core Connections Course 1* text.

Examples

1. Multiply 1.32 by 4.53.

Two methods are shown. Each method shows that $1.32 \times 4.53 = 5.9796$.

AREA MODEL

Write the parts of each factor along the dimensions of the area model.

	4	+	0.5	+	0.03
1					
+					
0.3					
+					
0.02					

Calculate the area of each piece by multiplying the length by the width.

	4	+	0.5	+	0.03
1	4		0.5		0.03
+					
0.3	1.2		0.15		0.009
+					
0.02	0.08		0.01		0.0006

The product is the sum of all the individual areas.

$$\begin{aligned} & 1.32 \times 4.53 \\ &= 4 + 0.5 + 0.03 + 1.2 + 0.15 + 0.009 + 0.08 + 0.01 + 0.0006 \\ &= 5.9796 \end{aligned}$$

STANDARD ALGORITHM

$$\begin{array}{r} 1.32 \\ \times\ 4.53 \\ \hline 0396 \\ 6600 \\ +\ 52800 \\ \hline 59796 \end{array} \longrightarrow \begin{array}{rl} 1.32 & \text{2 decimal places} \\ \times\ 4.53 & \text{2 decimal places} \\ \hline 0.0396 & \\ 0.6600 & \\ +\ 5.2800 & \\ \hline 5.9796 & \text{4 decimal places} \end{array}$$

2. Multiply (1.7) · (0.03).

In fractions, this means $\frac{17}{10} \cdot \frac{3}{100} = \frac{51}{1{,}000}$.

Knowing that the answer must be in the thousandth place tells you how many places to move the decimal point (to the left) without using the fractions.

(tenths)(hundredths) = thousandths

Therefore, move the decimal point three places.

1.7	1 decimal place
× 0.03	2 decimal places
0.051	3 decimal places

3. Determine 17% of 32.5.

Since 17% = $\frac{17}{100}$ = 0.17, 17% of 32.5 means (0.17) · (32.5) = 5.525.

(hundredths)(tenths) = thousandths

Therefore, move the decimal point three places.

32.5	1 decimal place
× 0.17	2 decimal places
2275	
+ 3250	
5.525	3 decimal places

Practice

Identify the number of places in the product. Do not calculate the product.

1. 0.3(0.5)

2. 1.5(0.12)

3. 1.23(2.6)

4. 0.126(3.4)

5. 17(32.016)

6. 4.32(3.1416)

Multiply.

7. 0.8(0.03)

8. 3.2(0.3)

9. 1.75(0.09)

10. 4.5(3.2)

11. (1.8) · (0.032)

12. 7.89(6.3)

Determine the value for each problem.

13. 8% of 540

14. 70% of 478

15. 37% of 4.7

16. 17% of 96

17. 15% of 4.75

18. 130% of 42

Answers

1. 2 **2.** 3 **3.** 3 **4.** 4 **5.** 3 **6.** 6

7. 0.024 **8.** 0.96 **9.** 0.1575 **10.** 14.4 **11.** 0.0576 **12.** 49.707

13. 43.2 **14.** 334.6 **15.** 1.739 **16.** 16.32 **17.** 0.7125 **18.** 54.6

Area of Polygons

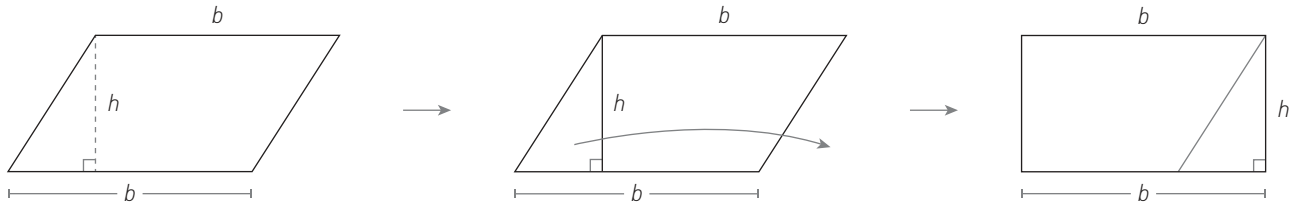
GUIDED SKILL SUPPORT

Overview

The area of a shape is the number of square units that fit inside the shape.

The area of a rectangle is the length of the base multiplied by the length of its height: $A = b \cdot h$. A square is a rectangle in which the base and height are of equal length. It can be calculated by multiplying the base times itself: $A = b^2$.

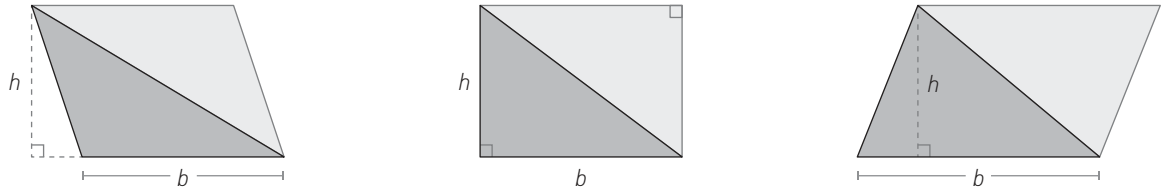
The area of a parallelogram is calculated using the same formula as for the area of a rectangle. This fact can be seen by separating a triangle from one end of the parallelogram and moving it to the other, forming a rectangle.



PARALLELOGRAM RECTANGLE

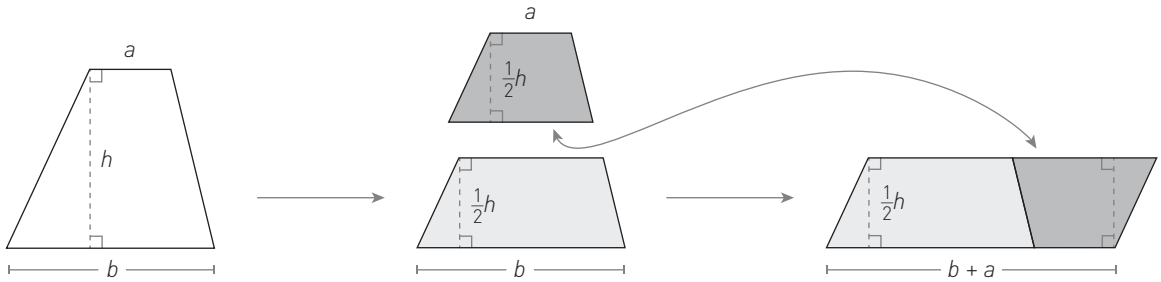
To calculate the area of a parallelogram, multiply the length of the base, b, by the length of the height, h: $A = b \cdot h$.

The area of a triangle is equal to one-half the area of a parallelogram. This fact can be seen by taking two copies of the same triangle and putting them together along a common side to form a parallelogram.



To calculate the area of a triangle, multiply the length of the base, b, by the length of the height, h, and divide the product by 2: $A = \frac{b \cdot h}{2} = \frac{1}{2}bh$.

A trapezoid is another shape that can be transformed into a parallelogram. This fact can be seen by cutting the trapezoid into two pieces, each having a height that is one-half the height of the original trapezoid. One piece is rotated and placed next to the other piece.



To calculate the area of a trapezoid, calculate the product of half of the height (h) and the sum of the two bases (a and b): $A = \frac{1}{2}h(a + b)$.

For additional information, see the Methods & Meanings boxes in Lessons 5.3.3, 5.3.4, and 6.1.1 of the *Core Connections Course 1* text.

Examples

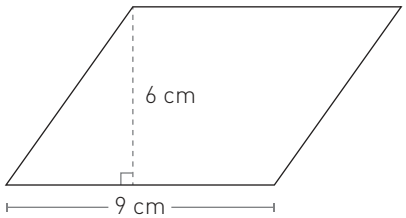
1. Calculate the area of the rectangle.

base = 8 units

height = 4 units

area = $8 \cdot 4 = 32$ square units

2. Calculate the area of the parallelogram.



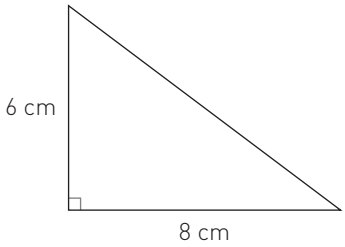
base = 9 cm

height = 6 cm

area = $9 \cdot 6 = 54$ square cm

3. Calculate the areas of the triangles.

a.

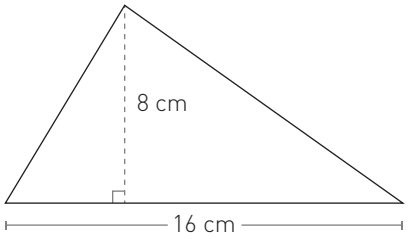


base = 8 cm

height = 6 cm

area = $\frac{8 \cdot 6}{2} = \frac{48}{2} = 24$ square cm

b.

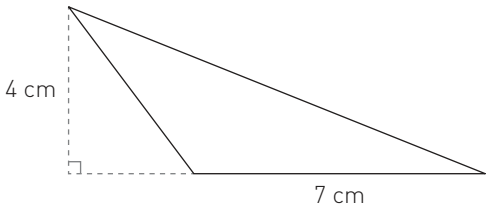


base = 16 cm

height = 8 cm

area = $\frac{16 \cdot 8}{2} = \frac{128}{2} = 64$ square cm

c.

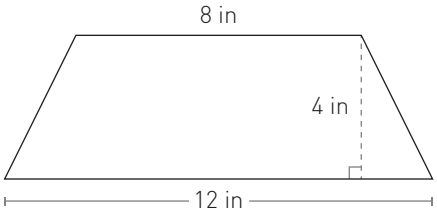


base = 7 cm

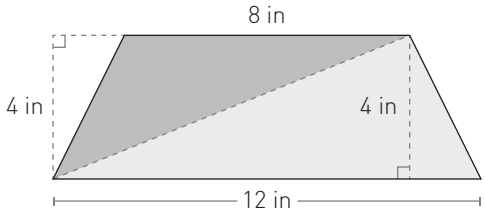
height = 4 cm

area = $\frac{1}{2} \cdot 7 \cdot 4 = 14$ square cm

4. Calculate the area of the trapezoid.



The trapezoid could be divided into two triangles.



Then, the area of the trapezoid is calculated by adding together the areas of the two triangles.

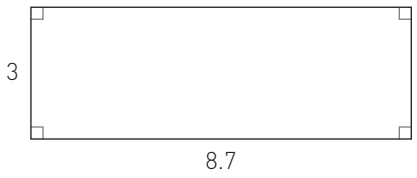
$$A = \left(\frac{1}{2} \cdot 8 \cdot 4\right) + \left(\frac{1}{2} \cdot 12 \cdot 4\right) = 16 + 24 = 40 \text{ square inches}$$

Practice

What is the area of each polygon?

1.

2.



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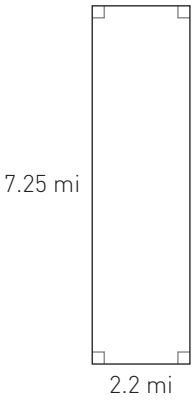

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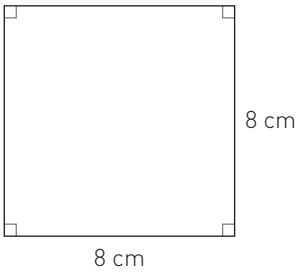

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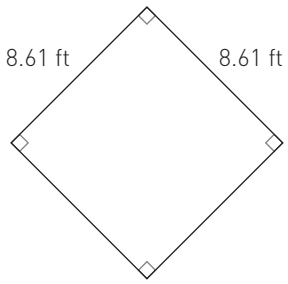

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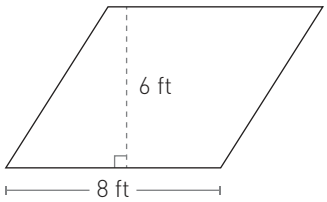
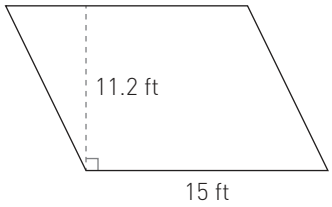


7.

8.



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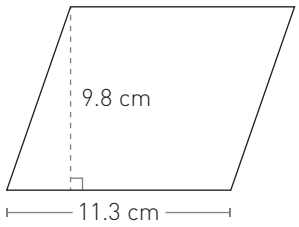

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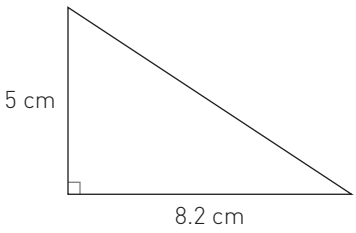

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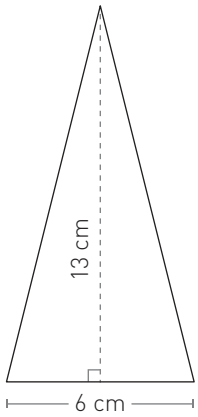

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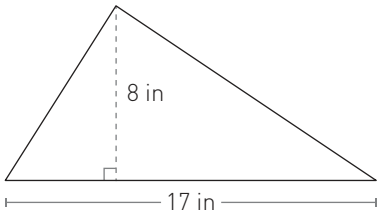

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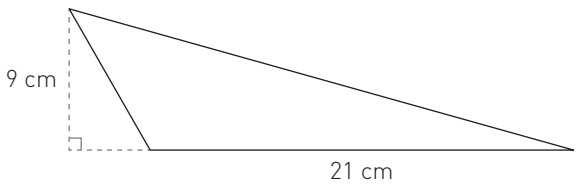

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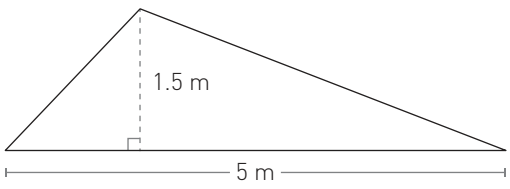

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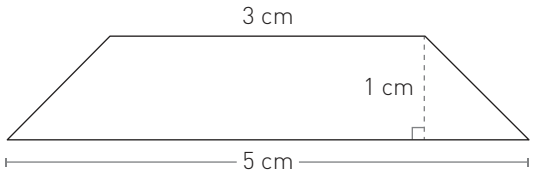

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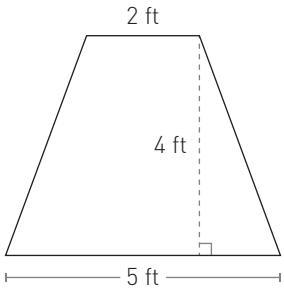

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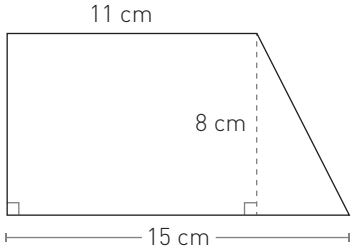
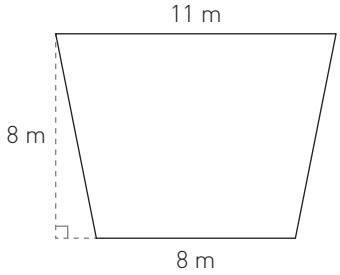


18.



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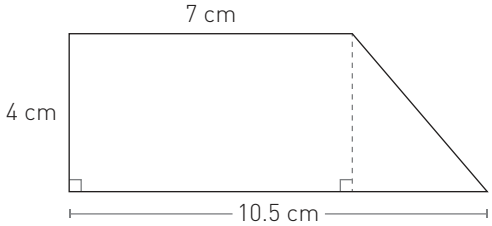

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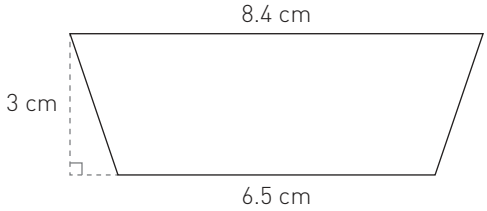

Answers

1.	8 mi^2	**2.**	26.1 square units	**3.**	15.95 mi^2	**4.**	64 cm^2
5.	74.1321 ft^2	**6.**	48 ft^2	**7.**	39 cm^2	**8.**	168 ft^2
9.	110.74 cm^2	**10.**	20.5 cm^2	**11.**	39 cm^2	**12.**	68 in^2
13.	94.5 cm^2	**14.**	3.75 m^2	**15.**	4 cm^2	**16.**	14 ft^2
17.	104 cm^2	**18.**	76 m^2	**19.**	35 cm^2	**20.**	22.35 cm^2