Comparing Quantities

GUIDED SKILL SUPPORT

Overview

Combining two Expression Mats into an Expression Comparison Mat creates a concrete model for representing (and later solving) inequalities and equations.

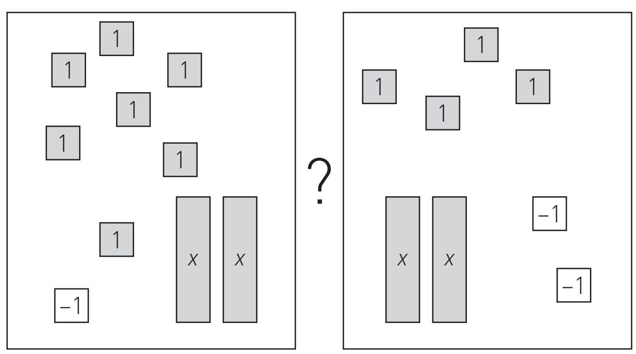
Tiles may be removed or moved on the mat in the following ways:

- removing the same number of opposite tiles (zeros) on the same side;
- removing an equal number of identical tiles (balanced sets) from both the left and right sides;
- adding the same number of opposite tiles (zeros) on the same side; and
- adding an equal number of identical tiles (balanced sets) to both the left and right sides.

These strategies are called "valid moves." After adding and/or removing tiles on the Expression Comparison Mat, students are asked to determine which side is greater. Sometimes it is only possible to determine which side is greater if you know the possible values of the variable.

Examples

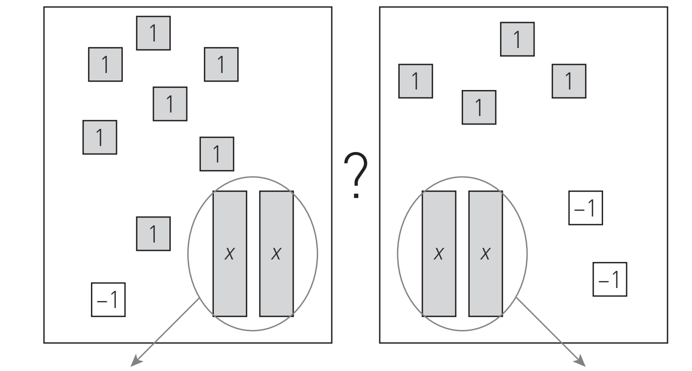
1. Determine which side is greater by using valid tile moves to rearrange the two expressions in fewer terms so you can compare them.



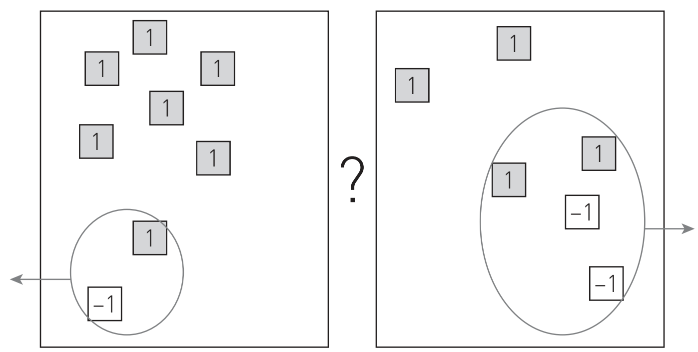
Solution shown on next page.

Example continued from previous page

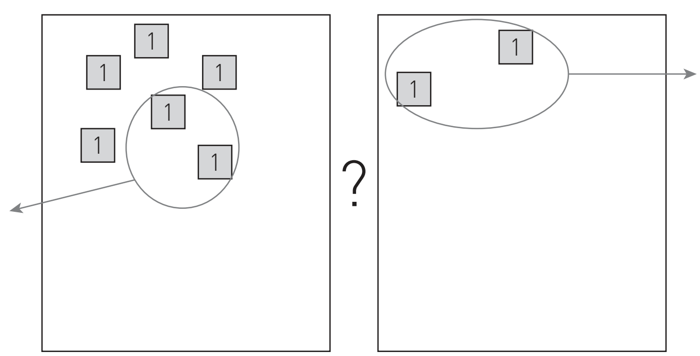
STEP 1: Remove balanced sets.



STEP 2: Remove zeros.

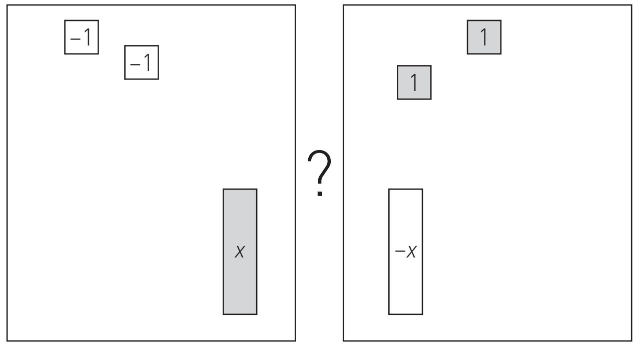


STEP 3: Remove balanced sets.



The left side is greater because after Step 3, 4 > 0. Also, after Step 2, 6 > 2. Note that this example shows only one of several possible strategies.

2. Use valid moves so that all the *x*-variables are on one side and all the unit tiles are on the other.



Solution shown on next page.

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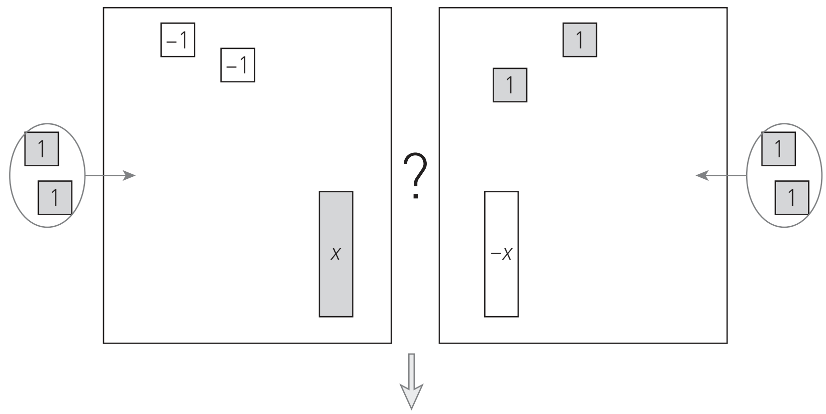
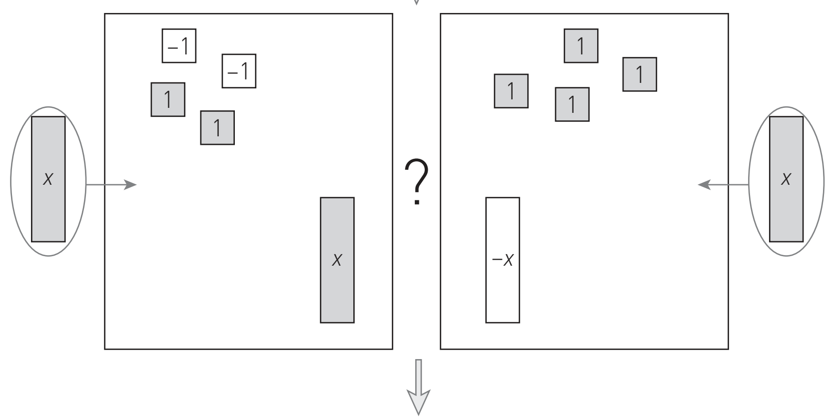
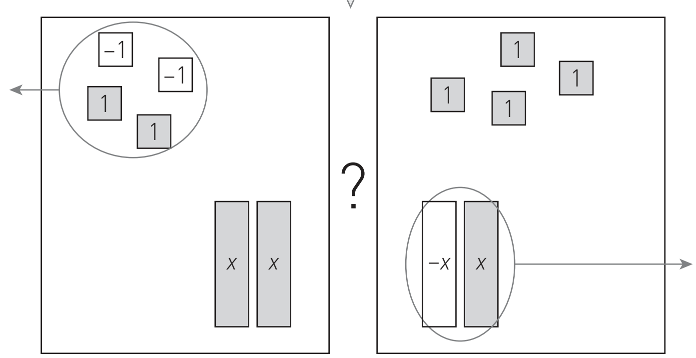
STEP 1: Add balanced sets.



STEP 2: Add balanced sets.



STEP 3: Remove zeros.



$2x$ remains on the left mat and 4 remains on the right mat. There are other possible arrangements. Whatever the arrangement, it is not possible to tell which side is greater because we do not know the value of x. Students are expected to record the results algebraically as demonstrated by the teacher. One possible method is shown.

LEFT EXPRESSION RIGHT EXPRESSION

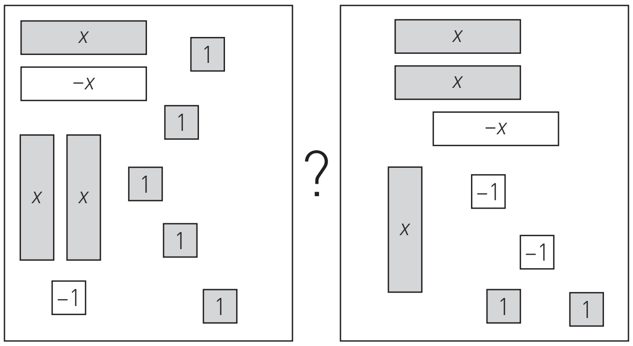
$$x + x - 2 + 2 \;\; \mathbf{?} \;\; -x + x + 2 + 2$$

$$2x \;\; \mathbf{?} \;\; 4$$

Practice

For problems 1 through 3, use the strategies of removing zeros or adding/removing balanced sets to determine which side is greater, if possible. Record your steps.

1.



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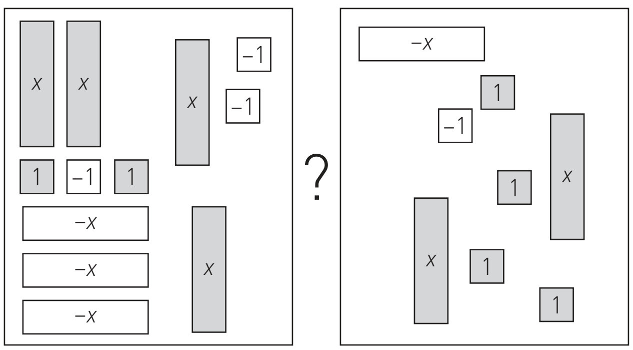

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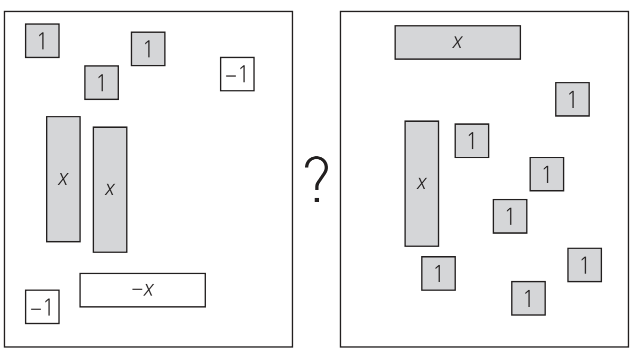


For problems 4 through 6, build or draw tiles to represent each expression and then use the strategies of removing zeros or adding/removing balanced sets to determine which side is greater, if possible. Record your steps.

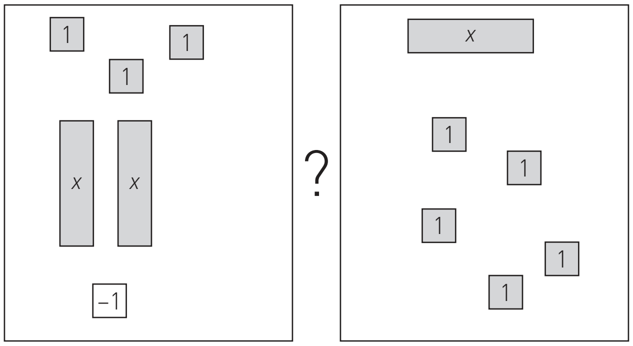
4. Left: $5 + (-8)$
Right: $-7 + 6$

5. Left: $2(x + 3) - 2$
Right: $4x - 2 - x + 4$

6. Left: $4 + (-2x) + 4x$
Right: $x^2 + 2x + 3 - x^2$

For problems 7 through 12, use the strategies of removing zeros or adding/removing balanced sets so that all the x-variables are on one side and the unit tiles are on the other. Record your steps.

7.



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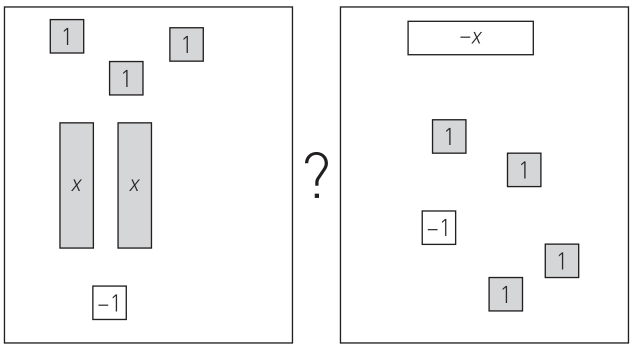
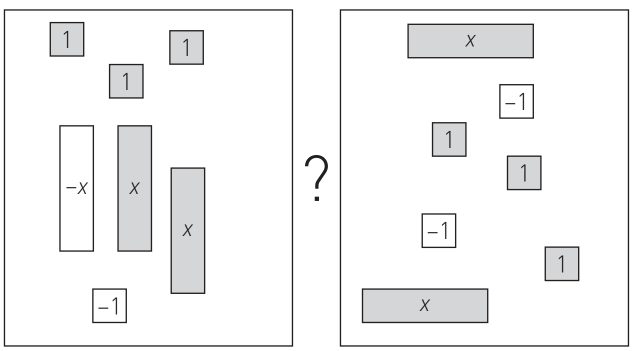


9.



10. Left: $3x - 2$
Right: $2x + 1$

11. Left: $4x + 2 + (-5)$
Right: $2x + 3 + (-8)$

12. Left: $2x + 3$
Right: $-x - 3$

Answers

Answers to problems 7 through 12 may vary, but the comparison cannot be determined.

1. left side is greater

2. right side is greater

3. not possible to tell

4. right side is greater

5. not possible to tell

6. left side is greater

7. left: x, right: 3

8. left: $3x$, right: 1

9. left: 1, right: x

10. left: x, right: 3

11. left: $2x$, right: -2

12. left: $3x$, right: -6

Graphing and Solving Inequalities

GUIDED SKILL SUPPORT

Overview

The solution to an inequality refers to all the values of the variable that make the inequality true. The solution can be represented as a ray or segment with solid or open endpoints on a number line. A solid endpoint indicates that the boundary point is included in the solution ($\leq$ or $\geq$), while an open endpoint indicates that it is not part of the solution ($<$ or $>$).

To solve an inequality, complete the following steps.

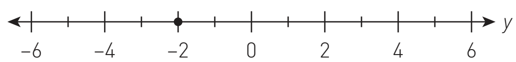
1. Rewrite the inequality as an equation.
2. Solve the equation by "undoing" operations to isolate the variable.
3. Use the solution of the equation as a boundary point on the number line.
4. Test a value from each side of the boundary point on the number line in the inequality. If the test number is true, then that part of the number line is part of the solution. If the test number is false, then that part of the number line is not part of the solution.

For additional information, see the Methods & Meanings boxes in Lessons 6.1.1 and 6.1.4 of the *Core Connections Course 2* text.

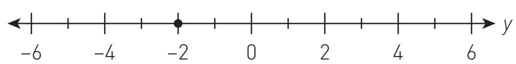
Examples

1. Graph the inequality $y \geq -2$.

The boundary point is -2 and the $\geq$ symbol indicates that -2 is included in the solution, so the boundary point is shaded.



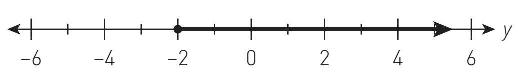
To verify the set of numbers that make the inequality true, test 0 and -3 by substituting each value for y in the inequality.



$y = 0$ $\downarrow$ $y \geq -2$, $0 \geq -2$ true

$y = -3$ $\downarrow$ $y \geq -2$, $-3 \geq -2$ false

Therefore, the number line has a boundary point at -2 and is shaded in the direction that includes the values that make the inequality true.



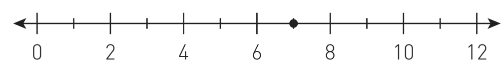
2. Solve the inequality $9 \geq m + 2$ and graph the solution.

To determine the boundary point, rewrite the inequality as an equation and solve it.

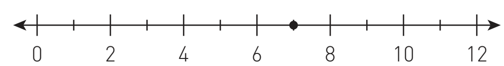
$$9 \geq m + 2$$
$$9 = m + 2$$
$$7 = m$$

7 is the boundary point.

Draw a number line. Since the original inequality includes the symbol ($\geq$), put a solid dot at 7.



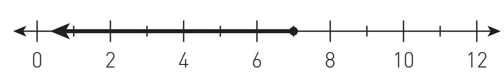
Next, test a number on each side of 7 in the original inequality, such as 0 and 10.



$m = 0$ $\downarrow$ $9 > m + 2$, $9 > 0 + 2$, $9 > 2$ true

$m = 10$ $\downarrow$ $9 > m + 2$, $9 > 10 + 2$, $9 > 12$ false

Therefore, the solution is $m \leq 7$. The number line has a boundary point at 7 and is shaded in the direction that includes the values that make the inequality true.



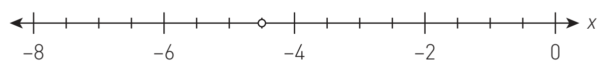
3. Solve the inequality −2x − 3 < 6 and graph the solution.

To determine the boundary point, rewrite the inequality as an equation and solve.

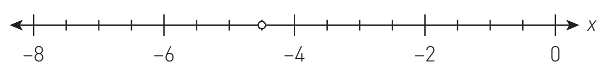
$\overline{-2} \quad \overline{-2}$

$x = -\frac{9}{2}$

Draw a number line. Since the original inequality did not include the solution (<), put an open dot at −4.5.

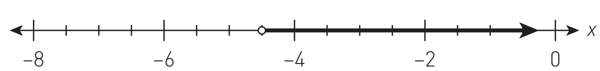


Next, test a number on each side of −4.5 in the original inequality, such as −8 and 0.



$x = \boxed{-8}$	$x = \boxed{0}$
$-2x - 3 < 6$	$-2x - 3 < 6$
$-2(-8) - 3 < 6$	$-2(0) - 3 < 6$
$13 < 6$	$-3 < 6$
false	true

Therefore, the solution is $x > -4.5$. The number line has a boundary point at −4.5, but not including −4.5, and is shaded in the direction that includes the values that make the inequality true.



Practice

Solve each inequality. Then graph the solution on a number line.

1. $x + 3 > -1$

2. $y + 3 \leq -5$

3. $3x \geq 6$

4. $2m + 1 \geq -7$

5. $-7 < -2y + 3$

6. $8 \geq -2m + 2$

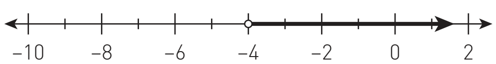
7. $-1 < -x + 8$

8. $2(m + 1) \geq -3$

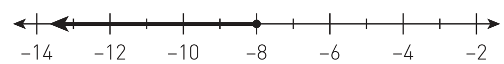
9. $3m + 1 \leq 7$

Answers

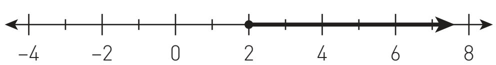
1. $x > -4$



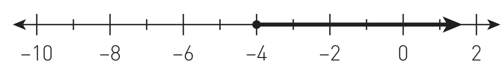
2. $y \leq -8$



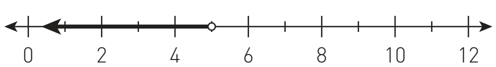
3. $x \geq 2$



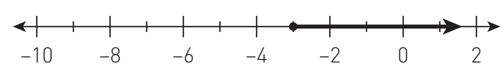
4. $m \geq -4$



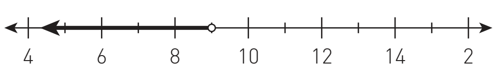
5. $y < 5$



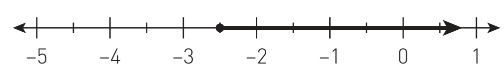
6. $m \geq -3$



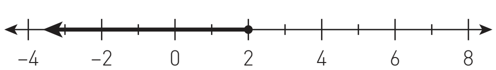
7. $x < 9$



8. $m \geq -2.5$



9. $m \leq 2$



Solving Algebraic Equations

GUIDED SKILL SUPPORT

Overview

Solving an equation means figuring out the value or values of the variable that makes the equation true. Previously two Expression Mats were combined to figure out what value(s) of the variable make(s) one expression greater than the other. Now two Expression Mats can be combined into an Equation Mat as a concrete model for solving equations. Practicing solving equations using this model will help students transition to solving equations abstractly with better accuracy and understanding.

In general, start by using valid tile moves such as removing zeros or balanced sets. Next, isolate the variables on one side of the Equation Mat and the non-variables (unit tiles) on the other. Then determine the value of the variable. Students should record and explain their steps.

Once the students understand how to solve equations using an Equation Mat, they may use the visual experience of moving the tiles to solve equations with variables and numbers. This process often involves "undoing" operations to isolate the variable. For example, addition undoes subtraction, and division undoes multiplication. A solution to an equation is a number that, when substituted into the equation for the variable, makes the left side of the equation equal to the right side.

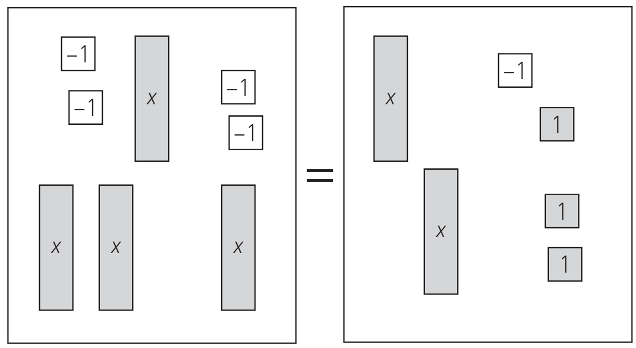
For additional information, see the Methods & Meanings boxes in Lesson 6.2.1 and 6.2.3 of the *Core Connections Course 2* text. For additional examples and practice, see the *Core Connections Course 2* Checkpoint 8 materials.

Examples

1. Solve the following equation using algebra tiles. Record symbols for each step.

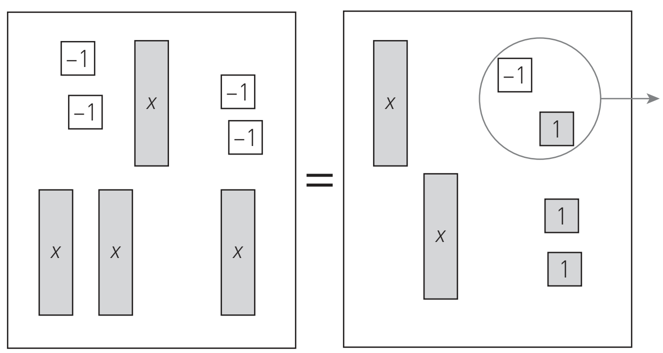
$$x + (-4) + 3x = 2x - 1 + 3$$

First build the equation on the Equation Mat.



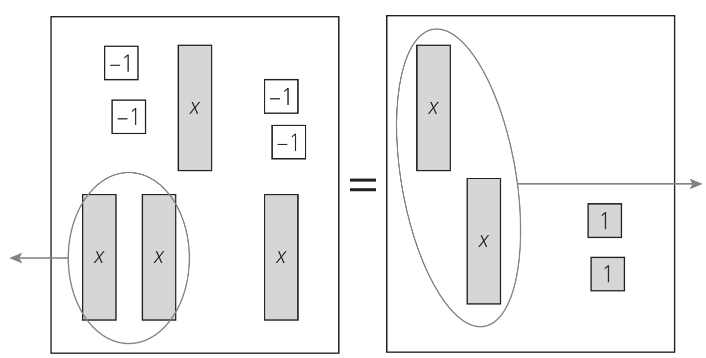
$x + (-4) + 3x$ $2x - 1 + 3$

Second, remove zeros (−1 and +1 on the right side of the mat).



$4x - 4$ $2x + 2$

Third, remove a balanced set (2*x*) from both sides.

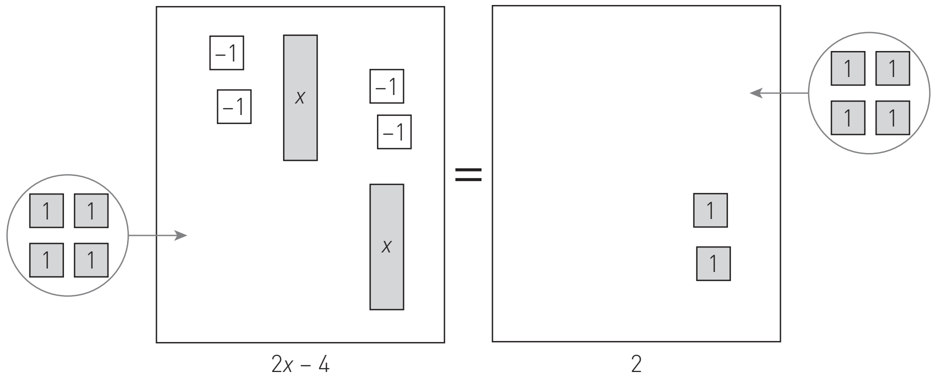


$4x - 4$ $2x + 2$

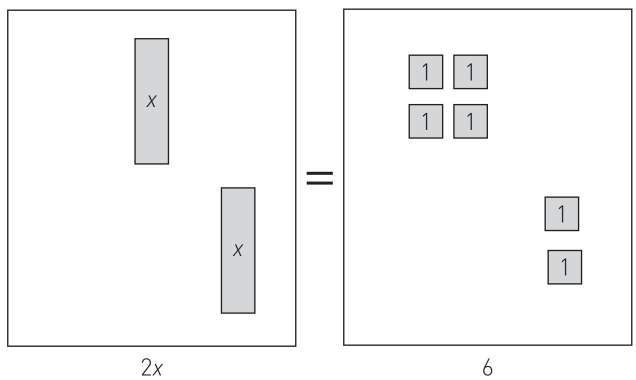
Solution continues on next page.

Solution continued from previous page.

Isolate the variable by adding a balanced set (+4) to both sides and removing the zeros on the left side.



Finally, since both sides of the equation are equal, determine the value of x by dividing.



$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

2. Solve the equation for x and record your steps. Check your solution.

$$3x + 2x - 1 = 9$$

A solution using symbolic notation is shown. You might want to also build and solve the equation using algebra tiles.

$3x + 2x - 1 = 9$	Original equation
$5x - 1 = 9$	Rewrite by combining like terms.
$5x = 10$	Add 1 on each side.
$x = 2$	Divide by 2.

Check the solution by substituting 2 for x in the original equation and making sure that the equation is true.

$$x = \boxed{2}$$

$$\begin{aligned} 3x + 2x - 1 &= 9 \\ 3(2) + 2(2) - 1 &= 9 \\ 6 + 4 - 1 &= 9 \\ 9 &= 9 \end{aligned}$$

true

3. Solve: $3(x - 4) = -3$

A solution using symbolic notation is shown. You might want to also build and solve the equation using algebra tiles.

$3(x - 4) = -3$	Original equation
$3(x) - 3(4) = -3$	Distribute the 3.
$3x - 12 = -3$	Multiply.
$3x = 9$	Add 12 to both sides.
$\frac{3x}{3} = \frac{9}{3}$	Divide both sides by 3.
$x = 3$	

Check the solution by substituting 3 for x in the original equation and making sure that the equation is true.

$$x = \boxed{3}$$

$$\begin{aligned} 3(x - 4) &= -3 \\ 3((3) - 4) &= -3 \\ 3(-1) &= -3 \\ -3 &= -3 \end{aligned}$$

true

Practice

Solve each equation for x.

1. $3x + 2 + x = 6$

2. $4x - 2 - x = -5$

3. $-x + 3 = -7$

4. $1 + 3x - x = -4 - 1$

5. $2(x - 2) + x = 5$

6. $10 = x + x + 5$

7. $3(2x - 2) = 2 + 4$

8. $\frac{1}{2}(4x + 2 + 2x) = 28$

9. $25x - 7 = 93$

10. $35 = -3x + 2$

11. $-(x + 4) = 11$

12. $x + x - 1 - 1 = -2$

Answers

1. 1

2. −1

3. 10

4. −3

5. 3

6. 2.5

7. 2

8. 9

9. 4

10. −11

11. −15

12. 0

Solving Equations in Context

GUIDED SKILL SUPPORT

Overview

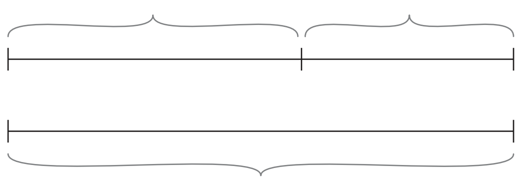
Equations can be used to make sense of situations as a way to solve for unknown information. Sometimes a diagram or geometric figure can be used to model the situation. First, define a variable to represent the unknown quantity. Then write and solve an equation.

Examples

1. Cecil the acrobat has a tightrope that is 32 feet long. Cecil's first leap was 10 feet. How many more feet does Cecil have to travel to reach the end of the rope?

Let x = the unknown length of rope.

In this case, a diagram can represent the situation.

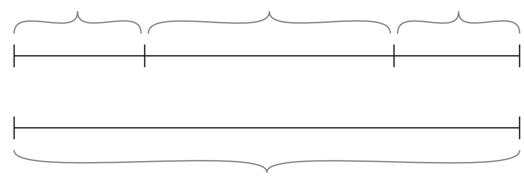


Next, write and solve an equation.

$$\begin{aligned} x + 10 &= 32 \\ x &= 22 \end{aligned}$$

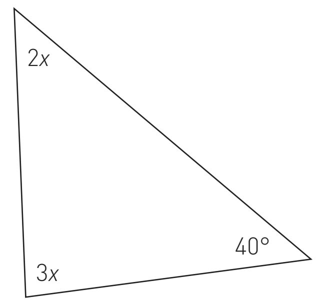
Cecil has 22 feet of rope remaining.

2. Write and solve an equation to represent the following diagram.



$$\begin{aligned} x + 2x + 8 &= 44 \\ x + 2x &= 36 \\ 3x &= 36 \\ x &= 12\text{, therefore } 2x = 24 \end{aligned}$$

3. The sum of the angles in a triangle is 180 degrees. Determine the measures of the missing angles in the diagram by writing and solving an equation.

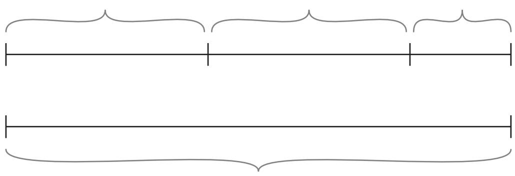


$$2x + 3x + 40° = 180°$$
$$2x + 3x = 140°$$
$$5x = 140°$$
$$x = 28°\text{, therefore } 2x = 56° \text{ and } 3x = 84°$$

Practice

Write an equation that represents each situation and then determine the value of the variable.

1.



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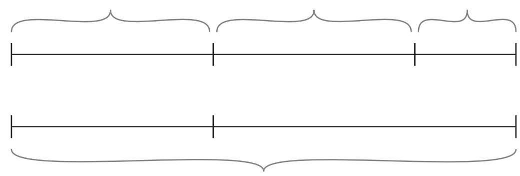

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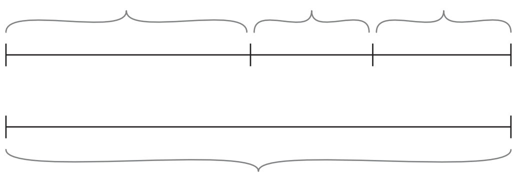
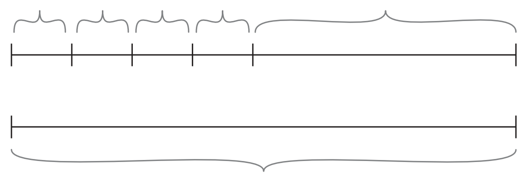


4.



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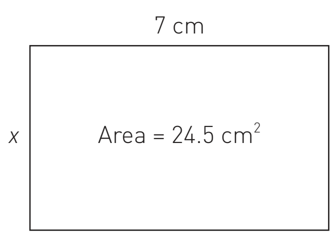

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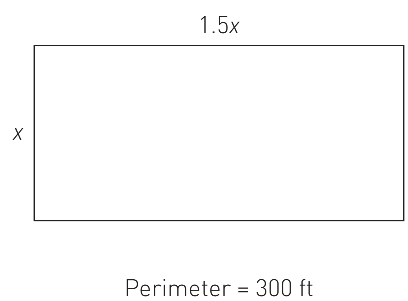

Answers

1. $2x + 3 = 25$, $x = 11$

2. $2x + 4 = 16$, $x = 6$

3. $3x + 7 = 25$, $x = 6$

4. $4n + 12 = 28$, $n = 4$

5. $7x = 24.5$, $x = 3.5$ cm

6. $x + x + 1.5x + 1.5x = 300$, $x = 60$ ft