

Rates and Unit Rates

GUIDED SKILL SUPPORT

Overview

A rate is a ratio that describes how one quantity changes with respect to another quantity. Unit rate is a rate that compares the change in one quantity to a 1-unit change in another quantity.

For additional information, see the Methods & Meanings boxes in Lesson 7.1.3 of the *Core Connections Course 1* text.

Examples

1. A rice recipe uses 6 cups of rice for 15 people. At the same rate, how much rice is needed for 40 people?

Three methods for calculating this rate are shown. Each method shows that 16 cups of rice are needed for 40 people.

GIANT ONE

To scale from 15 to 40 people using a Giant One, solve $15 \times ? = 40$, which is $\frac{40}{15}$.

$$\frac{6 \text{ cups}}{15 \text{ people}} = \frac{? \text{ cups}}{40 \text{ people}} \rightarrow \frac{6}{15} \cdot \frac{\boxed{\frac{40}{15}}}{\boxed{\frac{40}{15}}} = \frac{16 \text{ cups}}{40 \text{ people}}$$

RATIO TABLE

Rice (cups)	Number of People
6	15
2	5
16	40

$\div 3$ (from 6 to 2)
 $\times 8$ (from 2 to 16)
 $\div 3$ (from 15 to 5)
 $\times 8$ (from 5 to 40)

DOUBLE NUMBER LINE

RICE (cups)



NUMBER OF PEOPLE

2. A train in France traveled 932 miles in 5 hours. What is the unit rate in miles per hour?

Three methods for calculating this rate are shown. Each method shows that the unit rate is 186.4 miles per hour.

GIANT ONE

Unit rate means the denominator needs to be 1 hour: $\frac{932 \text{ miles}}{5 \text{ hour}} = \frac{? \text{ miles}}{1 \text{ hour}}$. To determine the numerator and denominator of the Giant One, solve $5 \times ? = 1$, which is $\frac{1}{5}$ or 0.2.

$$\frac{932}{5} \cdot \frac{0.2}{0.2} = \frac{186.4 \text{ miles}}{1 \text{ hour}}$$

RATIO TABLE

Distance (miles)	Time (hours)
932	5
1,864	10
186.4	1

$\times 2$ (from 932 to 1,864)
 $\div 10$ (from 5 to 1)
 $\times 2$ (from 5 to 10)
 $\div 10$ (from 10 to 1)

DOUBLE NUMBER LINE

DISTANCE (miles)



TIME (hours)

3. Arrange the following rates from least to greatest.

Rate A: 30 miles in 25 minutes

Rate B: 60 miles in 1 hour

Rate C: 70 miles in $1\frac{2}{3}$ hours

Changing each rate to have a common denominator will make them easier to compare. In this case, 60 minutes is chosen as the common denominator.

$$\text{Rate A: } \frac{30 \text{ miles}}{25 \text{ minutes}} \cdot \frac{2.4}{2.4} = \frac{72 \text{ miles}}{60 \text{ minutes}}$$

$$\text{Rate B: } \frac{60 \text{ miles}}{1 \text{ hour}} = \frac{60 \text{ miles}}{60 \text{ minutes}}$$

$$\text{Rate C: } \frac{70 \text{ miles}}{1\frac{2}{3} \text{ hours}} = \frac{70 \text{ miles}}{100 \text{ minutes}} \cdot \frac{0.6}{0.6} = \frac{42 \text{ miles}}{60 \text{ minutes}}$$

The order of the rates from least to greatest is: C, B, A.

Note that by using 60 minutes (1 hour) for the common unit to compare speeds, each rate can be expressed as a unit rate in miles per hour (mph): Rate A = 72 mph, Rate B = 60 mph, and Rate C = 42 mph.

Practice

Solve each rate problem. Explain your method.

1. Balvina knows that 6 cups of Spanish rice will make enough for 15 people. How many cups of rice are needed to feed 135 people?
2. Elaine enjoys working in her garden after school. She can plant 6 flowers in 15 minutes. How many flowers would she be able to plant in 135 minutes at the same rate?
3. Shane is a competitive cyclist. To stay competitive, he needs to practice biking long distances.
 - a. Shane rode his bike for 2 hours and traveled 32 miles. At this rate, how many miles would he ride in 5 hours?
 - b. On another day, Shane rode at the same rate as in part (a). At this rate, how long would it take him to travel 28 miles?
4. A plane travels 23 miles in 3 minutes.
 - a. How many miles does the plane travel in 1 hour?
 - b. At this rate, how long would it take the plane to travel 2,990 miles?
5. In some cities, it is more convenient and better for the environment to take public transportation to work than to drive. Alice spends \$25.50 a week on public transportation. Betty spends $\frac{1}{12}$ of her \$1,260 biweekly paycheck on public transportation every 2 weeks, and Cindy spends \$96 per month (4 weeks) on public transportation. Order the commuters from most to least spending per week.

Answers

1. 54 cups
2. 54 flowers
3. a. 80 miles b. 1.75 hours
4. a. 460 miles b. 6.5 hours
5. Cindy (\$24 per week), Alice (\$25.50 per week), Betty (\$52.50 per week)

Dividing Decimals

GUIDED SKILL SUPPORT

Overview

When dividing decimals or whole numbers by a decimal, multiply the divisor and dividend using a power of 10 in a Giant One. Then, follow the steps for division by a whole number. Sometimes, it is necessary to add zeros to the number being divided to complete the division.

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$$

For additional examples and practice, see the *Core Connections Course 1* Checkpoint 8B materials. For additional information on whole number division, see the Guided Skill Support in Chapter 6. For multiplying decimals, see the Guided Skill Support in Chapter 5.

Examples

1. Divide 32.4 by 8.

$$\begin{array}{r} \frac{32.4}{8} \cdot \frac{10}{10} = \frac{324}{80} \\ \begin{array}{r} 4.05 \\ 80 \overline{) 324.00} \\ \underline{- 320} \downarrow \\ 4.0 \downarrow \\ \underline{- 0.0} \downarrow \\ 4.00 \downarrow \\ \underline{- 4.00} \\ 0 \end{array} \end{array}$$

2. Divide 27.42 by 1.2.

First, multiply each number by 10^2 or 100.

$$\begin{array}{r} \frac{27.42}{1.2} \cdot \frac{100}{100} = \frac{2,742}{120} \\ 1.2 \overline{) 27.42} \rightarrow 120 \overline{) 2,742} \rightarrow 120 \overline{) 2,742.00} \\ \begin{array}{r} 22.85 \\ \underline{- 2,400} \downarrow \\ 342 \downarrow \\ \underline{- 240} \downarrow \\ 102.0 \downarrow \\ \underline{- 96.0} \downarrow \\ 6.00 \downarrow \\ \underline{- 6.00} \\ 0 \end{array} \end{array}$$

Practice

Divide. Round answers to the nearest hundredth, if necessary.

- | | | | |
|----------------------|------------------------|----------------------|-----------------------|
| 1. $18.32 \div 5$ | 2. $147.3 \div 6$ | 3. $100.32 \div 24$ | 4. $53.6 \div 0.004$ |
| 5. $1.32 \div 0.032$ | 6. $46.3 \div 0.011$ | 7. $25.46 \div 5.05$ | 8. $7.035 \div 0.005$ |
| 9. $306.4 \div 3.2$ | 10. $6.042 \div 0.006$ | 11. $500 \div 0.004$ | 12. $420 \div 0.05$ |

Answers

- | | | | |
|----------|-------------|-------------|-----------|
| 1. 3.664 | 2. 24.55 | 3. 4.18 | 4. 13,400 |
| 5. 41.25 | 6. 4,209.09 | 7. 5.04 | 8. 1,407 |
| 9. 95.75 | 10. 1,007 | 11. 125,000 | 12. 8,400 |

Using the Distributive Property With Variables

GUIDED SKILL SUPPORT

Overview

Variables are letters or symbols used to represent one or more numbers. Algebraic expressions contain variables, numbers, and operations (e.g., addition, division, exponents, etc.).

Algebraic expressions can be rewritten into equivalent forms. Two common strategies for rewriting algebraic expressions are using the Distributive Property and combining like terms.

To use the Distributive Property, rewrite $a(b + c)$ as $ab + ac$, or vice versa.

- To rewrite the product $a(b + c)$ as the sum, “distribute” the multiplier, a , by multiplying it by each term in the parentheses to get $ab + ac$.
- To rewrite the sum $ab + ac$ as a product, divide each term by a common factor, a , and write the factor outside the parentheses to get $a(b + c)$.

Examples

Rewrite each expression using the Distributive Property.

1. $3(d + 4)$

$$\begin{aligned} &3(d + 4) \\ &= (3 \cdot d) + (3 \cdot 4) \\ &= 3d + 12 \end{aligned}$$

2. $4(x + 3y + 1)$

$$\begin{aligned} &4(x + 3y + 1) \\ &= (4 \cdot x) + (4 \cdot 3y) + 4(1) \\ &= 4x + 12y + 4 \end{aligned}$$

3. $5k + 40$

$$\begin{aligned} &5k + 40 \\ &= 5 \cdot k + 5 \cdot 8 \\ &= 5(k + 8) \end{aligned}$$

Practice

Rewrite each expression using the Distributive Property.

1. $3(m + 6)$

2. $5(m + 6)$

3. $5(k + 6)$

4. $5k + 20$

5. $5(k + 6 + 3y)$

6. $5(k + 6 + 3k)$

7. $5k + 6 - 3k$

8. $10k - 15$

9. $3k + 15$

10. $10p + 15 + 3$

11. $2(15 + 3p)$

12. $12x + 15$

13. $10p + 15 - 5p$

14. $10p + 15 + 5p$

15. $7(2x + 3p)$

Answers

1. $3m + 18$

2. $5m + 30$

3. $5k + 30$

4. $5(k + 4)$

5. $5k + 30 + 15y$

6. $5k + 30 + 15k$ or $20k + 30$

7. $2(k + 3)$

8. $5(2k - 3)$

9. $3(k + 5)$

10. $2(5p + 9)$

11. $30 + 6p$

12. $3(4x + 5)$

13. $5(p + 3)$

14. $15(p + 1)$

15. $14x + 21p$

Writing and Solving Equations

GUIDED SKILL SUPPORT

Overview

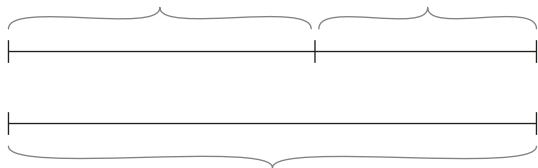
To solve an equation means to figure out the value or values of the variable that make the equation true. A solution to an equation is a number that, when substituted into the equation for the variable, makes the left side of the equation equal to the right side.

For additional information, see the Methods & Meanings box in Lesson 7.3.4 of the *Core Connections Course 1* text.

Examples

Write an equation that represents each situation and determine the value of the variable.

1.



$$x + 10 = 32$$

$$x + 10 - 10 = 32 - 10$$

$$x = 22$$



$$x + 10 = 32$$

$$22 + 10 = 32$$

$$32 = 32$$

true

Subtraction undoes addition.

The solution to the equation is $x = 22$.

Check the solution by substituting 22 for x in the original equation and making sure that the equation is true.

2. Zeke has 45 songs downloaded to his favorites playlist. His playlist includes only pop and hip-hop songs. He has twice as many hip-hop songs as pop songs on the playlist. How many hip-hop songs are on his favorites playlist?

Let x = number of pop songs

$2x$ = number of hip-hop songs

$$x + 2x = 45$$

$$3x = 45$$

$$3x \div 3 = 45 \div 3$$

$$x = 15$$



$$x + 2x = 45$$

$$(15) + 2(15) = 45$$

$$15 + 30 = 45$$

$$45 = 45$$

true

Combine like terms.

Division undoes multiplication.

The solution to the equation is $x = 15$.

Check the solution by substituting 15 for x in the original equation and making sure that the equation is true.

Since $x = 15$ and the number of hip-hop songs is equal to $2x$, Zeke has 30 hip-hop songs on his favorites playlist.

For this problem, you could also let x = the number of hip-hop songs, and then $12x$ would represent the number of pop songs.

Solve each equation.

3. $x + \frac{4}{11} = \frac{1}{2}$

The variable x is being added to $\frac{4}{11}$. The operation of subtraction undoes addition.

$$\begin{aligned} x + \frac{4}{11} &= \frac{1}{2} \\ -\frac{4}{11} &- \frac{4}{11} \\ x &= \frac{1}{2} - \frac{4}{11} \\ x &= \frac{3}{22} \end{aligned}$$

Check the solution by substituting $\frac{3}{22}$ for x in the original equation and making sure that the equation is true.

$$\begin{aligned} x &= \frac{3}{22} \\ \downarrow \\ x + \frac{4}{11} &= \frac{1}{2} \\ \frac{3}{22} + \frac{4}{11} &= \frac{1}{2} \\ \frac{3}{22} + \frac{8}{22} &= \frac{1}{2} \\ \frac{11}{22} &= \frac{1}{2} \\ \frac{1}{2} &= \frac{1}{2} \\ \text{true} \end{aligned}$$

4. $5.7x = 26.22$

The variable x is being multiplied by 5.7. The operation of division undoes multiplication.

$$\begin{aligned} 5.7x &= 26.22 \\ \frac{5.7x}{5.7} &= \frac{26.22}{5.7} \\ x &= 26.22 \div 5.7 \end{aligned}$$

Use long division to solve.

$$\begin{aligned} x &= 26.22 \div 5.7 \\ \downarrow \\ 5.7 \overline{)26.22} \\ \downarrow \\ 57 \overline{)262.2} \\ &\underline{-228.0} \\ &34.2 \\ &\underline{-34.2} \\ &0 \\ \downarrow \\ x &= 4.6 \end{aligned}$$

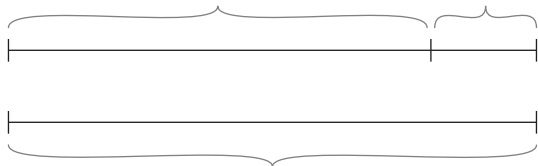
Check the solution by substituting 4.6 for x in the original equation and making sure that the equation is true.

$$\begin{aligned} x &= 4.6 \\ \downarrow \\ 5.7x &= 26.22 \\ 5.7(4.6) &= 26.22 \\ 26.22 &= 26.22 \\ \text{true} \end{aligned}$$

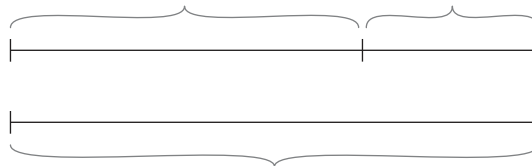
Practice

Write an equation that represents each situation, and then determine the value of the variable.

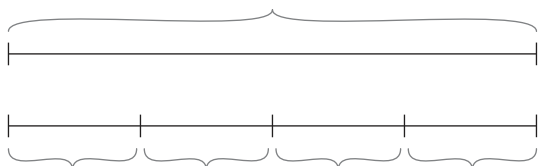
1.



2.



3.



Solve each equation for the variable.

4. $x + 7 = 9$

5. $y - 2 = 3$

6. $3y = 24$

7. $\frac{m}{2} = 6$

8. $2 = 11x$

9. $4x + x = 25$

10. $m - 7 = 16$

11. $x + 9 = 30$

12. $3 + y = 9$

13. $\frac{k}{4} = 7$

14. $\frac{x}{13} = 4$

15. $17 = m + 11$

16. $y + \frac{3}{2} = 12$

17. $9c = 72$

Answers

1. $x + 3 = 25$; $x = 22$

2. $2x + x = 18$; $x = 6$

3. $4n = 28$; $x = 7$

4. $x = 2$

5. $y = 5$

6. $y = 8$

7. $m = 12$

8. $x = \frac{2}{11}$

9. $5x = 25$; $x = 5$

10. $m = 23$

11. $x = 21$

12. $y = 6$

13. $k = 28$

14. $x = 52$

15. $m = 6$

16. $y = 10\frac{1}{2}$ or 10.5

17. $c = 8$

Graphing and Solving Inequalities

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Overview

The solution to an inequality refers to all the values of the variable that make the inequality true. The solution can be represented as a ray or segment on a number line with solid or open endpoints. A solid endpoint indicates that the endpoint is included in the solution ($\leq$ or $\geq$), while an open endpoint indicates that it is not part of the solution ($<$ or $>$).

Symbol	Meaning in Words
$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to

For additional information, see the Methods & Meanings box in Lesson 7.3.5 of the *Core Connections Course 1* text.

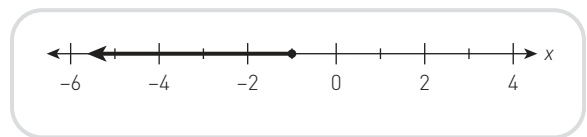
Examples

1. Represent the solution to each inequality on a number line.

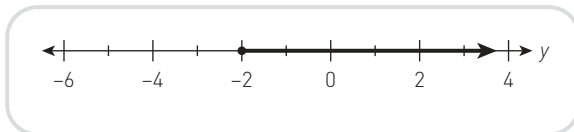
a. $x > 6$



b. $x \leq -1$



c. $y \geq -2$



2. Solve each inequality and represent the solution on a number line.

a. $m + 2 \leq 9$

b. $2x > 6$

Determine the boundary point:

$$\begin{aligned} m + 2 &= 9 \\ m &= 7 \end{aligned}$$

Draw a number line. Put a solid dot at 7.



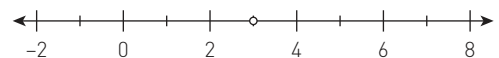
The solution is $m \leq 7$.



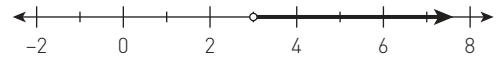
Determine the boundary point:

$$\begin{aligned} 2x &= 6 \\ x &= 3 \end{aligned}$$

Draw a number line. Put an open dot at 3.



The solution is $x > 3$.



In examples 3–5, write an inequality to represent each situation.

3. New models of yellow school buses can hold at most 48 passengers.

“At most” indicates the buses can hold 48 or fewer passengers. Therefore, the inequality sign $\leq$ should be used.

If x represents the number of passengers, then $x \leq 48$.

4. Children need to be taller than 36 inches to go on most amusement park rides.

“Taller than” indicates the inequality sign $>$ should be used.

If x represents height in inches, then $x > 36$.

5. There are fewer than 30 days left in the school year.

“Fewer than” indicates the inequality sign $<$ should be used.

If x represents the number of days, then $x < 30$.

Practice

Represent the solution to each inequality on a number line.

1. $m < 2$ 2. $x \leq -1$ 3. $y \geq 3$ 4. $m > -9$ 5. $x \leq 3$

Solve each inequality and represent the solution on a number line.

6. $x + 3 > 8$ 7. $y - 3 \leq 5$ 8. $3x \leq 6$ 9. $4m \geq 12$ 10. $17 < y + 13$
11. $8 \geq 2m$

Write an inequality to represent each situation. Graph the inequality.

12. This school year, Giona wants to read at least 18 books.
13. Tickets to the school football game cost at most \$10.
14. It takes no more than 3 hours to drive to the beach.
15. A phone battery lasts less than 14 hours.

Answers

