Measures of Central Tendency and Variability

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Overview

Measures of central tendency are numbers that locate or approximate the "center" of a data set—that is, a "typical" value that describes the set of data. Mean and median are the most common measures of central tendency.

The mean is the arithmetic average of a data set. To calculate the mean, add all the values in a set and divide this sum by the number of values in the set.

The median is the middle number in a set of data arranged numerically. If there is an even number of values, the median is the mean (average) of the two middle numbers.

Measures of variability are numbers that describe how the values in a data set vary. Range, interquartile range, mean absolute deviation, and outliers are used to describe variability.

The range of a data set is the difference between the highest and lowest values of the data set.

The interquartile range (IQR) represents the middle 50% of the data. It is calculated by subtracting the first quartile (the median of the lower half of the data) from the third quartile (the median of the upper half of the data).

The mean absolute deviation (MAD) is the average distance of each data point from the mean of a data set. It is appropriate to use for data sets with no outliers.

An outlier is a number that is much smaller or larger than most of the others in the data set. Outliers have a greater effect on a data set's mean than on its median.

For additional information, see the Methods & Meanings boxes in Lesson 8.1.3, 8.1.4, and 8.1.5 of the *Core Connections Course 1* text.

Examples

1. Use this data set to complete parts (a) through (e).

34, 31, 37, 44, 38, 34, 42, 34, 43, 81

a. Calculate the mean.

$34 + 31 + 37 + 44 + 38 + 34 + 42 + 34 + 43 + 81 = 418$

$\frac{418}{10} = 41.8$

The mean of this set of data is 41.8.

b. Calculate the median.

Arrange the data in order and identify the middle value(s).

31 34 34 34 (37 38) 42 43 44 81

median

Since there are two middle values, calculate their mean: $37 + 38 = 75$, $\frac{75}{2} = 37.5$. Therefore, the median of this data set is 37.5.

c. Calculate the range.

The highest value is 81. The lowest value is 31.

$81 - 31 = 50$, so the range of this data set is 50.

d. Calculate the interquartile range (IQR).

Arrange the data in order and divide the data in half. Calculate the first quartile and third quartile.

lower half: 31 34 (34) 34 37 | upper half: 38 42 (43) 44 81

first quartile third quartile

The difference between the third and first quartiles is $43 - 34 = 9$. The IQR of this data is 9.

Example continues on next page.

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e. Calculate the mean absolute deviation (MAD).

Data Value	Absolute Value of the Difference From Mean
31	10.8
34	7.8
34	7.8
34	7.8
37	4.8
38	3.8
42	0.2
43	1.2
44	2.2
81	39.2

The MAD is the sum of the absolute value of the differences from the mean divided by the number of data values.

$$\text{MAD} = \frac{10.8 + 7.8 + 7.8 + 7.8 + 4.8 + 3.8 + 0.2 + 1.2 + 2.2 + 39.2}{10} = \frac{85.6}{10} = 8.56$$

Note: For this data set, the interquartile range (IQR) is the better measure of variability due to the outlier of 81.

f. Identify any outliers.

81 is an outlier because it is much larger than the other numbers in the data set.

Practice

Calculate the mean, median, and range of each set of data.

1. 29, 28, 34, 30, 33, 26, 34

2. 25, 34, 35, 27, 31, 30

Identify the outlier in each set of data.

3. 70, 77, 75, 68, 98, 70, 72, 71

4. 14, 22, 17, 61, 20, 16, 15

5. 1,376, 1,645, 1,783, 1,455, 3,754, 1,790, 1,384, 1,643, 1,492, 1,776

6. 62, 65, 93, 51, 55, 14, 79, 85, 55, 72, 78, 83, 91, 76

7. Use the following data set to complete parts (a) through (e).

26, 28, 29, 30, 30, 33, 34, 54

a. Calculate the mean.

b. Calculate the median.

c. Calculate the range.

d. Calculate the interquartile range (IQR).

e. Identify any outliers.

8. Use this data set to complete parts (a) through (e).

116, 104, 101, 109, 100, 107, 113, 118, 113, 101, 100, 109, 105, 103, 91

a. Calculate the mean.

b. Calculate the median.

c. Calculate the range.

d. Calculate the mean absolute deviation (MAD).

e. Identify any outliers.

Answers

1. mean = 30.57, median = 30, range = 8

2. mean = $30.\overline{3}$, median = 30.5, range = 10

3. 98

4. 61

5. 3,754

6. 14

7. a. 33
b. 30
c. 28
d. 5
e. 54

8. a. 106
b. 105
c. 27
d. 5.73
e. none

Box Plots

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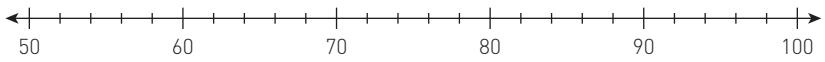
Overview

One way to display a distribution of one-variable numerical data is with a box plot. A box plot is the only display of data that clearly shows the median, quartiles, range, and outliers of a data set.

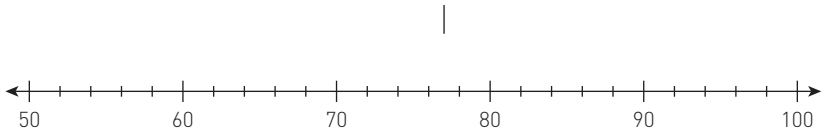
Examples

1. Display this data in a box plot: 51, 55, 55, 62, 65, 72, 76, 78, 79, 82, 83, 85, 91, and 93.

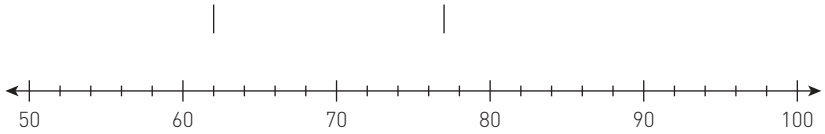
Since this data is already in order from least to greatest, the range is 93 – 51 = 42. A suitable number line could have equal intervals from 50 to 100.



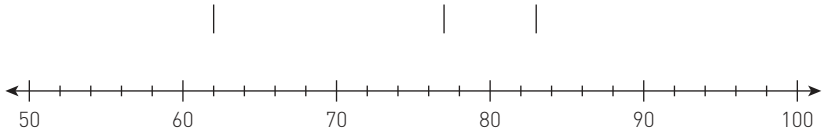
The median of the set of data is 77. Draw a vertical line segment at this value above the number line.



The median of the lower half of the data (the first quartile) is 62. Draw a vertical line segment at this value above the number line.



The median of the upper half of the data (the third quartile) is 83. Draw a vertical line segment at this value above the number line.



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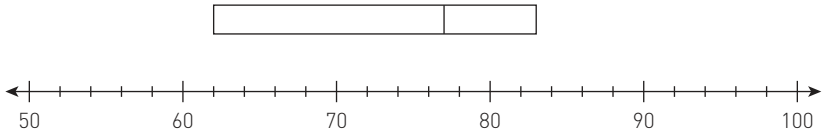
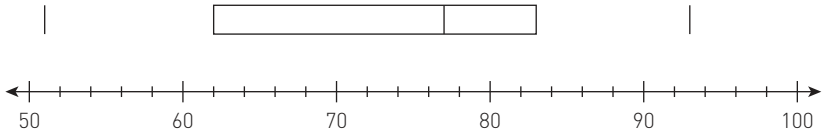
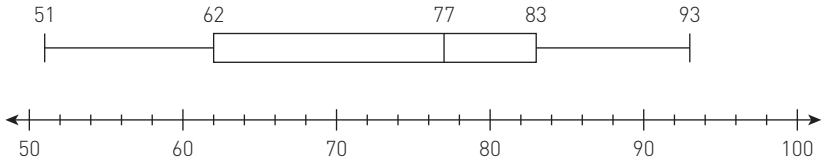
Draw horizontal lines connecting the first and third quartiles to form a box.



Place a vertical line segment at the minimum value (51) and one at the maximum value (93).



Use horizontal line segments to connect the minimum and maximum values to the box. Label values if needed.

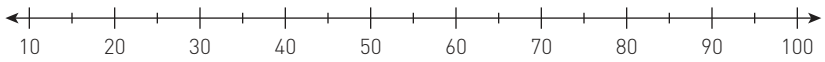


2. Display this data in a box plot: 62, 65, 93, 51, 12, 79, 85, 55, 72, 78, 83, 91, and 76.

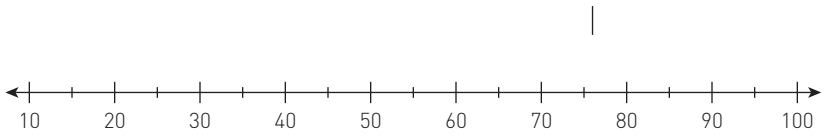
Place the data in order from least to greatest.

12, 51, 55, 62, 65, 72, 76, 78, 79, 83, 85, 91, 93

The range is 93 – 12 = 81. A suitable number line could have equal intervals from 10 to 100.



The median of the set of data is 76. Draw the line segment.



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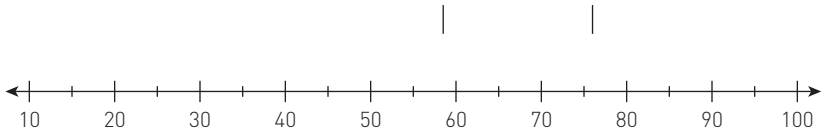
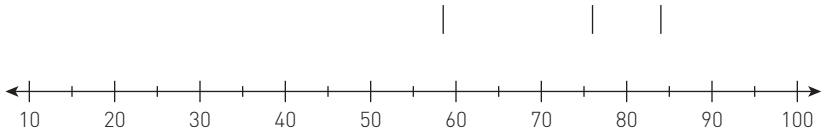
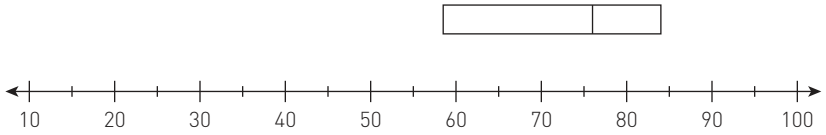
Since the lower half of the data has an even number of values, the first quartile is found by calculating the mean of the two middle values: 55 + 62 = 117; 117 ÷ 2 = 58.5. Draw the line segment.



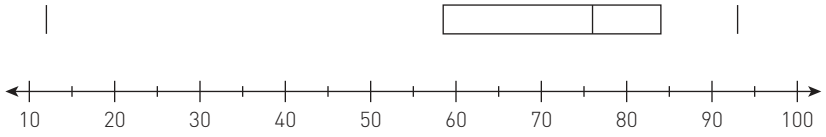
Since the upper half of the data also has an even number of values, the third quartile is found by calculating the mean of the two middle values: 83 + 85 = 168; 168 ÷ 2 = 84. Draw the line segment.



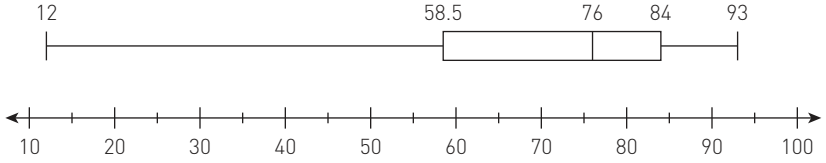
Draw horizontal lines connecting the first and third quartiles to form a box.



Place a vertical line segment at the minimum value (12) and one at the maximum value (93).



Use horizontal line segments to connect the minimum and maximum values to the box. Label values if needed.



Practice

Create a box plot for each set of data.

1. 45, 47, 52, 85, 46, 32, 83, 80, 75

2. 75, 62, 56, 80, 72, 55, 54, 80

3. 49, 54, 52, 58, 61, 72, 73, 78, 73, 82, 83, 73, 61, 67, 68

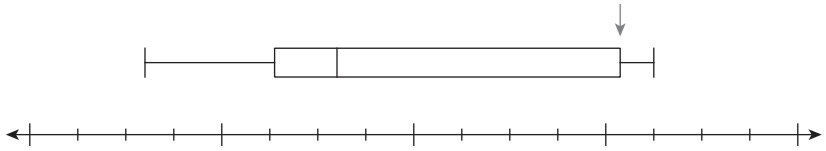
4. 65, 35, 48, 29, 57, 87, 94, 68, 86, 73, 58, 74, 85, 91, 88, 97

5. 265, 263, 269, 259, 267, 264, 253, 275, 264, 260, 273, 257, 291

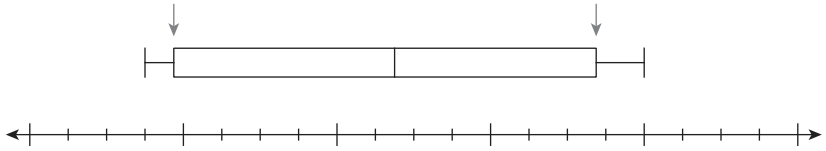
6. 48, 42, 37, 29, 49, 46, 38, 28, 45, 45, 35, 46.25, 34, 46, 46.5, 43, 46.5, 48, 41.25, 29, 47.75

Answers

1.



2.



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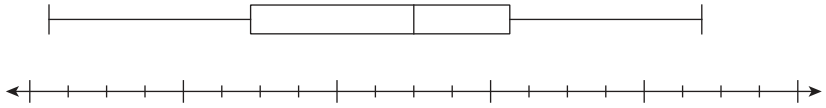

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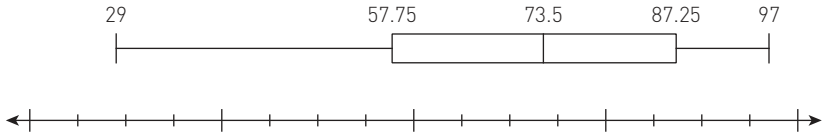

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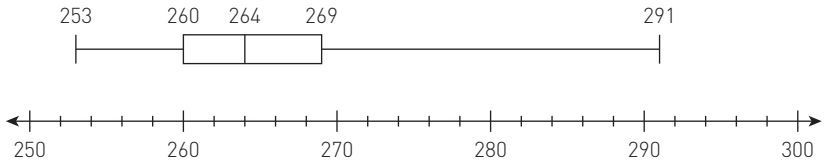

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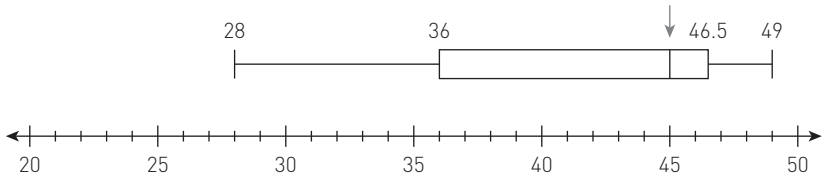

Distance, Rate, and Time

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Overview

Distance (d) equals the product of the rate of travel (r) and the time (t). This relationship is shown in three forms:

$$d = r \cdot t \qquad r = \frac{d}{t} \qquad t = \frac{d}{r}$$

It is important that the units of measure are consistent. For additional information, see the Methods & Meanings box in Lesson 8.2.2 of the *Core Connections Course 1* text.

Examples

1. Determine the rate of travel or speed of a passenger car if the distance traveled is 572 miles in 11 hours.

Since the problem asks for the rate of travel, r, the following equation can be used to solve for the rate: $r = \frac{d}{t}$.

$$r = \frac{572 \text{ miles}}{11 \text{ hours}}$$

$$r = \frac{52 \text{ miles}}{\text{hour}} \text{ or } 52 \text{ mph}$$

2. Determine the distance traveled by a train traveling at 135 miles per hour for 40 minutes.

The units of time are not the same, so one must be converted to match the other. 40 minutes can be converted into hours.

$$\frac{40}{60} = \frac{2}{3} \text{ hour}$$

Then, the relationship $d = r \cdot t$ can be used to determine the distance traveled.

$$d = \left(\frac{135 \text{ miles}}{\text{hour}}\right)\left(\frac{2}{3} \text{ hour}\right)$$

$$d = 90 \text{ miles}$$

3. The Central Middle School hamster race is fast approaching. Fred said that his hamster traveled 60 feet in 90 seconds.

a. Determine the unit rate (in feet per second) for Fred's hamster.

$\text{rate} = \frac{\text{distance}}{\text{time}} = \frac{60 \text{ feet}}{90 \text{ seconds}} = \frac{2}{3}$ ft/sec

b. Write an equation to represent how far Fred's hamster will travel, in d feet, after t seconds.

$d = \frac{2}{3}t$

c. How far will the hamster travel in 120 seconds?

$d = \frac{2}{3}(120) = 80$ feet

Practice

Solve the following problems.

1. A bus travels 5 miles in $\frac{1}{11}$ hours. Calculate the unit rate and write an equation to represent how far the bus will travel, in d miles, after t hours.

2. A plane travels 1,250 miles in 2.5 hours. Calculate the unit rate and write an equation to represent how far the plane will travel, in d miles, after t hours.

3. Calculate the time if the distance is 157.5 miles and the speed is 63 mph.

4. Calculate the distance if the speed is 67 mph and the time is 3.5 hours.

5. Calculate the rate if the distance is 247 miles and the time is 3.8 hours.

6. Calculate the distance if the speed is 60 mph and the time is 1 hour and 45 minutes.

7. Calculate the rate in miles per hour if the distance is 3.5 miles and the time is 0.5 hours.

8. Calculate the time in hours if the distance is 2 miles and the rate is 30 miles per hour.

Answers

1. 55 mph, $d = 55t$ **2.** 500 mph, $d = 500t$ **3.** 2.5 hours **4.** 234.5 miles

5. 65 mph **6.** 105 miles **7.** 7 mph **8.** $\frac{1}{15}$ hours

Converting Units

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Overview

Units of measurement define and describe quantities. They have been standardized to maintain consistency across measurements of the same types of quantities. For example, "1 centimeter" represents one specific length and is the same regardless of what you are measuring or where you are in the world.

Converting from one unit of measurement to another is sometimes necessary. Here are some common units of measurement and their relationships.

LENGTH

Within Same System	U.S. Customary Units to Metric
1 foot = 12 inches	1 inch = 2.54 centimeters
1 yard = 36 inches	1 foot = 30.48 centimeters
1 yard = 3 feet	1 foot = 0.3048 meters
1 mile = 5,280 feet	1 yard = 91.44 centimeters
1 centimeter = 10 millimeters	1 yard = 0.9144 meters
1 meter = 100 centimeters	1 mile = 1609.344 meters
1 kilometer = 1,000 meters	1 mile = 1.609344 kilometers

VOLUME

Within Same System	U.S. Customary Units to Metric
1 cup = 8 fluid ounces	1 ounce = 29.5735 milliliters
1 pint = 16 fluid ounces	1 pint = 0.473176 liters
1 quart = 2 pints	1 quart = 0.946353 liters
1 gallon = 4 quarts	1 gallon = 3.78541 liters
1 liter = 1,000 milliliters	

WEIGHT

Within Same System	U.S. Customary Units to Metric
1 pound = 16 ounces	1 ounce = 28.3495 grams
1 U.S. ton = 2,000 pounds	1 ounce = 0.0283495 kilograms
1 gram = 1,000 milligrams	1 pound = 0.453592 kilograms
1 kilogram = 1,000 grams	1 U.S. ton = 0.907185 metric tons
1 metric ton (tonne) = 1,000 kilograms	

TIME

Within Same System	U.S. Customary Units to Metric
1 minute = 60 seconds	1 week = 7 days
1 hour = 60 minutes	1 year = 365.2425 days (Gregorian calendar)
1 day = 24 hours	1 year ≈ 52.1775 weeks

Examples

1. Convert 15 yards to inches.

There are 3 feet in 1 yard and 12 inches in 1 foot.

$$15 \text{ yards} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} \cdot \frac{12 \text{ inches}}{1 \text{ foot}} = 540 \text{ inches}$$

2. Convert 7 ounces to grams.

There are 28.3495 grams in 1 ounce.

$$7 \text{ ounces} \cdot \frac{28.3495 \text{ grams}}{1 \text{ ounce}} = 198.4465 \text{ grams}$$

Practice

Complete the following unit conversions. Round to two decimal places when necessary.

1. Convert 4 kilometers to meters.

2. Convert 4 kilometers to centimeters.

3. Convert the number of centimeters from problem 2 to inches.

4. Convert 11 kilometers to centimeters.

5. Convert the number of centimeters from problem 4 to inches.

6. Convert 7 kilometers to inches.

7. Convert 6 years to weeks.

8. Convert 6 weeks to days.

9. Convert 6 ounces to pounds.

10. Convert 8 ounces to pounds.

11. Convert 8 fluid ounces to milliliters.

12. Convert 8 milliliters to liters.

13. Convert 22 gallons to liters.

14. Convert 22 quarts to liters.

15. Convert 22 kilograms to pounds.

16. Convert 660 ounces to grams.

17. Convert 660 days to weeks.

18. Convert 660 U.S. tons to metric tons.

Answers

1. 4,000 meters

2. 400,000 centimeters

3. 157,480.32 inches

4. 1,100,000 centimeters

5. 433,070.87 inches

6. 275,590.55 inches

7. about 312 weeks

8. 42 days

9. 0.375 pounds

10. 0.5 pounds

11. 236.59 milliliters

12. 0.008 liters

13. 83.28 liters

14. 20.82 liters

15. 48.50 pounds

16. 18,710.67 grams

17. 94.29 weeks

18. 598.74 metric tons