Areas of Quadrilaterals and Triangles

GUIDED SKILL SUPPORT

For additional skill support, see *Core Connections Course 1* Guided Skill Support, Chapter 9, for a review of this topic. For additional examples and practice, see the *Core Connections Course 2* Checkpoint 1 materials and problem 1-40.

Simple Probability

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Overview

The following terms are helpful when working with probability models.

- **Outcome**: Any possible or actual result of the action considered, such as rolling a specific number on a dice.
- **Event**: An outcome or group of outcomes from an experiment
- **Sample space**: All possible outcomes of a situation. For example, the sample space for rolling a standard dice has six possible outcomes (1, 2, 3, 4, 5, and 6).
- **Probability**: The likelihood that an event will occur. Probabilities may be written as fractions, decimals, or percents. An event that is certain to happen has a probability of 1 or 100%. An event that is impossible has a probability of 0 or 0%. Events that are possible but not guaranteed have probabilities between 0 and 1 or between 0% and 100%. Generally, the more likely an event is to happen, the greater its probability.
- **Experimental probability**: The probability based on data collected in experiments.

$$\text{Experimental probability} = \frac{\text{number of successful outcomes in the experiment}}{\text{total number of outcomes in the experiment}}$$

- **Theoretical probability**: A calculated probability based on the possible outcomes when they all have the same chance of occurring.

$$\text{Theoretical probability} = \frac{\text{number of possible successful outcomes (events)}}{\text{total number of possible outcomes}}$$

- **Success**: A desired or specified outcome.

Examples

1. If you roll a fair, 6-sided dice, what is $P(3)$? That is, what is the probability that you will roll a 3?

> Because the six sides are equally likely to come up and there is only one 3, $P(3) = \frac{1}{6}$.

2. There are 12 marbles in a bag: 2 clear, 4 green, 5 yellow, and 1 blue. If one marble is chosen randomly from the bag, what is the probability that it will be yellow?

> $P(\text{yellow}) = \frac{\text{number of yellow marbles}}{\text{number of marbles}} = \frac{5}{12}$

3. Joe flipped a coin 50 times. When he recorded his tosses, his result was 30 heads and 20 tails. Joe's activity provided data to calculate the experimental probability of flipping a coin.

a. What is the *theoretical* probability of Joe flipping heads?

> The theoretical probability is 50% or $\frac{1}{2}$, because there are only two possibilities (heads and tails), and each is equally likely to occur.

b. What was the *experimental* probability of flipping a coin and getting heads based on Joe's activity?

> The experimental probability is $\frac{30}{50}$, $\frac{3}{5}$, or 60%. These are the results Joe actually got when he flipped the coin.

4. Decide whether these statements describe theoretical or experimental probabilities.

a. The chance of rolling a 6 on a fair die is $\frac{1}{6}$.

> This probability is theoretical.

b. I rolled the dice 12 times, and 5 came up three times.

> This probability is experimental.

c. There are 15 marbles in a bag: 5 blue, 6 yellow, and 4 green. The probability of getting a blue marble is $\frac{1}{3}$.

> This probability is theoretical.

d. When Veronika pulled three marbles out of the bag, she got 2 yellow and 1 blue, or $\frac{2}{3}$ yellow, $\frac{1}{3}$ blue.

> This probability is experimental.

Practice

1. There are 24 crayons in a box: 5 black, 3 white, 7 red, 2 yellow, 3 blue, and 4 green. What is the probability of randomly choosing a green crayon? Is this an experimental or theoretical probability?

2. A spinner is divided into four equal sections numbered 2, 4, 6, and 8. What is the probability of spinning an 8?

3. A fair dice marked 1, 2, 3, 4, 5, and 6 is rolled. Tyler tossed the dice 40 times and noted that 26 times, an even number showed. What is the experimental probability that an even number will be rolled? What is the theoretical probability?

4. Sara is at a picnic and reaches into an ice chest, without looking, to grab a drink can. If there are 14 cans of orange soda, 12 cans of fruit punch, and 10 cans of cola, what is the probability that she will grab a can of fruit punch? Is this an experimental probability or a theoretical one?

5. In baseball, a batting average is the probability that a baseball player will hit the ball when batting. If a baseball player has a batting average of 266, it means the player's probability of getting a hit is 0.266. Is a batting average an experimental probability or a theoretical probability?

6. In 2011, 26 people died by being struck by lightning, and 187 people were injured. There were about 314,000,000 people in the United States. What was the probability of being one of the people struck by lightning?

7. In a medical study, 107 people were given a new vitamin pill. If a participant got sick, they were removed from the study. 10 of the participants caught a common cold, 2 came down with the flu, 18 got sick to their stomach, and 77 never got sick. What was the probability of a participant getting sick during this study? Is this an experimental probability or theoretical?

8. Insurance companies use probabilities to determine the rate they will charge for an insurance policy. In a study of 300 people who had life insurance policies, 111 people were over 80 years old when they died, 82 people died when they were between 70 and 80 years old, 52 died between 60 and 70 years old, and 55 died when they were younger than 60 years old. In this study, what was the probability of dying younger than 70 years old? Is this an experimental probability or a theoretical one?

Answers

1. $\frac{4}{24} = \frac{1}{6}$; theoretical

2. $\frac{1}{4}$; theoretical

3. experimental = $\frac{26}{40} = \frac{13}{20}$; theoretical = $\frac{20}{40} = \frac{1}{2}$

4. $\frac{12}{36} = \frac{1}{3}$; theoretical

5. experimental

6. $\frac{213}{314{,}000{,}000} \approx 0.00000068$; experimental

7. $\frac{10 + 2 + 18}{107} \approx 0.28$; experimental

8. $\frac{55 + 52}{300} \approx 35.7\%$; experimental

Measures of Central Tendency

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Overview

Measures of central tendency are numbers that locate or approximate the center of a set of data—that is, a "typical" value that describes the set of data. The mean and the median are the most common measures of center.

The mean is the arithmetic average of a data set. To calculate the mean, add all the values in a set and divide this sum by the number of values in the set.

The median is the middle number in a set of data arranged in numerical order. If there is an even number of values, the median is the mean (average) of the two middle numbers.

For additional information, see the Methods & Meanings boxes in Lessons 1.1.3 and 1.1.4 of the *Core Connections Course 2* text.

Examples

1. Calculate the *mean* of this set of data: 34, 31, 37, 44, 38, 34, 42, 34, 43, and 41.

$$34 + 31 + 37 + 44 + 38 + 34 + 42 + 34 + 43 + 41 = 378$$

$$\frac{378}{10} = 37.8$$

The mean of this set of data is 37.8.

2. Calculate the *median* of this set of data: 34, 31, 37, 44, 38, 34, 43, and 41.

Arrange the data in order: 31, 34, 34, 37, 38, 41, 43, 44.

Determine the middle value(s): 37 and 38.

Since there are two middle values, calculate their mean: 37 + 38 = 75, then $\frac{75}{2}$ = 37.5. The median of this data set is 37.5.

Practice

Calculate the mean of each set of data.

1. 29, 28, 34, 30, 33, 26, 34

2. 25, 34, 35, 27, 31, 30

3. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, 123

4. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, 91

Calculate the median of each set of data.

5. 29, 28, 34, 30, 33, 26, 34

6. 25, 34, 27, 25, 31, 30

7. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, 123

8. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, 91

Answers

1. ≈ 30.57 **2.** $30.\overline{3}$ **3.** $86.\overline{13}$ **4.** $106.\overline{6}$

5. 30 **6.** 28.5 **7.** 82 **8.** 107

Choosing a Scale

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Overview

The axis (or axes) of a graph must be marked with equal-sized spaces called intervals. Marking the uniform intervals on the axes is called scaling the axes. The difference between consecutive markings tells the size (scale) of each interval. Note that each axis of a two-dimensional graph may use a different scale.

Sometimes, the axis or set of axes is not provided. Follow these steps to scale each axis of a graph.

1. Calculate the difference between the smallest and largest numbers (the range) you need to display on an axis.
2. Count the number of intervals (spaces) you have on your axis.
3. Divide the range by the number of intervals to determine the interval size.
4. Label the marks on the axis by adding the interval size to the smallest number in the range and repeating for each interval.

Sometimes, dividing the range by the number of intervals produces an interval size that makes it difficult to interpret the location of points on the graph. The interval may need to be rounded by sizing up (always up, if rounded at all) to a number that is convenient to use. Interval sizes such as 1, 2, 5, 10, 20, 25, 50, 100, etc., work well.

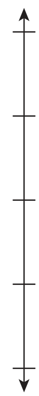
For additional information, see the Methods & Meanings box in Lesson 1.2.2 of the *Core Connections Course 2* text.

Examples

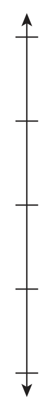
1.

1. The difference between 0 and 60 is 60.
2. The number line is divided into 5 equal intervals.
3. 60 divided by 5 is 12.
4. The marks are labeled with multiples of the interval size, 12.

2.



1. The difference between 300 and 0 is 300.
2. There are 4 intervals.
3. 300 ÷ 4 = 75
4. The axis is labeled with multiples of 75.



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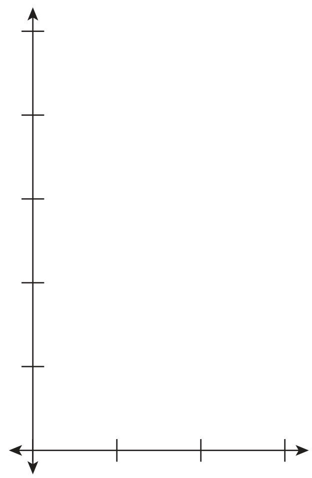
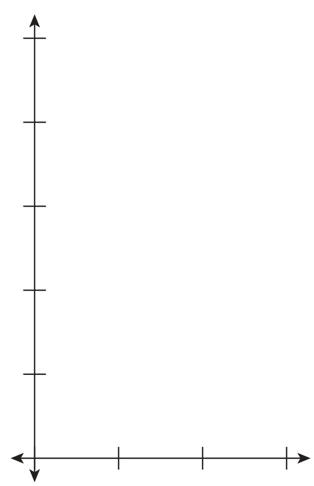


1. The difference on the vertical y-axis is $750 - 0 = 750$. (The origin is (0, 0).) The difference on the horizontal x-axis is $6 - 0 = 6$.
2. There are 5 spaces on the y-axis and 3 spaces on the x-axis.
3. The vertical interval size is $750 \div 5 = 150$. The horizontal interval size is $6 \div 3 = 2$.
4. The y-axis is labeled with multiples of 150, and the x-axis is labeled with multiples of 2.



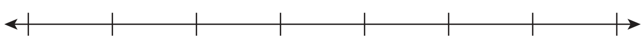
4. 

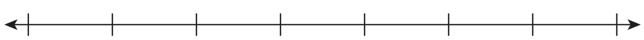
Sometimes, the axes extend in the negative direction.

1. The range is 20 − (−15) = 35.
2. There are 7 intervals along the line.
3. 35 ÷ 7 = 5
4. Label the axes with multiples of 5.

Practice

Complete the scale for each axis.

1. 

2. 

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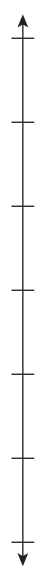
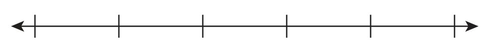
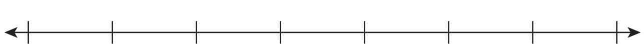
4. 

5. 

6. 

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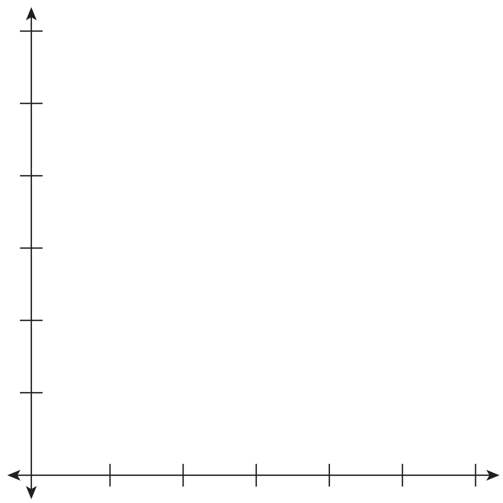

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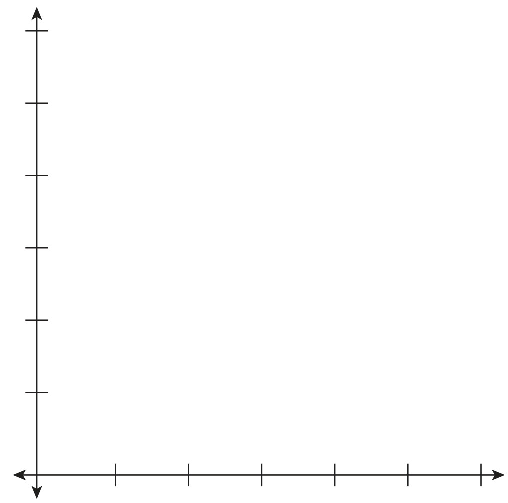

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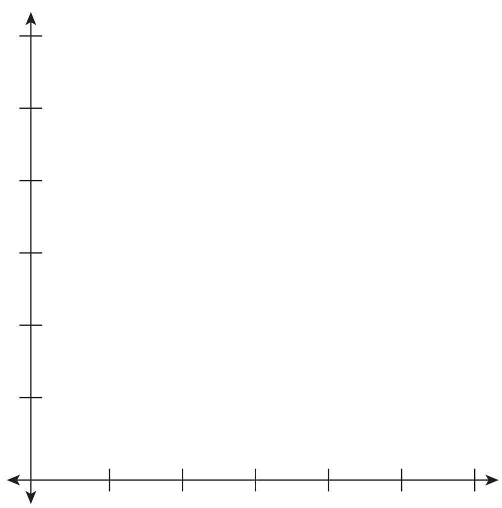

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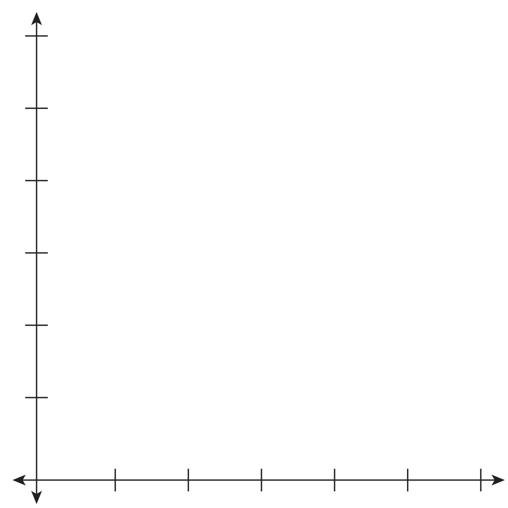
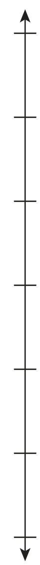


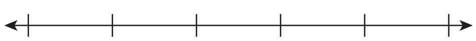
Answers

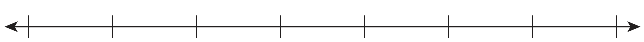
1. 

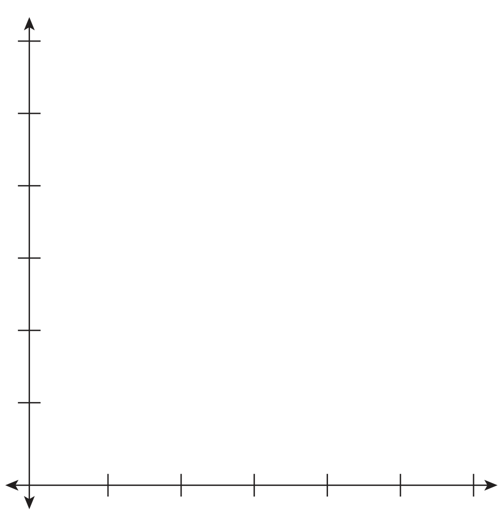
2. 

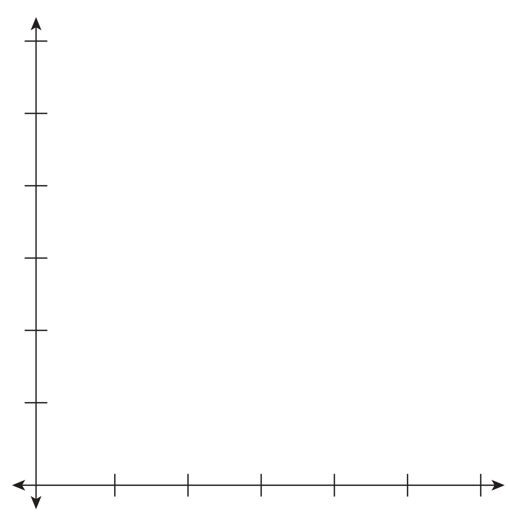
3. 

4. 

5. 

6. 

7. 

8. 

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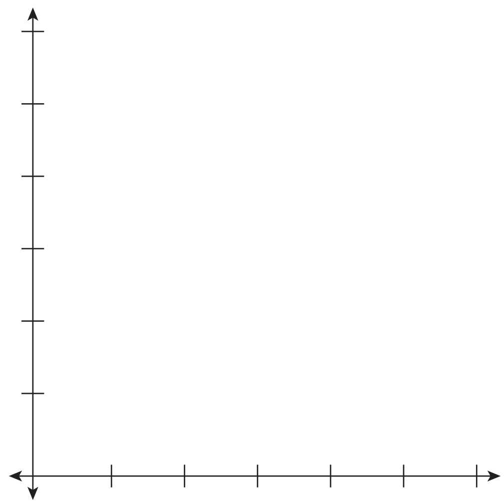

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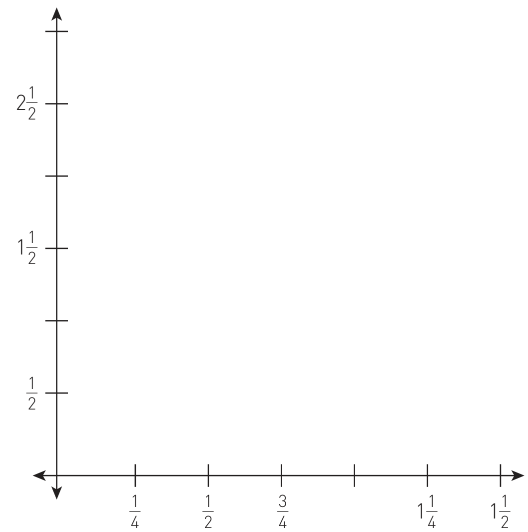
$2\frac{1}{2}$
$1\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{4}$
$\frac{1}{2}$
$\frac{3}{4}$
$1\frac{1}{4}$
$1\frac{1}{2}$

Equivalent Fractions

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Overview

Fractions that represent the same value are called equivalent fractions, such as $\frac{2}{3}$ and $\frac{6}{9}$. One method for writing equivalent fractions is to use the Multiplicative Identity, that is, multiplying the given fraction by a form of the number 1, such as $\frac{2}{2}$, $\frac{5}{5}$, etc. In this course, a fraction like this is called a "Giant One." Multiplying by 1 does not change the value of a number; this is stated in the Identity Property of Multiplication.

For additional information, see the Methods & Meanings boxes in Lessons 1.2.5 and 1.2.8 of the *Core Connections Course 2* text.

Examples

1. Write three equivalent fractions for $\frac{1}{2}$.

$$\frac{1}{2} \cdot \frac{2}{2} = \frac{2}{4}$$

$$\frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6}$$

$$\frac{1}{2} \cdot \frac{4}{4} = \frac{4}{8}$$

2. Use the Giant One to write an equivalent fraction to $\frac{7}{12}$ that has a denominator of 96. Which Giant One do you use?

$$\frac{7}{12} \cdot \frac{\ }{\ } = \frac{?}{96}$$

Since $\frac{96}{12} = 8$, the Giant One is $\frac{8}{8}$.

$$\frac{7}{12} \cdot \frac{8}{8} = \frac{56}{96}$$

Practice

Use the Giant One to write the specified equivalent fraction. Your answer should include the Giant One you use and the equivalent numerator.

1. $\frac{4}{3} \cdot \frac{\ }{\ } = \frac{\ }{15}$

2. $\frac{5}{9} \cdot \frac{\ }{\ } = \frac{\ }{36}$

3. $\frac{9}{2} \cdot \frac{\ }{\ } = \frac{\ }{38}$

4. $\frac{3}{7} \cdot \frac{\ }{\ } = \frac{\ }{28}$

5. $\frac{5}{3} \cdot \frac{\ }{\ } = \frac{\ }{18}$

6. $\frac{6}{5} \cdot \frac{\ }{\ } = \frac{\ }{15}$

Answers

1. $\frac{4}{3}\cdot\frac{5}{5}=\frac{20}{15}$

2. $\frac{5}{9}\cdot\frac{4}{4}=\frac{20}{36}$

3. $\frac{9}{2}\cdot\frac{19}{19}=\frac{171}{38}$

4. $\frac{3}{7}\cdot\frac{4}{4}=\frac{12}{28}$

5. $\frac{5}{3}\cdot\frac{6}{6}=\frac{30}{18}$

6. $\frac{6}{5}\cdot\frac{3}{3}=\frac{18}{15}$

Adding and Subtracting Fractions

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Overview

To add or subtract two fractions that are written with the same denominator (the number on the bottom), simply add or subtract the numerators (the numbers on the top) and keep the denominator the same. For example, $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$.

If the fractions have different denominators, rewrite them first as fractions with the same denominator. Two methods for adding and subtracting two fractions with different denominators are using an area model and using the Multiplicative Identity (Giant One).

AREA MODEL METHOD

Fractions can be modeled as shaded portions of a rectangle. The area model method involves creating common denominators by using rectangles to represent each fraction by dividing the rectangles into equal portions and shading them.

The steps of the area model method are:

1. Model each fraction using a shaded rectangle. Cut one rectangle vertically into an equal number of pieces based on the first denominator (bottom). Cut the other rectangle horizontally, using the second denominator. Shade a number of pieces in each rectangle to represent the numerator (top number). Label each rectangle with the fraction it represents.
2. Superimpose the lines from each rectangle onto the other rectangle, as if one rectangle is placed on top of the other one. Rewrite the fractions by changing the numerators to match the number of shaded portions in each figure and changing the denominators to match the new number of total equal parts.
3. Add or subtract the equivalent parts to determine the sum or difference.

IDENTITY PROPERTY OF MULTIPLICATION (GIANT ONE) METHOD

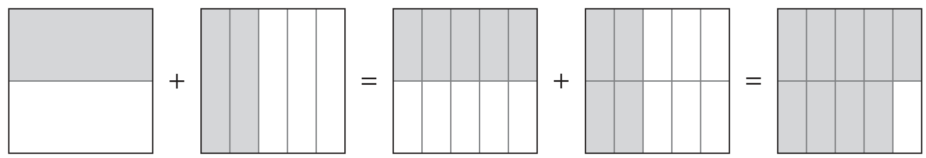
The Giant One, a representation of the number 1, is known in mathematics as the Multiplicative Identity. It is a fraction with the same numerator and denominator ($\frac{3}{3}$, for example). Multiplying by a Giant One can create common denominators.

The steps of the Giant One method are:

1. Rewrite the fractions as equivalent fractions that have a common denominator.
2. Add or subtract only the numerators, keeping the common denominator.
3. Rewrite in the lowest terms possible.

Examples

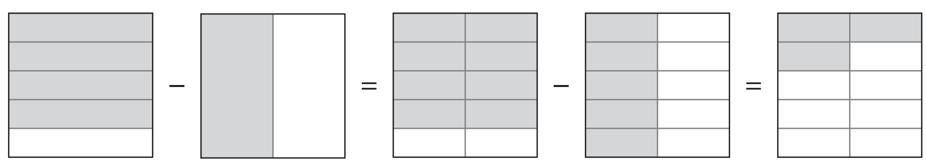
1. Add $\frac{1}{2}+\frac{2}{5}$ using an area model.



$$\frac{1}{2}+\frac{2}{5}=\frac{5}{10}+\frac{4}{10}=\frac{9}{10}$$

2. Use an area model to calculate the difference.

$$\frac{4}{5}-\frac{1}{2}$$



$$\frac{4}{5}-\frac{1}{2}=\frac{8}{10}-\frac{5}{10}=\frac{3}{10}$$

3. Use the Giant One method to calculate the sum.

$$\frac{1}{3}+\frac{1}{4}$$

Multiply both $\frac{1}{3}$ and $\frac{1}{4}$ by Giant Ones to get a common denominator.

$$\frac{1}{3}\cdot\frac{4}{4}+\frac{1}{4}\cdot\frac{3}{3}=\frac{4}{12}+\frac{3}{12}$$

Add the numerators of both fractions to get the sum.

$$\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$$

4. Use the Giant One method to calculate the difference.

$$\frac{5}{6}-\frac{1}{4}$$

Multiply both $\frac{5}{6}$ and $\frac{1}{4}$ by Giant Ones to get a common denominator.

$$\frac{5}{6}\cdot\frac{2}{2}-\frac{1}{4}\cdot\frac{3}{3}=\frac{10}{12}-\frac{3}{12}$$

Subtract the numerators of both fractions to get the answer.

$$\frac{10}{12}-\frac{3}{12}=\frac{7}{12}$$

Practice

Use the area model method to add or subtract the following fractions.

1. $\frac{3}{4}+\frac{1}{5}$ **2.** $\frac{1}{3}+\frac{2}{7}$ **3.** $\frac{3}{4}-\frac{2}{3}$

Use the Giant One to add or subtract the following fractions.

4. $\frac{1}{3}+\frac{2}{5}$ **5.** $\frac{4}{5}-\frac{1}{3}$ **6.** $\frac{3}{4}-\frac{1}{5}$

Determine each sum or difference. Use the method of your choice.

7. $\frac{1}{6}+\frac{2}{3}$ **8.** $\frac{3}{8}+\frac{2}{5}$ **9.** $\frac{1}{4}+\frac{3}{7}$ **10.** $\frac{2}{9}+\frac{3}{4}$

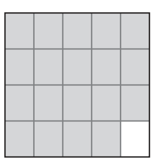
11. $\frac{5}{12}+\frac{1}{3}$ **12.** $\frac{7}{9}-\frac{2}{3}$ **13.** $\frac{3}{4}+\frac{1}{3}$ **14.** $\frac{5}{6}+\frac{2}{3}$

15. $\frac{7}{8}+\frac{1}{4}$ **16.** $\frac{6}{7}-\frac{2}{3}$ **17.** $\frac{1}{3}-\frac{1}{4}$ **18.** $\frac{3}{5}+\frac{3}{4}$

19. $\frac{5}{5}-\frac{3}{4}$ **20.** $\frac{3}{4}-\frac{1}{3}$ **21.** $\frac{2}{5}+\frac{9}{15}$ **22.** $\frac{2}{3}-\frac{3}{5}$

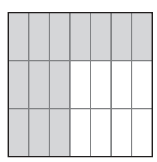
Answers

1.



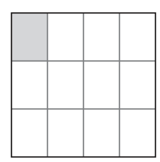
$\frac{19}{20}$

2.



$\frac{13}{21}$

3.



$\frac{1}{12}$

4. $\frac{11}{15}$ **5.** $\frac{7}{15}$ **6.** $\frac{11}{20}$

7. $\frac{5}{6}$ **8.** $\frac{31}{40}$ **9.** $\frac{19}{28}$ **10.** $\frac{35}{36}$

11. $\frac{9}{12}=\frac{3}{4}$ **12.** $\frac{1}{9}$ **13.** $\frac{13}{12}=1\frac{1}{12}$ **14.** $\frac{9}{6}=\frac{3}{2}=1\frac{1}{2}$

15. $\frac{9}{8}=1\frac{1}{8}$ **16.** $\frac{4}{21}$ **17.** $\frac{1}{12}$ **18.** $\frac{27}{20}=1\frac{7}{20}$

19. $\frac{1}{4}$ **20.** $\frac{5}{12}$ **21.** $\frac{15}{15}=1$ **22.** $\frac{1}{15}$

