Order of Operations

GUIDED SKILL SUPPORT

Overview

Mathematicians decided on a method to evaluate an expression that uses more than one operation so that everyone can agree on the answer. Otherwise, for an expression such as $3 + 4 \cdot 2$, some people may think the answer is 14 while others think the answer is 11.

The Order of Operations is a set of rules to follow that provides a consistent way for everyone to evaluate expressions. These rules state the order in which the mathematical operations are to be completed to arrive at the correct answer.

The first step is to organize the numerical expression into parts called terms, which are single numbers or products of numbers. A numerical expression is made up of a sum or difference of terms. Examples of numerical terms are 4, $3(6)$, $6(9-4)$, $2 \cdot 3^2$, $3(5+2^3)$, and $\frac{16-4}{6}$. For the expression $3 + 4 \cdot 2$, the terms can be circled as shown.

$$\boxed{3} + \boxed{4 \cdot 2}$$

Each term is evaluated separately, giving $3 + 8$. Then the terms are added: $3 + 8 = 11$. Thus, $3 + 4 \cdot 2 = 11$, and not 14, when the Order of Operations is followed.

For additional information, see the Methods & Meanings box in Lesson 3.1.2 of the *Core Connections Course 2* text. For additional examples and practice, see the *Core Connections Course 2* Checkpoint 5 materials.

Examples

1. Evaluate the following expression.

$$2 \cdot 3^2 + 3(6-3) + 10$$

$2 \cdot 3^2 + 3(6-3) + 10$	
$= \boxed{2 \cdot 3^2} + \boxed{3(6-3)} + \boxed{10}$	Circle each term in the expression.
$= \boxed{2 \cdot 3^2} + \boxed{3(3)} + \boxed{10}$	Evaluate the expressions within the parentheses.
$= \boxed{2 \cdot 9} + \boxed{3(3)} + \boxed{10}$	Evaluate each exponential part (e.g., 3^2).
$= \boxed{18} + \boxed{9} + \boxed{10}$	Multiply and divide from left to right.
$= 27 + 10$	Finally, combine terms by adding or subtracting from left to right.
$= 37$	

2. Evaluate the following expression.

$$5 - 8 \div 2^2 + 6(5 + 4) - 5^2$$

$5 - 8 \div 2^2 + 6(5 + 4) - 5^2$	
$= \boxed{5} - \boxed{8 \div 2^2} + \boxed{6(5 + 4)} - \boxed{5^2}$	Circle the terms.
$= \boxed{5} - \boxed{8 \div 2^2} + \boxed{6(9)} - \boxed{5^2}$	Evaluate inside the parentheses.
$= \boxed{5} - \boxed{8 \div 4} + \boxed{6(9)} - \boxed{25}$	Evaluate the exponents.
$= 5 - 2 + 54 - 25$	Multiply and divide from left to right.
$= 32$	Finally, add and subtract from left to right.

3. Evaluate the following expression.

$$20 + \frac{5 + 7}{3} - 4^2 + 12 \div 4$$

$20 + \frac{5 + 7}{3} - 4^2 + 12 \div 4$	
$= \boxed{20} + \boxed{\frac{5 + 7}{3}} - \boxed{4^2} + \boxed{12 \div 4}$	Circle the terms.
$= 20 + 4 - 16 + 3$	Multiply and divide from left to right, including exponents.
$= 11$	Add or subtract from left to right.

Practice

1. $5 \cdot 3 + 4$

2. $10 \div 5 + 3$

3. $2(9 - 4) \cdot 7$

4. $6(7 + 3) + 8 \div 2$

5. $15 \div 3 + 7(8 + 1) - 6$

6. $\frac{9}{3} + 5 \cdot 3^2 - 2(14 - 5)$

7. $\frac{20}{6 + 4} + 7 \cdot 2 \div 2$

8. $\frac{5 + 30}{7} + 6^2 - 18 \div 9$

9. $2^3 + 8 - 16 \div 8 \cdot 2$

10. $25 - 5^2 + 9 - 3^2$

11. $5(17 - 7) + 4 \cdot 3 - 8$

12. $(5 - 2)^2 + (9 + 1)^2$

13. $4^2 + 9(2) \div 6 + (6 - 1)^2$

14. $\frac{18}{3^2} + \frac{5 \cdot 3}{5}$

15. $3(7 - 2)^2 + 8 \div 4 - 6 \cdot 5$

16. $14 \div 2 + 6 \cdot 8 \div 2 - (9 - 3)^2$

17. $\frac{27}{3} + 18 - 9 \div 3 - (3 + 4)^2$

18. $26 \cdot 2 \div 4 - (6 + 4)^2 + 3(5 - 2)^3$

19. $\left(\frac{42 + 3}{5}\right)^2 + 3^2 - (5 \cdot 2)^2$

Answers

1.	19	**2.**	5	**3.**	70	**4.**	64
5.	62	**6.**	30	**7.**	9	**8.**	39
9.	12	**10.**	0	**11.**	54	**12.**	109
13.	44	**14.**	5	**15.**	47	**16.**	–5
17.	–25	**18.**	–6	**19.**	–10		

Subtracting Integers

GUIDED SKILL SUPPORT

Overview

Subtraction of integers can be represented using the concrete models of integer tiles and number lines. Subtraction is the opposite of addition, so it makes sense to do the opposite actions of addition.

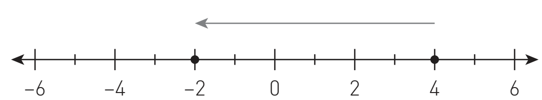
To model addition using integer tiles, start with a diagram of the first integer, add the second integer to the diagram, eliminate zeros, and then record what is left. One method of subtracting integers is to do the same, except that instead of adding the second integer, you remove the second integer. Sometimes this removal will require adding extra zeros to the diagram first.

When using the number line, adding a positive integer moves to the right, so subtracting a positive integer moves to the left. Adding a negative integer moves to the left, so subtracting a negative integer moves to the right.

For additional information, see the Methods & Meanings box in Lesson 3.2.3 of the *Core Connections Course 2* text.

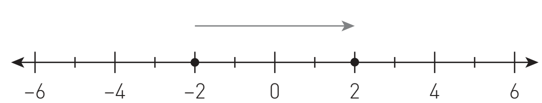
Examples

1. $4 - 6$



$4 - 6 = -2$

2. $-2 - (-4)$



$-2 - (-4) = 2$

3. −6 − (−3)

−6 − (−3)

↓ Build the first integer.

− − − − − −

↓ Remove three negatives.

− − − − − −

↓ Three negatives are left.

−6 − (−3) = −3

4. −2 − 3

−2 − 3

↓ Build the first integer.

− −

↓ It is not possible to remove three positives, so add some zero pairs.

\+ + + +
− − − − − −

↓ Now remove three positives and circle any zero pairs.

\+ + + +
− − − − − −

↓ Five negatives remain.

−2 − 3 = −5

Practice

Calculate each difference. Use one of the models for at least the first five differences.

1. −6 − (−2)

2. 2 − (−3)

3. 6 − (−3)

4. 3 − 7

5. 7 − (−3)

6. 7 − 3

7. −5 − 3

8. −12 − (−10)

9. −12 − 10

10. 12 − (−10)

11. −6 − (−3) − 5

12. 6 − (−3) − 5

13. 8 − (−8)

14. −9 − 9

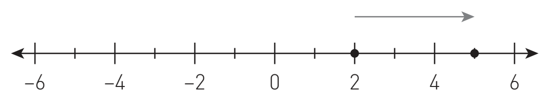
15. −9 − 9 − (−9)

Answers

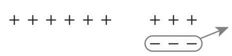
Models may vary.

1. −4

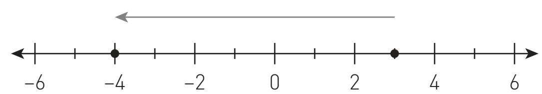
2. 5



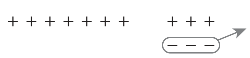
3. 9



4. −4



5. 10



6. 4

7. −8

8. −2

9. −2

10. 22

11. −8

12. 4

13. 16

14. −18

15. −9

Connecting Addition and Subtraction

GUIDED SKILL SUPPORT

Overview

Another method for subtracting integers is to notice the relationship between addition problems and subtraction problems, as shown:

$$3 - 2 = 1 \text{ and } 3 + (-2) = 1$$

$$-3 - (-2) = -1 \text{ and } -3 + 2 = -1$$

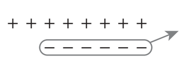
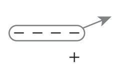
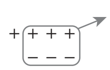
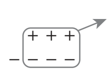
These relationships happen because removing a negative amount gives the same result as adding the same positive amount and vice versa. Subtracting two integers is the same as adding the first integer and the *opposite* (more formally, the additive inverse) of the second integer.

That is, $a - b = a + (-b)$, for any rational numbers a and b.

For additional information, see the Methods & Meanings box in Lesson 3.2.4 of the *Core Connections Course 2* text.

Examples

1. For parts (a) through (c), compare and contrast each pair of expressions, the corresponding integer tile diagrams, and the completed equations. What do you notice?

a. $2 - (-6)$		$2 - (-6) = 8$
$2 + 6$	+ + + + + + + +	$2 + 6 = 8$
b. $-3 - (-4)$		$-3 - (-4) = 1$
$-3 + 4$		$-3 + 4 = 1$
c. $-4 - (-3)$		$-3 - (-4) = 1$
$-3 + 4$		$-3 + 4 = 1$

For each pair of expressions, one is a subtraction problem, and the other is an addition problem. The answers to each pair of examples are the same. Also, notice that the second integers in the pairs are opposites (that is, they are the same distance from zero on opposite sides of the number line), while the first integers in each pair are the same. For example, in part (a), the first expression has (−6) after the operation (subtraction), while the second expression has 6 after the operation (addition).

2. Rewrite each subtraction expression as addition of the opposite and then determine the sum.

a. $9 - (-12)$

$9 + (+12) = 21$

b. $-9 - (-12)$

$-9 + (+12) = 3$

c. $-9 - 12$

$-9 + (-12) = -21$

d. $9 - 12$

$9 + (-12) = -3$

Practice

Calculate each difference.

1. $9 - (-3)$

2. $9 - 3$

3. $-9 - 3$

4. $-9 - (-3)$

5. $-14 - 15$

6. $-16 - (-15)$

7. $-40 - 62$

8. $-40 - (-62)$

9. $40 - 62$

10. $40 - (-62)$

11. $-5 - (-3) - 5 - 6$

12. $-5 - 3 - (-5) - (-6)$

13. $5 - 3 - (-5) - 6$

14. $5 - (-4) - 6 - (-7)$

15. $-125 - (-125) - 125$

16. $5 - (-6)$

17. $12 - 14$

18. $20 - 25$

19. $-3 - 2$

20. $-7 - 3$

21. $-10 - 5$

22. $-30 - 7$

23. $-3 - (-3)$

24. $-3 - (-4)$

25. $10 - (-3)$

26. $5 - (-9)$

27. $27 - (-7)$

28. $15 - 32$

29. $-58 - 37$

30. $-79 - (-32)$

31. $-62 - 81$

32. $-106 - 242$

33. $47 - (-55)$

34. $257 - 349$

35. $-1{,}010 - (-1{,}010)$

Answers

1.	12	**2.**	6	**3.**	−12
4.	−6	**5.**	−29	**6.**	−1
7.	−102	**8.**	22	**9.**	−22
10.	102	**11.**	−13	**12.**	3
13.	1	**14.**	10	**15.**	−125
16.	11	**17.**	−2	**18.**	−5
19.	−5	**20.**	−10	**21.**	−15
22.	−37	**23.**	0	**24.**	1
25.	13	**26.**	14	**27.**	34
28.	−17	**29.**	−95	**30.**	−47
31.	−143	**32.**	−348	**33.**	102
34.	−92	**35.**	0		

Arithmetic Operations With Decimals

GUIDED SKILL SUPPORT

See Guided Skill Support Chapter 2 for a review of this topic.

Multiplying Decimals and Percents

GUIDED SKILL SUPPORT

Overview

When multiplying multi-digit decimals, it is important to pay attention to place value. The process is similar to multiplying whole numbers. The number of decimal places in the product is equal to the total number of decimal places in the factors (the numbers you multiplied). Sometimes zeros need to be added to place the decimal point. When computing "a percent of a number," the percent can be changed to a decimal; then determine the product of the two numbers. Multiple methods for multiplying decimals are shown in the examples.

For additional information, see the Methods & Meanings box in Lesson 3.3.2 of the *Core Connections Course 2* text.

Examples

1. Multiply (0.2) · (0.3).

In fractions, this means $\frac{2}{10} \cdot \frac{3}{10} = \frac{6}{100}$.

Without using fractions, the place value of the numbers being multiplied indicates how many places to move the decimal point (to the left).

(tenths)(tenths) = hundredths

Therefore, move two places.

$$\begin{array}{r} 0.2 \\ \times\ 0.3 \\ \hline 0.06 \end{array}$$

2. Multiply (1.7) · (0.03).

In fractions, this means $\frac{17}{10} \cdot \frac{3}{100} = \frac{51}{1{,}000}$.

Knowing that the answer must be in the thousandth place tells you how many places to move the decimal point (to the left) without using the fractions.

(tenths)(hundredths) = thousandths

Therefore, move three places.

$$\begin{array}{r} 1.7 \\ \times\ 0.03 \\ \hline 0.051 \end{array}$$

3. Calculate 17% of 32.5.

$17\% = \frac{17}{100} = 0.17$, so 17% of 32.5 is (0.17) · (32.5).

AREA MODEL

	30	+ 2	+ 0.5
0.1	3	0.2	0.05
+ 0.07	2.1	0.14	0.035

3 + 2.1 + 0.2 + 0.14 + 0.05 + 0.035 = 5.525

STANDARD ALGORITHM

$$\begin{array}{rl} 32.5 & \text{1 decimal place} \\ \times\ 0.17 & \text{2 decimal places} \\ \hline 2275 & \\ +\ 3250 & \\ \hline 5.525 & \text{3 decimal places} \end{array}$$

Practice

Identify the number of places to the left to move the decimal point in the product. Do not compute the product.

1. (0.3) · (0.5)

2. (1.5) · (0.12)

3. (1.23) · (2.6)

4. (0.126) · (3.4)

5. 17 · (32.016)

6. (4.32) · (3.1416)

Calculate the product.

7. (0.8) · (0.03)

8. (3.2) · (0.3)

9. (1.75) · (0.09)

10. (4.5) · (3.2)

11. (1.8) · (0.032)

12. (7.89) · (6.3)

13. 8% of 540

14. 70% of 478

15. 37% of 4.7

16. 17% of 96

17. 15% of 4.75

18. 130% of 42

Answers

1. 2

2. 3

3. 3

4. 4

5. 3

6. 6

7. 0.024

8. 0.96

9. 0.1575

10. 14.40

11. 0.0576

12. 49.707

13. 43.2

14. 334.6

15. 1.739

16. 16.32

17. 0.7125

18. 54.6

Multiplying Integers

GUIDED SKILL SUPPORT

See Guided Skill Support Chapter 2 for a review of this topic.

Dividing Integers

GUIDED SKILL SUPPORT

Overview

Division of integers is developed as the inverse of integer multiplication. Division can be thought of as partitive, that is, creating equal groups from a set. Integer division can be modeled by writing and reasoning about a multiplication equation or with an integer tile model. Examples of both methods are shown.

In all cases, when dividing integers, the following is true. If the divisor and the dividend have the same sign, the quotient is *positive*; if the divisor and the dividend have different signs, the quotient is *negative* (with the dividend not equaling zero). This understanding should be developed by observing patterns rather than memorizing rules.

Examples

1. $-10 \div 2$

METHOD 1

Write a multiplication equation.

$$2 \cdot ? = -10$$

2 sets of what number = – 10?

$$2 \cdot (-5) = -10$$

Therefore, $-10 \div 2 = -5$.

METHOD 2

Build the –10 using integer tiles.

– – – – – – – – – –

Circle the number of sets indicated by the divisor, which is 2.

– – – – – – – – – –

The quotient is the number in each set.

$$-10 \div 2 = -5$$

2. $-10 \div (-2)$

METHOD 1

Write a multiplication equation.

$$-2 \cdot ? = -10$$

-2 multiplied by what number $= -10$?

$$-2 \cdot 5 = -10$$

Therefore, $-10 \div (-2) = 5$.

METHOD 2

When using an integer tile model to represent a negative divisor, the number of sets is the absolute value of the divisor and the negative sign is interpreted as "the opposite of."

Build a tile model to represent the *opposite* of -10, which is positive 10.

\+ + + + + + + + + +

Circle the number of sets indicated by the absolute value of the divisor, which is 2.

\+ + + + + + + + + +

The quotient is the number in each set.

$$-10 \div (-2) = 5$$

Practice

1. $10 \div (-5)$	**2.** $18 \div (-3)$	**3.** $96 \div (-3)$	**4.** $282 \div (-6)$
5. $-18 \div 6$	**6.** $-48 \div 4$	**7.** $-121 \div 11$	**8.** $-85 \div 85$
9. $-76 \div (-4)$	**10.** $-175 \div (-25)$	**11.** $-108 \div (-12)$	**12.** $-161 \div 23$
13. $-223 \div (-223)$	**14.** $354 \div (-6)$	**15.** $-1{,}992 \div (-24)$	**16.** $-1{,}819 \div (-17)$
17. $-1{,}624 \div 29$	**18.** $1{,}007 \div (-53)$	**19.** $994 \div (-14)$	**20.** $-2{,}241 \div 27$

Answers

1.	–2	**2.**	–6	**3.**	–32	**4.**	–47
5.	–3	**6.**	–12	**7.**	–11	**8.**	–1
9.	19	**10.**	7	**11.**	9	**12.**	–7
13.	1	**14.**	–59	**15.**	83	**16.**	107
17.	–56	**18.**	–19	**19.**	–71	**20.**	–83

Dividing Fractions

GUIDED SKILL SUPPORT

Overview

In general, division for a problem like 8 ÷ 2 can be thought of as, "In 8, how many groups of 2 are there?" Similarly, $\frac{1}{2} \div \frac{1}{4}$ means, "In $\frac{1}{2}$, how many fourths are there?" Three methods for fraction division are introduced to help students understand how dividing by fractions works: diagrams, common denominators, and the Super Giant One. The Super Giant One method develops the reasoning behind multiplication by the reciprocal as a fraction division method.

To divide by a fraction using a diagram, create a model of the situation using rectangles, a linear model, or a visual representation of it. Then, break that model into the fractional parts named.

To divide a number by a fraction using common denominators, express both numbers as fractions with the same denominator. Then, divide the first numerator by the second.

To use a Super Giant One, write the division problem in fraction form, with a fraction in both the numerator and the denominator. Use the reciprocal of the denominator for the numerator and the denominator in the Super Giant One, multiply the fractions as usual, and write the resulting fraction.

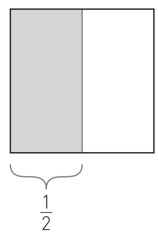
Often, mathematicians record fractions as easier-to-visualize equivalent fractions, so if the result of this calculation is something like $\frac{9}{18}$, mathematicians might record $\frac{1}{2}$. As you work through each of these fractions, you can decide whether you want to rewrite your answer as an equivalent fraction.

For more information, see the Methods & Meanings box in Lesson 3.3.1 of the *Core Connections Course 2* text.

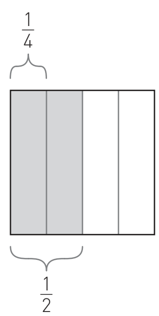
Examples

1. Use the rectangular model to divide: $\frac{1}{2} \div \frac{1}{4}$.

Divide the rectangle into 2 equal pieces. Each piece represents $\frac{1}{2}$. Shade $\frac{1}{2}$ of the rectangle.



Then divide the *original* rectangle into four equal pieces. Each section represents $\frac{1}{4}$. In the shaded section, $\frac{1}{2}$, there are two fourths.

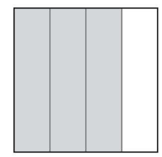


Write the equation.

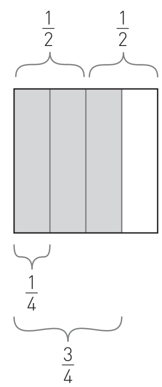
$$\frac{1}{2} \div \frac{1}{4} = 2$$

2. In $\frac{3}{4}$, how many $\frac{1}{2}$s are there? That is, $\frac{3}{4} \div \frac{1}{2} = ?$

Start with $\frac{3}{4}$.



In $\frac{3}{4}$, there is one full $\frac{1}{2}$ shaded and half of another one (that is, half of one half).



So $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$, or one and one-half halves.

3. Use common denominators to divide: $\frac{4}{5} \div \frac{2}{3}$.

Express both fractions with a common denominator, then divide the first numerator by the second.

$$\frac{4}{5} \div \frac{2}{3}$$
$$= \frac{12}{15} \div \frac{10}{15}$$
$$= \frac{12}{10}$$
$$= \frac{6}{5} \text{ or } 1\frac{1}{5}$$

4. Use common denominators to divide: $1\frac{1}{3} \div \frac{1}{6}$.

Express both numbers as fractions with a common denominator, then divide the first numerator by the second.

$$1\frac{1}{3} \div \frac{1}{6}$$
$$= \frac{4}{3} \div \frac{1}{6}$$
$$= \frac{8}{6} \div \frac{1}{6}$$
$$= \frac{8}{1} \text{ or } 8$$

5. Use a Super Giant One to divide: $\frac{1}{2} \div \frac{1}{4}$.

$$\frac{1}{2} \div \frac{1}{4} = \frac{\frac{1}{2}}{\frac{1}{4}} \cdot \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{\frac{4}{2}}{1} = \frac{4}{2} = 2$$

6. Use a Super Giant One to divide: $\frac{3}{4} \div \frac{1}{6}$.

$$\frac{3}{4} \div \frac{1}{6} = \frac{\frac{3}{4}}{\frac{1}{6}} \cdot \frac{\frac{6}{1}}{\frac{6}{1}} = \frac{\frac{18}{4}}{1} = \frac{9}{2} = 4\frac{1}{2}$$

7. $1\frac{1}{3} \div 1\frac{1}{2}$

$$1\frac{1}{3} \div 1\frac{1}{2} = \frac{1\frac{1}{3}}{1\frac{1}{2}} = \frac{\frac{4}{3}}{\frac{3}{2}} \cdot \frac{\frac{2}{3}}{\frac{2}{3}} = \frac{\frac{8}{9}}{1} = \frac{8}{9}$$

8. $\frac{2}{3} \div \frac{3}{5}$

$$\frac{2}{3} \div \frac{3}{5} = \frac{\frac{2}{3}}{\frac{3}{5}} \cdot \frac{\frac{5}{3}}{\frac{5}{3}} = \frac{\frac{10}{9}}{1} = \frac{10}{9} = 1\frac{1}{9}$$

Practice

Use the rectangular model to divide.

1. $1\frac{1}{3} \div \frac{1}{6}$

2. $\frac{3}{2} \div \frac{3}{4}$

3. $1 \div \frac{1}{4}$

4. $1\frac{1}{4} \div \frac{1}{2}$

5. $2\frac{2}{3} \div \frac{1}{9}$

Complete the division problems using any method.

6. $\frac{3}{7} \div \frac{1}{8}$

7. $1\frac{3}{7} \div \frac{1}{2}$

8. $\frac{4}{7} \div \frac{1}{3}$

9. $1\frac{4}{7} \div \frac{1}{3}$

10. $\frac{6}{7} \div \frac{5}{8}$

11. $\frac{3}{10} \div \frac{5}{7}$

12. $2\frac{1}{3} \div \frac{5}{8}$

13. $7 \div \frac{1}{3}$

14. $1\frac{1}{3} \div \frac{2}{5}$

15. $2\frac{2}{3} \div \frac{3}{4}$

16. $3\frac{1}{3} \div \frac{5}{6}$

17. $1\frac{1}{2} \div \frac{1}{2}$

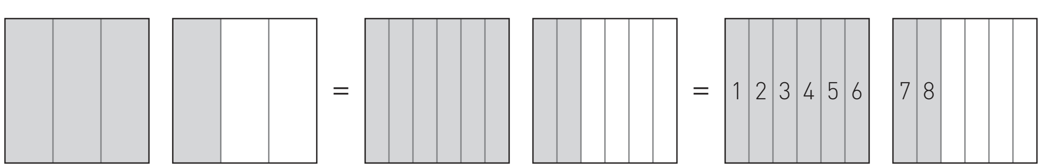
18. $\frac{5}{8} \div 1\frac{1}{4}$

19. $10\frac{1}{3} \div \frac{1}{6}$

20. $\frac{3}{6} \div 6$

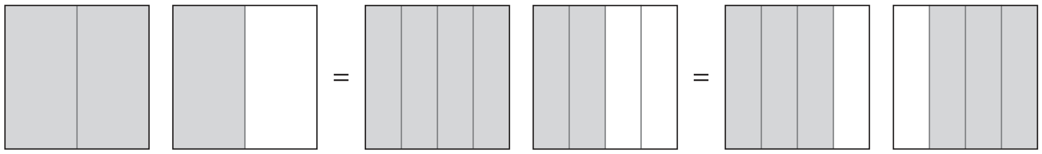
Answers

1.



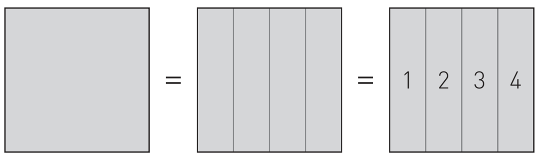
$1\frac{1}{3} \div \frac{1}{6} = 8$

2.



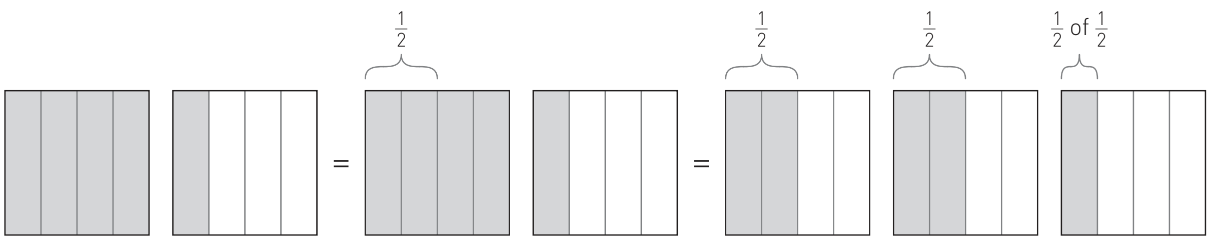
$\frac{3}{2} \div \frac{3}{4} = 2$

3.



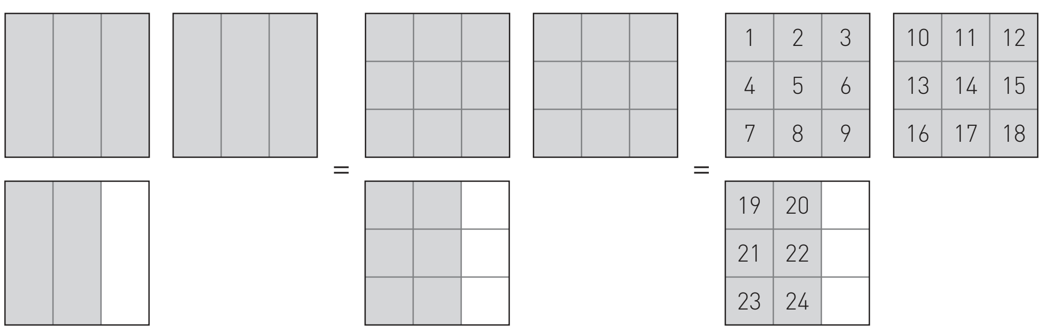
$1 \div \frac{1}{4} = 4$

4.



$1\frac{1}{4} \div \frac{1}{2} = 2\frac{1}{2}$

5.



$2\frac{2}{3} \div \frac{1}{9} = 24$

6.	$3\frac{3}{7}$	**7.**	$2\frac{6}{7}$	**8.**	$1\frac{5}{7}$	**9.**	$4\frac{5}{7}$
10.	$1\frac{13}{35}$	**11.**	$\frac{21}{50}$	**12.**	$3\frac{11}{15}$	**13.**	21
14.	$3\frac{1}{3}$	**15.**	$3\frac{5}{9}$	**16.**	4	**17.**	3
18.	$\frac{1}{2}$	**19.**	62	**20.**	$\frac{1}{12}$		

Properties of Addition and Multiplication

GUIDED SKILL SUPPORT

Overview

When two numbers or variables are combined using addition or multiplication, the order in which they are added or multiplied does not matter. This is called the Commutative Property. For example, $7 + 5 = 5 + 7$ shows the Commutative Property of Addition, and $5 \cdot 10 = 10 \cdot 5$ shows the Commutative Property of Multiplication.

These properties can be generalized using variables: $a + b = b + a$ shows the Commutative Property of Addition, and $a \cdot b = b \cdot a$ shows the Commutative Property of Multiplication. Subtraction and division do not satisfy the Commutative Property, since $7 - 5 \neq 5 - 7$ and $10 \div 5 \neq 5 \div 10$.

When adding three numbers or multiplying three numbers, the grouping can be changed. This is called the Associative Property. For example, $(2 + 3) + 5 = 2 + (3 + 5)$ shows the Associative Property of Addition, and $(2 \cdot 3) \cdot 5 = 2 \cdot (3 \cdot 5)$ shows the Associative Property of Multiplication.

These properties can be generalized using variables: $(a + b) + c = a + (b + c)$ shows the Associative Property of Addition, and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ shows the Associative Property of Multiplication. Subtraction and division do not satisfy the Associative Property since $(7 - 5) - 1 \neq 7 - (5 - 1)$ and $(10 \div 2) \div 5 \neq 10 \div (2 \div 5)$.

The Distributive Property distributes one operation over another. Students have been introduced to multiplication distributed over addition. For example, $2(3 + 5) = 2 \cdot 3 + 2 \cdot 5$. The Distributive Property can be generalized using variables. For all numbers a, b, and c, $a(b + c) = a \cdot b + a \cdot c$.

For additional information, see the Methods & Meanings box in Lesson 4.1.1 of the *Core Connections Course 2* text.

Examples

Identify the property or reason that justifies each step.

1. $4 \cdot (17 \cdot 25)$
$= 4 \cdot (25 \cdot 17)$
$= (4 \cdot 25) \cdot 17$
$= (100) \cdot 17$
$= 1{,}700$

$4 \cdot (17 \cdot 25)$	
$= 4 \cdot (25 \cdot 17)$	Commutative Property of Multiplication
$= (4 \cdot 25) \cdot 17$	Associative Property of Multiplication
$= (100) \cdot 17$	multiplication
$= 1{,}700$	multiplication

2. $8(56)$
$= 8(50 + 6)$
$= 8(50) + 8(6)$
$= 400 + 48$
$= 448$

$8(56)$	
$= 8(50 + 6)$	represent 56 as 50 + 6
$= 8(50) + 8(6)$	Distributive Property
$= 400 + 48$	multiplication
$= 448$	addition

Practice

Possible steps used to mentally calculate a problem are shown. Give the missing reasons that justify the steps.

1.

$15(29)$	
$= 15(30 + (-1))$	represent 29 as 30 + (–1)
$= 15(30) + 15(-1)$	____________________
$= 450 + (-15)$	multiplication
$= 450 + (-10 + -5)$	represent (–15) as (–10) + (–5)
$= (450 + (-10)) + (-5)$	____________________
$= 440 + (-5)$	
$= 435$	addition

2. $386 + 177 + 214$

$= 386 + 214 + 177$	________________
$= (386 + 214) + 177$	________________
$= 600 + 177$	
$= 777$	addition

3. $49(12)$

$= 12(49)$	________________
$= 12(50 - 1)$	represent 49 as 50 – 1
$= 12(50) - 12(1)$	________________
$= (6 \cdot 2)(50) - 12$	represent 12 as 6 · 2
$= 6(2 \cdot 50) - 12$	________________
$= 6(100) - 12$	multiplication
$= 600 - 12$	
$= 588$	multiplication and subtraction

Answers

1. Distributive Property, Associative Property

2. Commutative Property, Associative Property

3. Commutative Property, Distributive Property, Associative Property