Distance, Rate, and Time

GUIDED SKILL SUPPORT

Overview

There is a proportional relationship between distance and time. Distance (*d*) equals the product of the rate of travel (*r*) and the time (*t*). This relationship can be represented in three forms:

$$d = r \cdot t \qquad r = \frac{d}{t} \qquad t = \frac{d}{r}$$

It is important that the units of measure are consistent.

Examples

1. Determine the rate of travel (speed) of a passenger car if the distance traveled is 572 miles in 11 hours.

Start with the equation $d = r \cdot t$ and substitute the values for distance and time given in the problem.

$$d = r \cdot t$$
$$572 \text{ miles} = r \cdot 11 \text{ hours}$$

Next, divide both sides by 11 hours to determine the rate.

$$r = \frac{572 \text{ miles}}{11 \text{ hours}}$$
$$r = \frac{52 \text{ miles}}{\text{hour}} \text{ or } 52 \text{ mph}$$

2. A train traveled 135 miles per hour for 40 minutes. How many miles did the train travel?

The units of time are not the same, so one must be converted to match the other. 40 minutes can be converted into hours.

$$\frac{40}{60} = \frac{2}{3} \text{ hour}$$

Then, the relationship $d = r \cdot t$ can be used to determine the distance traveled.

$$d = r \cdot t$$
$$d = \frac{135 \text{ miles}}{\text{hour}} \cdot \frac{2}{3} \text{ hour}$$
$$d = 90 \text{ miles}$$

3. The Central Middle School hamster race is fast approaching. Francisco said that his hamster traveled 60 feet in 90 seconds and Willow said she timed for one minute and her hamster traveled 12 yards. Which hamster has the faster rate?

> All measurements need to be in the same units. In this example, both hamster rates can be converted to feet per minute.
>
> $$\text{rate} = \frac{\text{distance}}{\text{time}}$$
>
> $$\text{Francisco's hamster: } \frac{60 \text{ feet}}{90 \text{ seconds}} = \frac{60 \text{ feet}}{1.5 \text{ minutes}} \cdot \frac{\frac{2}{3}}{\frac{2}{3}} = \frac{40 \text{ feet}}{1 \text{ minute}}$$
>
> $$\text{Willow's hamster: since 1 yard = 3 feet, } \frac{12 \text{ yards}}{1 \text{ minute}} = \frac{36 \text{ feet}}{1 \text{ minute}}$$
>
> Francisco's hamster is faster by 4 feet per minute.

Practice

Solve the following problems.

1. How long would it take to travel 157.5 miles at a speed of 63 mph?

2. If you traveled for 3.5 hours at 67 mph, how many miles would you travel?

3. Determine the rate in miles per hour if the distance is 247 miles and the time is 3.8 hours.

4. What distance could you travel at 60 mph in 1 hour and 45 minutes?

5. Determine the rate in miles per hour if the distance is 3.5 miles and the time is 20 minutes.

6. How many minutes would it take to travel 2 miles at the rate of 30 mph?

7. Which rate is faster: 60 feet in 90 seconds, or 60 inches in 5 seconds?

8. Which distance is longer: 4 feet/second for a minute, or 3 inches/minute for an hour?

9. Which time is shorter: 4 miles at 60 mph, or 6 miles at 80 mph?

Answers

1.	2.5 hours	**2.**	234.5 miles	**3.**	65 mph
4.	105 miles	**5.**	10.5 mph	**6.**	4 minutes
7.	60 inches in 5 seconds	**8.**	4 feet/second for a minute	**9.**	4 miles at 60 mph

Scaling to Solve Problems

GUIDED SKILL SUPPORT

Overview

Scale factors (multipliers) can be used to increase and decrease quantities or to enlarge and reduce figures. When this change happens, every original quantity or length of a figure is multiplied by the scale factor to get the new quantities or lengths. The original and new values are proportional.

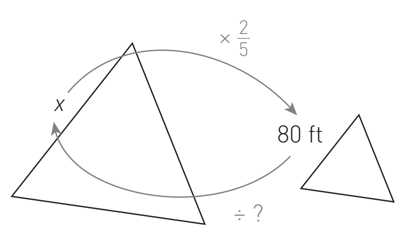
To reverse this process and scale from the new situation back to the original, divide by the scale factor. Division by a scale factor is the same as multiplying by the reciprocal of the scale factor. Recall that a reciprocal is the multiplicative inverse of a number; that is, the product of a number and its reciprocal is 1. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, of $\frac{1}{2}$ is $\frac{2}{1}$, and of 5 is $\frac{1}{5}$.

Scaling may also be used with percentage problems where a quantity is increased or decreased by a certain percent. Scaling by a factor of 1 does not change the quantity. Increasing by a certain percent may be found by multiplying by (1 + the percent), and decreasing by a certain percent may be found by multiplying by (1 – the percent).

For additional information, see the Methods & Meanings box in Lesson 7.1.4 of the *Core Connections Course 2* text.

Examples

1. A large triangle is reduced by a scale factor of $\frac{2}{5}$. If the side labeled x now has a length of 80 feet in the new figure, what was the original length?



To undo the reduction, multiply 80 feet by the reciprocal of $\frac{2}{5}$, which is $\frac{5}{2}$, or divide 80 feet by $\frac{2}{5}$.

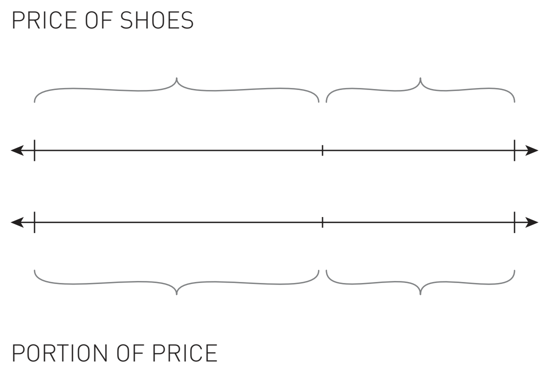
$80 \div \frac{2}{5}$ is the same as $80 \cdot \frac{5}{2}$, so $x = 200$ feet.

2. Samantha wants to leave a 15% tip on her lunch bill of $12.50. What scale factor can be used to determine how much money she should leave to cover the bill and the tip?

Since tipping increases the total by 15%, the scale factor is (1 + 0.15) = 1.15.

She should leave (1.15)(12.50) = $14.38.

3. Carlos sees that all of the shoes at his favorite store are on sale at 40% off. If the regular price of a pair of shoes is $124.00, what scale factor can be used to determine the sale price? What price would Carlos pay for the shoes?



If items are reduced 40%, the scale factor is (100% – 40%) = 60%.

The sale price is (0.60)($124.00) = $74.40.

Practice

1. A rectangle was enlarged by a scale factor of $\frac{5}{2}$. If the original width is 16 cm, what is the new width?

2. A side of a triangle was reduced by a scale factor of $\frac{2}{3}$. If the new side is now 18 inches, what was the original side?

3. The scale factor used to create the design for a backyard is 2 inches for every 75 feet. On the design, the fire pit is 6 inches away from the house. How far from the house, in feet, should the fire pit be dug?

4. After a very successful year, a rental company raised salaries by a scale factor of $\frac{11}{10}$. If Luan makes $18.00 per hour, what will she earn after the salary increase?

5. What is the total cost of an $89.50 family dinner after adding a 20% tip?

6. If the current cost to attend Magicland Park is now $49.50 per person, what will be the cost after an 8% increase?

7. Winter coats are on clearance at 35% off. If the regular price is $79, what is the sale price?

8. The company president has offered to reduce her salary by 10% to cut expenses. If she now earns $175,000, what will be her new salary?

Answers

1.	40 cm	**2.**	27 in	**3.**	225 ft	**4.**	\$19.80
5.	\$107.40	**6.**	\$53.46	**7.**	\$51.35	**8.**	\$157,500

Equations With Fractional Coefficients

GUIDED SKILL SUPPORT

Overview

When solving equations with fractional coefficients, you may divide each term in the equation by the coefficient or multiply each term by the reciprocal of the coefficient. Alternatively, multiplying all of the terms of an equation by a common denominator (the "Fraction Buster" method) will remove all of the fractions from the equation. Then, the equation can be solved using the standard method.

For additional information, see the Methods & Meanings box in Lesson 7.1.6 of the *Core Connections Course 2* text.

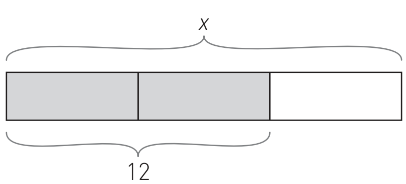
Examples

1. Solve for x.

$$\frac{2}{3}x = 12$$

USING A DIAGRAM

Creating a diagram can help make sense of the situation and sometimes lead to a mental math solution.



Since $\frac{2}{3}$ of x is 12, $\frac{1}{3}$ of x is 6 and therefore $x = 18$.

USING DIVISION AND A GIANT ONE

$$\frac{2}{3}x = 12$$

$$\frac{\frac{2}{3}}{\frac{2}{3}}x = \frac{12}{\frac{2}{3}}$$

$$x = \frac{12}{\frac{2}{3}}$$

$$x = 12 \div \frac{2}{3}$$

$$x = \frac{36}{3} \div \frac{2}{3}$$

$$x = \frac{36}{2}$$

$$x = 18$$

USING RECIPROCALS

Multiply both sides of the equation by the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$.

$$\frac{2}{3}x = 12$$

$$\left(\frac{3}{2}\right)\frac{2}{3}x = 12\left(\frac{3}{2}\right)$$

$$x = 18$$

2. Solve for x.

$$\frac{x}{2} + \frac{x}{5} = 6$$

The "Fraction Busters" method was used in this example. Multiplying by 10 (the common denominator) will eliminate the fractions.

$$\frac{x}{2} + \frac{x}{5} = 6$$

$$(10)\left(\frac{x}{2} + \frac{x}{5}\right) = (6)(10)$$

$$(10)\left(\frac{x}{2}\right) + (10)\left(\frac{x}{5}\right) = (6)(10)$$

$$5x + 2x = 60$$

$$7x = 60$$

$$x = \frac{60}{7}$$

$$x \approx 8.57$$

Practice

Solve each equation for the variable using any method.

1. $\frac{3}{4}x = 60$	**2.** $\frac{2}{5}x = 42$	**3.** $\frac{3}{5}x = 40$	**4.** $-\frac{8}{3}x = 6$
5. $\frac{3x+1}{2} = 5$	**6.** $\frac{x}{3} - \frac{x}{5} = 3$	**7.** $\frac{y+7}{3} = 10$	**8.** $\frac{m}{3} - \frac{2m}{5} = \frac{1}{5}$
9. $-\frac{3}{5}x = \frac{2}{3}$	**10.** $\frac{x}{2} + \frac{x-3}{5} = 3$	**11.** $\frac{1}{3}x + \frac{1}{4}x = 4$	**12.** $\frac{2x}{5} + \frac{x-1}{3} = 4$

Answers

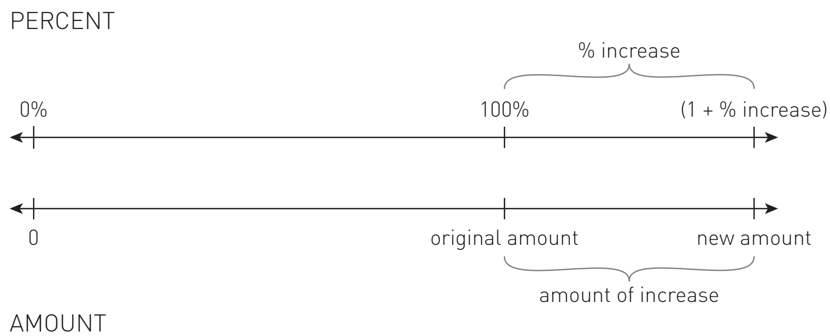
1. $x = 80$	**2.** $x = 105$	**3.** $x = 66\frac{2}{3}$	**4.** $x = -2\frac{1}{4}$
5. $x = 3$	**6.** $x = 22.5$	**7.** $y = 23$	**8.** $m = -3$
9. $x = -\frac{10}{9}$	**10.** $x = \frac{36}{7}$	**11.** $x = \frac{48}{7}$	**12.** $x = \frac{65}{11}$

Percent Increase or Decrease

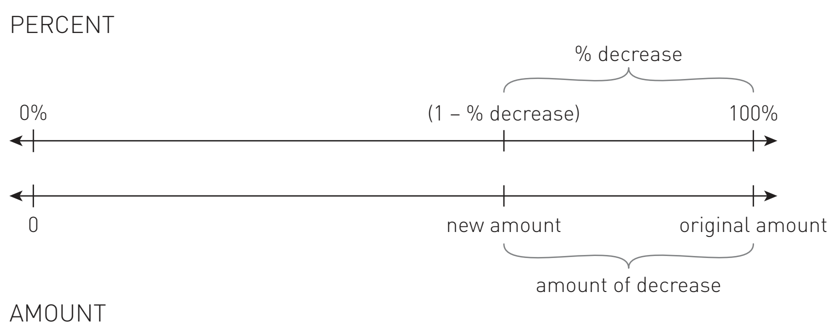
GUIDED SKILL SUPPORT

Overview

A percent increase is the amount that a quantity has increased based on a percent of the original amount. To increase by a certain percent, multiply the original amount by (1 + the percent).



A percent decrease is the amount that a quantity has decreased based on a percent of the original amount. To decrease by a certain percent, multiply the original amount by (1 – the percent).

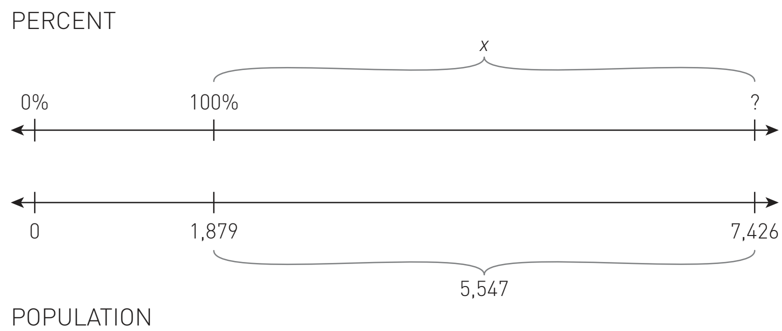


An equation that represents either situation is (amount of increase or decrease) = (% change)(original amount).

For additional information, see the Methods & Meanings box in Lesson 7.1.7 of the *Core Connections Course 2* text.

Examples

1. A town's population grew from 1,879 to 7,426 over five years. What percent is the increase in the population?



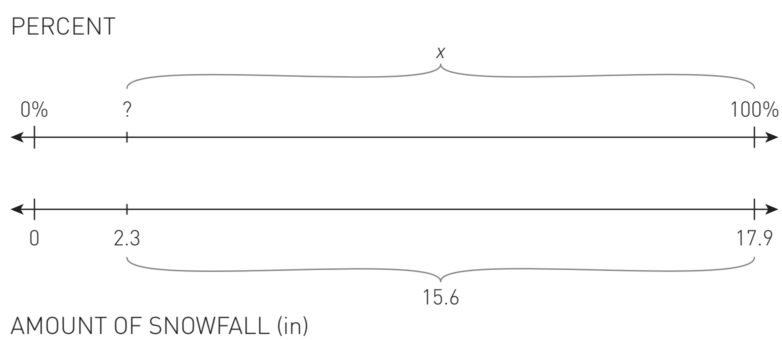
(amount of increase) = 7,426 − 1,879 = 5,547

(amount of increase or decrease) = (% change)(original amount)

$$5{,}547 = (x)(1{,}879)$$
$$\frac{5{,}547}{1{,}879} = x$$
$$x \approx 2.952$$

Therefore, the population increased by approximately 295.2%.

2. The amount of snowfall in Central Park in New York City over the 2021–2022 season was 17.9 inches. Over the 2023–2024 season, there was 2.3 inches of snow. What percent is the decrease in the amount of snowfall?



(amount of decrease) = 17.9 − 2.3 = 15.6

(amount of increase or decrease) = (% change)(original amount)

$$15.6 = x(17.9)$$
$$\frac{15.6}{17.9} = x$$
$$x \approx 0.8715$$

Therefore, the snowfall decreased by approximately 87.15%.

Practice

1. 50 years ago, gasoline cost \$0.30 per gallon on average. 10 years ago, gasoline averaged about \$2.50 per gallon. What percent is the increase in the cost of gasoline?

2. When Spencer was 5, he was 39 inches tall. Today, he is 5 feet 3 inches tall. What percent is the increase in Spencer's height?

3. The cars of the early 1900s cost about \$500. Today, a new car costs an average of \$50,000. What percent is the increase in the cost of a car?

4. The population of the United States at the first census in 1790 was 3,929,214 people. By 2020, the population had increased to 331,449,281! What is the percent increase in the population?

5. In 2024, the rate for a first-class U.S. postage stamp increased to \$0.69. This represents a \$0.66 increase since 1917. What percent is the increase in cost since 1917?

6. In 1906, Americans consumed an average of 26.85 gallons of whole milk per year. By 1998, the yearly average consumption was 8.32 gallons. What percent is the decrease in consumption of whole milk?

7. In 1984, there were 125 students for each computer in U.S. public schools. By 1998, there were 6.1 students for each computer. What percent is the decrease in the ratio of students to computers?

8. Sara bought a dress on sale for \$33. She saved 45%. What was the original cost?

9. Pat was shopping and found a jacket with an original price of \$120 on clearance for \$9.99. What percent was the decrease in the cost?

10. The price of a pair of pants decreased from \$49.99 to \$19.95. What percent was the decrease in the price?

Answers

1.	$733.\overline{3}\%$	**2.**	125%	**3.**	9,900%	**4.**	≈ 8,335.51%
5.	2,200%	**6.**	≈ 69.01%	**7.**	95.12%	**8.**	\$60
9.	≈ 91.7%	**10.**	≈ 60.1%				

Simple Interest

GUIDED SKILL SUPPORT

Overview

Simple interest is interest paid or earned only on the original amount of the principal at each specified interval (such as annually or monthly). The formula for simple interest is $I = Prt$, where P is the principal, I is the interest, r is the rate, and t is the time. The total amount, including interest, would be $A = P + I$.

For additional information, see the Methods & Meanings box in Lesson 7.1.8 of the *Core Connections Course 2* text.

Examples

1. Wayne earns 5.3% simple interest for 5 years on $3,000. How much interest does he earn and what is the total amount in the account?

$P = 3{,}000 \qquad r = 5.3\% \qquad t = 5$

$I = Prt$	Substitute the numbers into the formula $I = Prt$.
$I = 3{,}000(5.3\%)5$	
$I = 3{,}000(0.053)5$	Change the percent to a decimal.
$I = 795$	Multiply. Wayne would earn $795 interest.
$3{,}000 + 795 = 3{,}795$	Add principal and interest. Wayne would have $3,795 in the account.

Practice

1. Tong loaned Jody $50. He charged 5% simple interest for the month. How much did Jody have to pay Tong?

2. Jessica's grandparents gave her $2,000 for college to put in a savings account until she finishes high school, which will be in four years. Her grandparents agreed to pay her an additional 7.5% simple interest on the $2,000 for every year. How much extra money will her grandparents give her at the end of four years?

3. David read an ad offering $8\frac{3}{4}\%$ simple interest on accounts with at least $500 for a minimum of 5 years. He has $500 and thinks this sounds like a great deal. How much money will he earn in 5 years?

4. Jair's parents set an amount of money aside when he was born. They earned 4.5% simple interest on that money each year. When Jair was 15, the account had a total of $1,012.50 interest paid on it. How much did Jair's parents set aside when he was born?

5. Kristina received \$125 for her birthday. Her parents offered to pay her 3.5% simple interest per year if she would save it for at least one year. How much interest could Kristina earn?

Answers

1. $I = 50(0.05)1 = \$2.50$; Jody paid back \$52.50.

2. $I = 2{,}000(0.075)4 = \$600$

3. $I = \$500(0.0875)5 = \$218.75$

4. $\$1{,}012.50 = x(0.045)15$; $x = \$1{,}500$

5. $I = 125(0.035)1 \approx \$4.38$

Solving Proportions

GUIDED SKILL SUPPORT

Overview

An equation stating that two ratios are equal is called a **proportion**. Some examples of proportions are shown.

$$\frac{6 \text{ miles}}{2 \text{ hours}} = \frac{9 \text{ miles}}{3 \text{ hours}} \qquad \frac{5}{7} = \frac{50}{70}$$

Proportions can be used to solve for an unknown part of one ratio when two ratios are equal. A variety of strategies can be used to reason about and solve proportions. The methods shown in the following examples include using a Giant One, undoing division, and using cross multiplication (Fraction Buster method). Depending on the proportion you are solving, some methods may be more efficient than others.

For additional information, see the Methods & Meanings boxes in Lessons 4.2.3, 4.2.4, and 7.2.2 of the *Core Connections Course 2* text. For additional examples and practice, see the *Core Connections Course 2* Checkpoint 9 materials.

Examples

1. Solve the proportion for *x*.

$$\frac{x}{24} = \frac{15}{4}$$

GIANT ONE

Since $4 \cdot 6 = 24$, use $\frac{6}{6}$ for the Giant One.

$$\frac{15}{4} = \frac{x}{24}$$

$$\frac{15}{4} \cdot \frac{6}{6} = \frac{x}{24}$$

$$\frac{15}{4} \cdot \frac{6}{6} = \frac{90}{24}$$

$$x = 90$$

UNDOING DIVISION

Since *x* is divided by 24, multiply both sides by 24 to undo the division.

$$\frac{x}{24} = \frac{15}{4}$$

$$\left(\frac{24}{1}\right)\frac{x}{24} = \frac{15}{4}\left(\frac{24}{1}\right)$$

$$x = \frac{360}{4}$$

$$x = 90$$

CROSS MULTIPLICATION (FRACTION BUSTER)

The Cross Multiplication strategy is based on the process of multiplying each side of the equation by the denominators of each ratio and setting the two sides equal. It is a shortcut for using a Fraction Buster (multiplying each side of the equation by the denominators).

FRACTION BUSTER

$$\frac{x}{24} = \frac{15}{4}$$

$$24 \cdot \frac{x}{24} = \frac{15}{4} \cdot 24$$

$$x = 15 \cdot 6$$

$$x = 90$$

CROSS-MULTIPLICATION

$$\frac{x}{24} \times \frac{15}{4}$$

$$4 \cdot x = 15 \cdot 24$$

$$4x = 360$$

$$x = 90$$

Practice

Set up a proportion for each problem and solve it using any method.

1. If a box of 100 pencils costs \$4.75, what should you expect to pay for 225 pencils?

2. When Amber does her math homework, she finishes 10 problems every 7 minutes. How long will it take for her to complete 35 problems?

3. Ben and his friends are having a TV marathon, and after 4 hours they have watched 5 episodes of a show. About how long will it take to complete the season, which has 24 episodes?

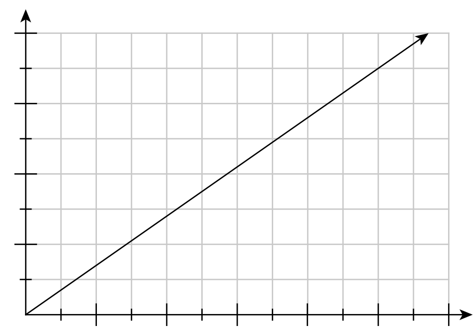
4. The tax on a \$600 vase is \$54. At this rate, what should be the tax on a \$1,700 vase?

5. Use the table shown to determine how long it will take the Spirit Club to wax 60 cars.

Number of Cars Waxed	8	16	32
Time (hours)	3	6	12

6. While baking, Evan discovered a recipe that required $\frac{1}{2}$ cups of walnuts for every $2\frac{1}{4}$ cups of flour. How many cups of walnuts will he need for 4 cups of flour?

7. Based on the graph, what would the cost be to refill 50 bottles?



8. Sam grew $1\frac{3}{4}$ inches in $4\frac{1}{2}$ months. If he continues to grow at the same rate, how much will he grow in one year?

9. On his afternoon jog, Chris took 42 minutes to run $3\frac{3}{4}$ miles. How many miles can he run in 60 minutes?

10. If Caitlin needs $1\frac{1}{3}$ cans of paint for each room in her house. How many cans of paint will she need to paint the entire 7-room house?

11. Stephen gets 20 minutes of video game time for every 45 minutes of dog walking he does. If he wants 90 minutes of game time, how many hours will he need to walk dogs?

12. Sarah's grape vine grew 15 inches in 6 weeks. How many weeks would it take the grape vine to grow 3 feet at this rate?

Answers

1.	≈ \$10.69	**2.**	24.5 minutes	**3.**	19.2 hours	**4.**	\$153
5.	22.5 hours	**6.**	$\frac{8}{9}$ cup	**7.**	\$175	**8.**	$4\frac{2}{3}$ inches
9.	≈ 5.36 miles	**10.**	$9\frac{1}{3}$ cans	**11.**	$3\frac{3}{8}$ hours	**12.**	14.4 weeks