Long Division

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Overview

Long division is one method for dividing two numbers. It is a standard strategy that can be completed by hand. Place value is an important piece to consider when using long division. The notation for long division is shown.

$$\text{divisor}\overset{\displaystyle\text{quotient}}{\overline{\big)\,\text{dividend}}}$$

The quotients can be recorded as a mixed number or a decimal when it is not a whole number. The result of a division problem can be written in the form dividend ÷ divisor = quotient + $\frac{\text{remainder}}{\text{divisor}}$ or as a decimal.

For additional information and examples, see the Methods & Meanings box in Lesson 6.1.2 of the *Core Connections Course 1* text.

Examples

1. Determine the quotient: 1,860 ÷ 8

MIXED NUMBER QUOTIENT

The first step is to consider the divisor, 8, multiplied by 1,000 since the thousands is the largest place value in the dividend. Multiplying 8 by 1,000 gives 8,000, which is greater than the dividend, so the next place value is considered, which is the hundreds place.

Since 8 multiplied by 100 is 800, determine how many 800s are in 1,860. There are more than 2 (since $2 \cdot 800 = 1{,}600$) but less than 3 (since $3 \cdot 800 = 2{,}400$), so a 2 is placed in the hundreds place of the quotient.

$$\begin{array}{r} 200 \\ 8\overline{)1,860} \end{array}$$

Next, the divisor, 8, is multiplied by 200, and the product, 1,600, is placed below 1,860. Then 1,600 is subtracted from 1,860 to get 260.

$$\begin{array}{r} 200 \\ 8\overline{)1,860} \\ \underline{-\,1,600} \\ 260 \end{array}$$

Repeat the same process to determine how many times 8 divides 260: more than 30 but less than 40. The 3 is placed in the tens place of the quotient.

$$\begin{array}{r} 30 \\ 200 \\ 8\overline{)1,860} \\ \underline{-\,1,600} \\ 260 \end{array}$$

Multiplying 8 by 30 gives 240, and after subtracting 240 from 260, there is 20 left.

$$\begin{array}{r} 30 \\ 200 \\ 8\overline{)1,860} \\ \underline{-\,1,600} \\ 260 \\ \underline{-\,240} \\ 20 \end{array}$$

Then determine how many times 8 divides 20: more than 2 but less than 3. The 2 is placed in the ones place of the quotient.

$$\begin{array}{r} 2 \\ 30 \\ 200 \\ 8\overline{)1,860} \\ \underline{-\,1,600} \\ 260 \\ \underline{-\,240} \\ 20 \\ \underline{-\,16} \\ 4 \end{array}$$

Since 4 cannot be divided by 8 a whole number of times, the quotient is $200 + 30 + 2$ with a remainder of 4 divided by 8, which is shown as the fraction $\frac{4}{8}$. The final quotient is $232\frac{4}{8}$ or $232\frac{1}{2}$.

$$\begin{array}{r} \left.\begin{array}{r} 2 \\ 30 \\ 200 \end{array}\right\} + \frac{4}{8} \\ 8\overline{)1,860} \\ \underline{-\,1,600} \\ 260 \\ \underline{-\,240} \\ 20 \\ \underline{-\,16} \\ 4 \end{array}$$

A more condensed method of long division is shown.

$$\begin{array}{r} 232 + \frac{4}{8} \\ 8\overline{)1,860} \\ \underline{-\,1,600} \\ 260 \\ \underline{-\,240} \\ 20 \\ \underline{-\,16} \\ 4 \end{array}$$

Solution continues on next page.

Solution continued from previous page.

DECIMAL QUOTIENT

To calculate the quotient as a decimal number, use the same long division process and continue dividing past the decimal point. Place value is still important. The remainder of 4 can be represented as 40 tenths and a decimal point is added to show that there are 5 × 8 tenths in 4.0. The quotient is written as a decimal, 232.5, with a remainder of 0.

$$\begin{array}{r} 232 \\ 8\overline{)1{,}860} \\ \underline{-\,1{,}600} \\ 260 \\ \underline{-\,240} \\ 20 \\ \underline{-\,16} \\ 4 \end{array} \quad \Rightarrow \quad \begin{array}{r} 232. \\ 8\overline{)1{,}860.0} \\ \underline{-\,1{,}600} \\ 260 \\ \underline{-\,240} \\ 20 \\ \underline{-\,16} \\ 4.0 \end{array} \quad \Rightarrow \quad \begin{array}{r} 232.5 \\ 8\overline{)1{,}860} \\ \underline{-\,1{,}600} \\ 260 \\ \underline{-\,240} \\ 20 \\ \underline{-\,16} \\ 4.0 \\ \underline{-\,4.0} \\ 0 \end{array}$$

Practice

1.	154 ÷ 8	**2.**	626 ÷ 4	**3.**	7,344 ÷ 5	**4.**	955 ÷ 4	**5.**	216 ÷ 18
6.	459 ÷ 18	**7.**	780 ÷ 60	**8.**	3,597 ÷ 60	**9.**	4,870 ÷ 40	**10.**	9,534 ÷ 600

Answers

1.	19.25	**2.**	156.5	**3.**	1,468.8	**4.**	238.75	**5.**	12
6.	25.5	**7.**	13	**8.**	59.95	**9.**	121.75	**10.**	15.89

Order of Operations

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Overview

A numerical expression is made up of a sum or difference of terms. You might see groups of numbers, exponents, multiplication, division, addition, and subtraction. Examples of numerical terms are: 4, 3(6), 6(9 – 4), $2 \cdot 3^2$, $3(5 + 2^3)$, and $\frac{16-4}{6}$.

For the expression $3 + 4 \cdot 2$, the terms can be circled as shown.

$$\boxed{3} + \boxed{4 \cdot 2}$$

Each term is evaluated separately, giving 3 + 8. Then the terms are added: 3 + 8 = 11. Thus, $3 + 4 \cdot 2 = 11$.

The Order of Operations is an agreed-upon set of steps for interpreting and evaluating numerical expressions. Numerical expressions should be evaluated in the following order.

1. Identify and circle all terms
2. Evaluate inside groups (such as parentheses, brackets, and fraction bars).
3. Evaluate exponents.
4. Complete multiplication and division from left to right.
5. Complete addition and subtraction from left to right.

For additional information, see the Methods & Meanings box in Lesson 6.2.1 of the *Core Connections Course 1* text.

Examples

1. Evaluate the expression $2 \cdot 3^2 + 3(6 - 3) + 10$.

$2 \cdot 3^2 + 3(6 - 3) + 10$	
$= \boxed{2 \cdot 3^2} + \boxed{3(6-3)} + \boxed{10}$	Circle each term in the expression.
$= \boxed{2 \cdot 3^2} + \boxed{3(3)} + \boxed{10}$	Rewrite the expressions within the parentheses.
$= \boxed{2 \cdot 9} + \boxed{3(3)} + \boxed{10}$	Evaluate each exponential part (e.g., 3^2).
$= \boxed{18} + \boxed{9} + \boxed{10}$	Multiply and divide from left to right.
$= 27 + 10$	Finally, combine terms by adding or subtracting from left to right.
$= 37$	

2. Evaluate the expression $5 - 8 \div 2^2 + 6(5 + 4) - 5^2$.

$5 - 8 \div 2^2 + 6(5 + 4) - 5^2$	
$= \boxed{5} - \boxed{8 \div 2^2} + \boxed{6(5+4)} - \boxed{5^2}$	Circle the terms.
$= \boxed{5} - \boxed{8 \div 2^2} + \boxed{6(9)} - \boxed{5^2}$	Rewrite inside the parentheses.
$= \boxed{5} - \boxed{8 \div 4} + \boxed{6(9)} - \boxed{25}$	Rewrite the exponents.
$= 5 - 2 + 54 - 25$	Multiply and divide from left to right.
$= 32$	Finally, add and subtract from left to right.

3. Evaluate the expression $20 + \frac{5+7}{3} - 4^2 + 12 \div 4$.

$20 + \frac{5+7}{3} - 4^2 + 12 \div 4$	
$= \boxed{20} + \boxed{\frac{5+7}{3}} - \boxed{4^2} + \boxed{12 \div 4}$	Circle the terms.
$= 20 + 4 - 16 + 3$	Multiply and divide left to right, including exponents.
$= 11$	Add or subtract from left to right.

Practice

Circle the terms, then evaluate each expression.

1. $5 \cdot 3 + 4$
2. $10 \div 5 + 3$
3. $2(9 - 4) \cdot 7$
4. $6(7 + 3) + 8 \div 2$
5. $15 \div 3 + 7(8 + 1) - 6$
6. $\frac{9}{3} + 5 \cdot 3^2 - 2(14 - 5)$
7. $\frac{20}{6+4} + 7 \cdot 2 \div 2$
8. $\frac{5+30}{7} + 6^2 - 18 \div 9$
9. $2^3 + 8 - 16 \div 8 \cdot 2$
10. $25 - 5^2 + 9 - 3^2$
11. $5(17 - 7) + 4 \cdot 3 - 8$
12. $(5 - 2)^2 + (9 + 1)^2$
13. $4^2 + 9(2) \div 6 + (6 - 1)^2$
14. $\frac{18}{3^2} + \frac{5 \cdot 3}{5}$
15. $3(7 - 2)^2 + 8 \div 4 - 6 \cdot 5$
16. $14 \div 2 + 6 \cdot 8 \div 2 - (8 - 3)^2$
17. $\frac{27}{3} + 18 - 9 \div 3 + (3 + 2)^2$
18. $26 \cdot 2 \div 4 + (6 + 4)^2 + 3(5 - 2)^3$
19. $\left(\frac{42+3}{5}\right)^2 + 3^2 - (4 \cdot 2)^2$

Answers

1. 19
2. 5
3. 70
4. 64
5. 62
6. 30
7. 9
8. 39
9. 12
10. 0
11. 54
12. 109
13. 44
14. 5
15. 47
16. 6
17. 49
18. 194
19. 26

Area and Perimeter With Algebra Tiles

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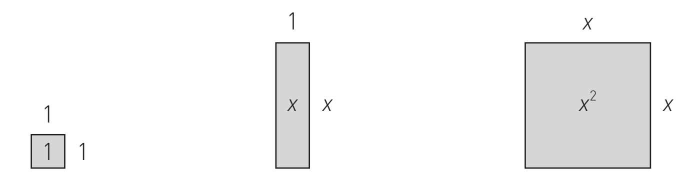
Overview

Algebraic expressions can be represented by the areas and perimeters of algebra tiles (rectangles and squares) and combinations of algebra tiles. The dimensions of each tile are shown along its sides, and the tile is named by its area, as shown.



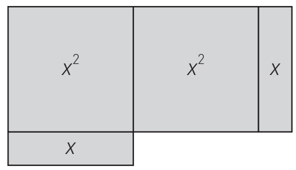
unit tile x-tile x^2-tile

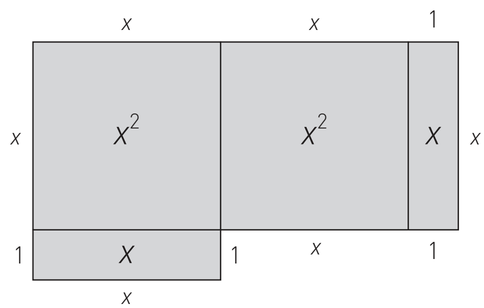
When several tiles are put together to form a more complicated figure, the area of the new figure is the sum of the areas of the individual pieces. The perimeter is the sum of the lengths around the outside of the figure, not the sum of the perimeters of the individual tiles. The expressions to represent the area and perimeter of these more complicated figures can be rewritten by combining like terms.

For additional information, see the Methods & Meanings boxes in Lessons 6.2.3 and 6.2.4 of the *Core Connections Course 1* text.

Examples

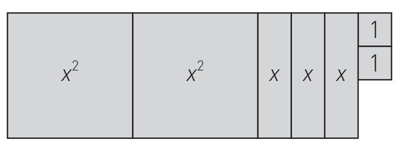
1. Label the side lengths on the algebra tile figure shown and then write an expression to represent the perimeter of the figure.

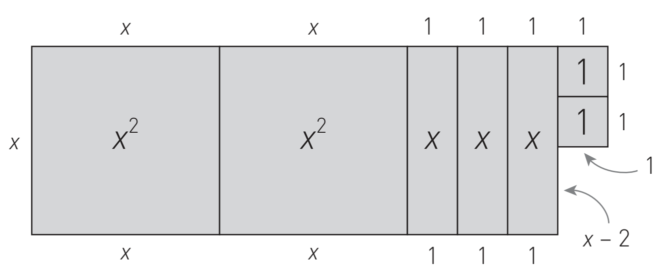




Perimeter = $x + x + 1 + x + 1 + x + 1 + x + 1 + x = 6x + 4$ units

2. Label the side lengths on the algebra tile figure shown and then write an expression to represent the perimeter of the figure.



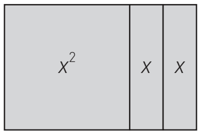


Perimeter = $x + x + 1 + 1 + 1 + 1 + 1 + 1 + 1 + (x - 2) + 1 + 1 + 1 + x + x + x = 6x + 8$ units

Practice

Write an expression to represent the perimeter of each figure.

1.



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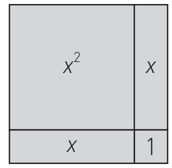
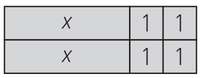
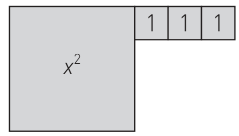


3.



4.



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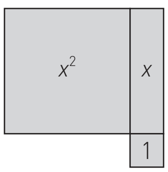

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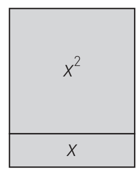
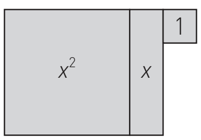
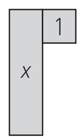


7.



8.



Answers

1. $4x + 4$ units

2. $4x + 4$ units

3. $2x + 8$ units

4. $4x + 6$ units

5. $4x + 4$ units

6. $4x + 2$ units

7. $4x + 4$ units

8. $2x + 4$ units

Combining Like Terms

GUIDED SKILL SUPPORT

Overview

Algebraic expressions can be rewritten into equivalent forms. Two common strategies for rewriting algebraic expressions are using the Distributive Property and combining like terms. To combine like terms, identify the terms that have the same variable(s) raised to the same power(s). Then, combine these terms into one term by adding or subtracting.

For additional information, see the Methods & Meanings box in Lesson 6.2.4 of the *Core Connections Course 1* text.

Examples

1. Combine like terms to rewrite the expression $3x + 5x + 7x$.

All of the terms have the same variable, x, raised to the power of 1, so they are combined into one term, $15x$.

2. Combine like terms to rewrite the expression $3x + 12 + 7x + 5$.

The terms with x can be combined. The terms without variables (the constants) can also be combined, but separately from those terms with variables.

$$\begin{aligned} &3x + 12 + 7x + 5 \\ &= 3x + 7x + 12 + 5 \\ &= 10x + 17 \end{aligned}$$

Note that in the rewritten form, the term with the variable is listed before the constant term.

3. Combine like terms to rewrite the expression $5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$.

$$\begin{aligned} &5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1 \\ &= 4x^2 + 2x^2 + 5x + 2x + x + 10 - 6 - 1 \\ &= 6x^2 + 8x + 3 \end{aligned}$$

Note that terms with the same variable but with different exponents are not combined and are written in order of decreasing power of the variable, with the constant term last.

4. Combine like terms to rewrite the expression $2x^2 + 5x + 6 + 3x^2 - 4x - 2$.

Sometimes it is helpful to take an expression that is written horizontally, circle the terms with their signs, and rewrite like terms in vertical columns before you combine them:

$$2x^2 + 5x + 6 + 3x^2 - 4x - 2$$

$$\boxed{2x^2}\boxed{+5x}\boxed{+6}\boxed{+3x^2}\boxed{-4x}\boxed{-2}$$

$$\Downarrow$$

$$\begin{array}{r} 2x^2 + 5x + 6 \\ \underline{+\ 3x^2 - 4x - 2} \\ 5x^2 \quad + x + 4 \end{array}$$

This procedure may make it easier to identify the terms as well as the sign of each term.

Practice

Combine the following sets of terms.

1. $(2x^2 + 6x + 10) + (4x^2 + 2x + 3)$

2. $(3x^2 + x + 4) + (x^2 + 4x + 7)$

3. $(8x^2 + 3) + (4x^2 + 5x + 4)$

4. $(4x^2 + 6x + 5) + (3x^2 + 2x + 4)$

5. $(4x^2 + 7x + 3) + (2x^2 - 2x + 5)$

6. $(3x^2 + 7x) + (x^2 + 3x + 9)$

7. $(5x + 6) + (5x^2 + 6x - 2)$

8. $2x^2 + 3x + x^2 + 4x - 3x^2 + 2$

9. $5c^2 + 4c + 7x + 12 - 4c^2 + 9 + 6x$

10. $2a^2 + 3a^3 + 4a^2 + 6a + 12 - 4a + 2$

Answers

1. $6x^2 + 8x + 13$

2. $4x^2 + 5x + 11$

3. $12x^2 + 5x + 7$

4. $7x^2 + 8x + 9$

5. $6x^2 + 5x + 8$

6. $4x^2 + 10x + 9$

7. $5x^2 + 11x + 4$

8. $7x + 2$

9. $c^2 + 4c + 13x + 21$

10. $3a^3 + 6a^2 + 2a + 14$

Evaluating Expressions

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Overview

Variables are letters or symbols used to represent one or more numbers. Algebraic expressions contain variables, numbers, and operations (e.g., addition, division, exponents, etc.). Algebraic expressions can be evaluated for specific values of the variable(s). Replace the number for the variable and evaluate, following the Order of Operations.

Examples

Evaluate each variable expression for $x = 3$.

1. $5x$

$5(3) = 15$

2. $x + 10$

$(3) + 10 = 13$

3. $\frac{18}{x}$

$\frac{18}{3} = 6$

4. $\frac{x}{3}$

$\frac{3}{3} = 1$

5. $3x - 5$

$3(3) - 5 = 9 - 5 = 4$

6. $5x^2 + 3x$

$5(3)^2 + 3(3) = 5(9) + 3(3) = 45 + 9 = 54$

Practice

Evaluate each variable expression for $y = 4$.

1. $y - 1$

2. $6y - 1$

3. $6(y - 1)$

4. $6\left(\frac{y}{2} - 1\right)$

5. $6(y^2 - 1)$

6. $6(y^2 - y)$

7. $6(y^2) - y$

Evaluate each variable expression for $p = 1.5$.

8. $2 + p$

9. $2 + p \cdot 10$

10. $(2 + p) \cdot 10$

11. $(2 + p)^2 \cdot 10$

12. $4 + (2 + p)^2 \cdot 10$

Evaluate each variable expression for $x = \frac{1}{2}$.

13. $x \cdot (14 - 11)^3$

14. $20 + x \cdot (14 - 11)^3$

15. $x \cdot 20 + x \cdot (14 - 11)^3$

16. $8 \div x \cdot 20 + x \cdot (14 - 11)^3$

17. $8 \div x \cdot [20 + x \cdot (14 - 11)^3]$

Answers

1. 3

2. 23

3. 18

4. 6

5. 90

6. 72

7. 92

8. 3.5

9. 17

10. 35

11. 122.5

12. 126.5

13. $\frac{27}{2} = 13\frac{1}{2}$

14. $\frac{67}{2} = 33\frac{1}{2}$

15. $\frac{47}{2} = 23\frac{1}{2}$

16. $\frac{667}{2} = 333\frac{1}{2}$

17. 536