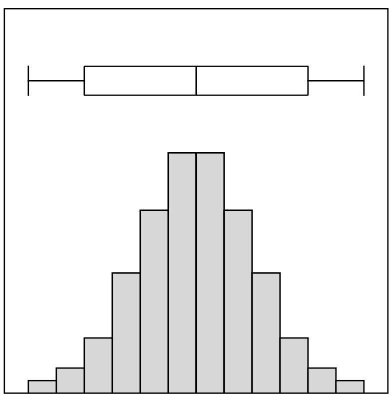
Comparing Distributions

GUIDED SKILL SUPPORT

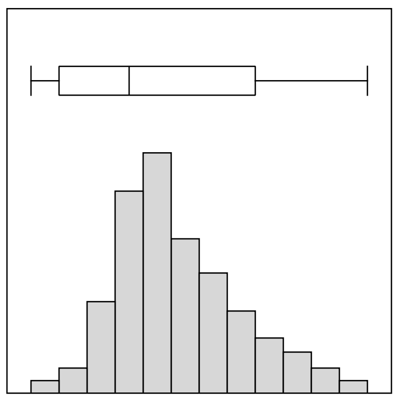
Overview

Statisticians collect and analyze data to answer questions. A data distribution shows all the values in the data set and how often they occur. Histograms and box plots are ways to display data distributions. Two distributions can be compared by analyzing their centers, shapes, variability, and outliers to understand the similarities and differences between them.

The **shape** is the overall appearance of the data in a data display. The shape of a distribution can be described as symmetric or skewed (non-symmetric).



SYMMETRIC



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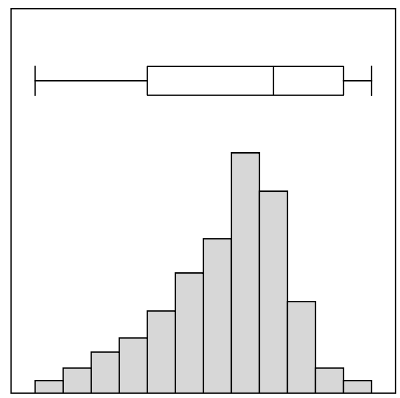

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The **center** describes the typical value of a data distribution. Generally, if the distribution is symmetric and has no outliers, the mean can be used to describe the center. If the data distribution is skewed, or there are outliers, the median might be a better way to describe the center.

An **outlier** is a data value that is far away from the rest of the data. The **variability** of the data describes how spread out the data is. The range measures the difference between the maximum and minimum values. The **mean absolute deviation** (MAD) is the measure of variability used when using the mean as a measure of center. When using the median, the interquartile range (IQR) might better express the variability of the data.

For additional information, see the Methods & Meanings boxes in Lessons 7.1.3 and 8.1.2 of the *Core Connections Course 2* text.

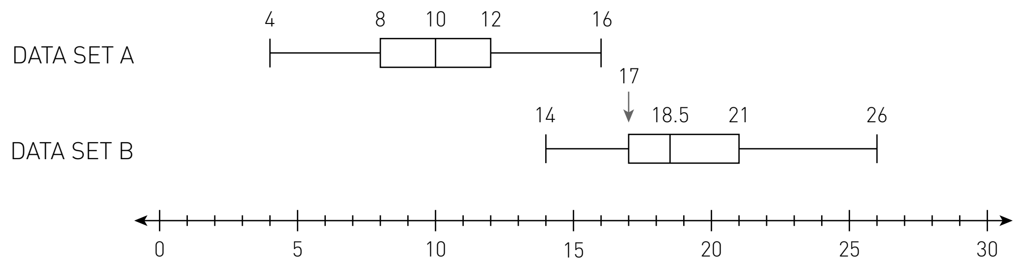
Examples

1. Compare data sets A and B.

DATA SET A
4, 4, 8, 10, 10, 10, 11, 12, 12, 16

DATA SET B
14, 14, 17, 18, 18, 19, 19, 21, 22, 26

Visually compare the two data distributions using parallel box plots.



Both data distributions appear symmetric. Most of the points in data set A are less than the points in data set B. The variability (spread) of each data set is about the same.

Measure the difference between the two centers. Since the distributions are symmetric and there are no outliers, the mean is a helpful measure of center.

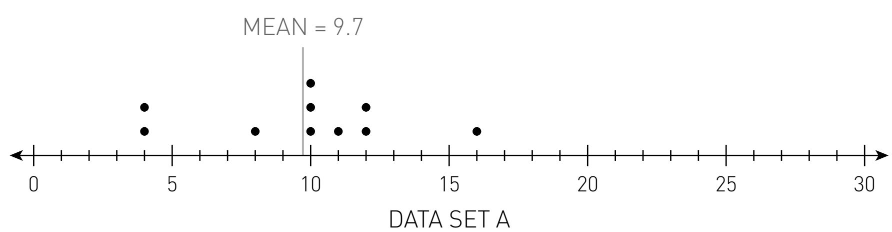
Mean of data set A = (4 + 4 + 8 + 10 + 10 + 10 + 11 + 12 + 12 + 16) ÷ 10 = 9.7

Mean of data set B = (14 + 14 + 17 + 18 + 18 + 19 + 19 + 21 + 22 + 26) ÷ 10 = 18.8

Difference between centers = 18.8 − 9.7 = 9.1

Since the two distributions have similar variability, it can be helpful to express the difference between the centers as a multiple of a measure of variability.

Since the mean was used as the measure of center, mean absolute deviation (MAD) is an appropriate measure of variability. Recall that the MAD is the average of the distances from the mean.



MAD of data set A = (5.7 + 5.7 + 1.7 + 0.3 + 0.3 + 0.3 + 1.3 + 2.3 + 2.3 + 6.3) ÷ 10 = 2.62

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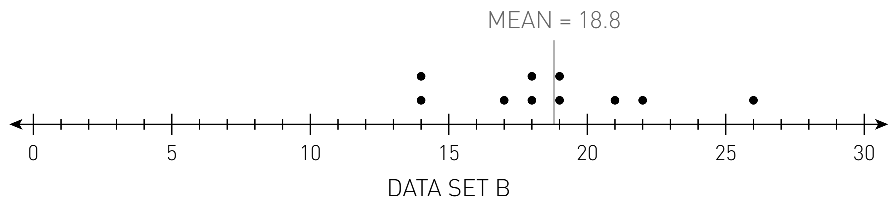
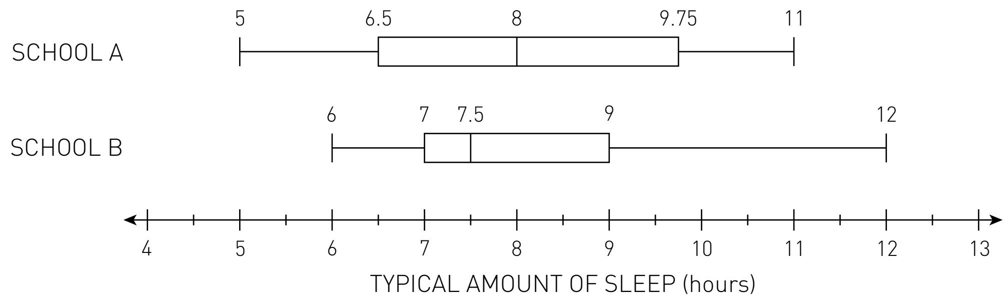


MAD of data set B = (4.8 + 4.8 + 1.8 + 0.8 + 0.8 + 0.2 + 0.2 + 2.2 + 3.2 + 7.2) ÷ 10 = 2.6

The MAD is approximately 2.6 for both distributions, and the difference between the centers is 9.1; 9.1 ÷ 2.6 = 3.5.

Therefore, the mean of data set A is 9.1 less than the mean of data set B, which is about 3.5 times the variability (mean absolute deviation) of either data set.

2. Students from two schools were randomly selected to participate in a survey. They were asked, "How many hours of sleep do you typically get on a school night?" Their responses are shown in the following box plots. How do the students' sleep times compare in the two schools?



Compare the shapes of the two data distributions.

The data representation is symmetric for School A and skewed right for School B. Therefore, sleep time appears to be evenly distributed for students in School A; whereas in School B, a few of the students get a lot of sleep, while most get less than 9 hours a night.

Compare the average sleep time for students at the two schools.

The median for School A is 8 hours, and the median for School B is 7.5 hours. Therefore, the students in School A get, on average, a half-hour more sleep than the students in School B (assuming that the random samples are representative of the populations).

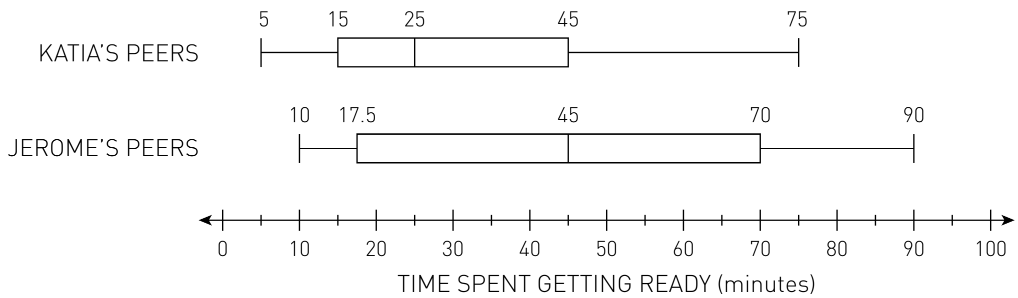
Compare the variability in the sleep data.

The IQR for School A is 3.25 hours, and the IQR for School B is 2 hours. Therefore, there is more variability in the amount of sleep the students in School A get than the students in School B.

Note that the range (6 hours) is the same for both schools.

Practice

1. Katia and Jerome are childhood friends who attend different schools. They are interested in knowing how much time their peers spend getting ready for school each morning. They asked everyone in their classes, "About how much time (in minutes) do you spend getting ready for school each day?" The box plots shown represent the responses.



a. What is the difference between the two medians? Describe the difference in context.

b. Compare the variability of the time Katia's peers spent getting ready and the time Jerome's peers spent using the interquartile range (IQR). What does this tell you about the two groups of friends?

2. After comparing time getting prepared for school, Katia and Jerome were curious about the amount of time, in hours, their peers spent each week on out-of-school activities. They each surveyed 10 random students. The student responses are listed.

KATIA'S PEERS
4, 6, 12, 12, 14, 16, 18, 22, 24, 26

JEROME'S PEERS
4, 6, 8, 10, 12, 15, 17, 18, 20, 25

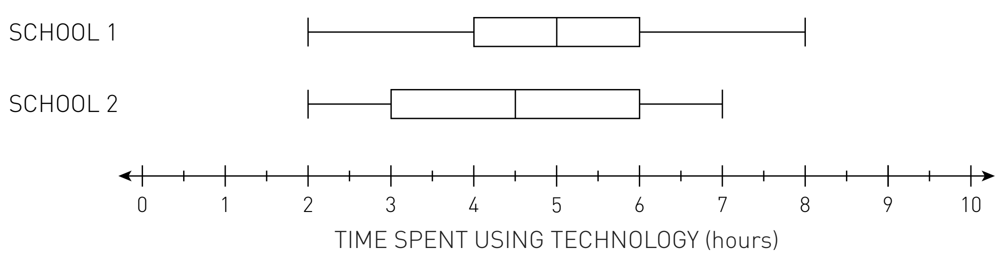
a. Calculate the mean for each group. How do they compare? Describe what this means in terms of time spent on out-of-school activities by each group of peers.

b. Compare the variability of Katia's peers and Jerome's peers using the mean absolute deviation (MAD). How do they compare? Describe what this means.

3. Students from two schools were randomly selected to participate in a survey. They were asked, "How many hours do you spend using technology each day?" Their responses are shown in the following lists and in the parallel box plot.

SCHOOL 1
2, 3, 4, 4, 5, 5, 6, 6, 7, 8

SCHOOL 2
2, 2, 3, 4, 4, 5, 5, 6, 6, 7



Examine the two data sets and describe their similarities and differences. Use the measures of center and variability to justify your answer.

Answers

Responses may vary.

1. a. The median for Katia's peers is 25 minutes. The median for Jerome's peers is 45 minutes. The difference between the medians is 20 minutes. This means that Jerome's peers spend on average 20 more minutes getting ready for school each morning than Katia's peers.

b. The interquartile range (IQR) for Katia's peers is 30 minutes, and the IQR for Jerome's peers is 52.5 minutes. This means that there is more variability in the amount of time Jerome's friends take to get ready than between Katia's friends.

2. a. The mean for Katia's peers is 15.4 hours, and the mean for Jerome's peers is 13.5 hours. The difference between the means is 1.9 hours. This means that on average, Katia's peers spend about 1.9 more hours each week on out-of-school activities than Jerome's peers.

b. The mean absolute deviation (MAD) for Katia's peers is 5.8 hours and the MAD for Jerome's peers is 5.5 hours. Both groups show about the same variability in time spent on out-of-school activities.

3. Methods may vary. Since the distributions are symmetric and there are no apparent outliers, the mean and mean absolute deviation can be used to compare the data sets. The mean and MAD for School 1 are 5 and 1.4 hours, respectively. The mean and MAD for School 2 are 4.4 and 1.4 hours, respectively. The average time spent using technology each day is slightly higher for School 1 than School 2, and the variability is about the same.

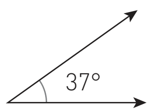
Angles and Angle Pair Relationships

GUIDED SKILL SUPPORT

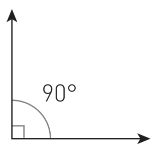
Overview

NAMING ANGLES

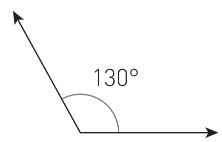
Acute angles are angles with measures between (but not including) 0° and 90°, right angles measure 90°, and obtuse angles measure between (but not including) 90° and 180°. A straight angle measures 180°.



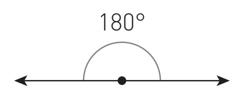
ACUTE ANGLE



RIGHT ANGLE



OBTUSE ANGLE

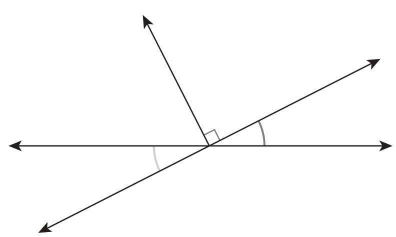


STRAIGHT ANGLE

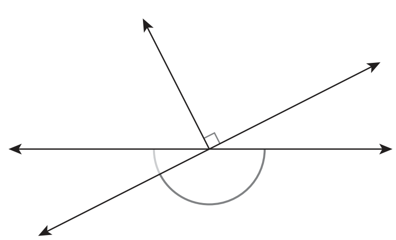
Protractors can be used to measure the size of angles.

ANGLE RELATIONSHIPS

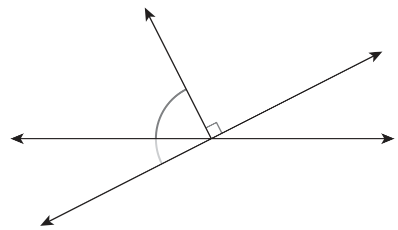
Intersecting lines form four angles. **Vertical** angles are the two opposite (that is, non-adjacent) angles formed by two intersecting lines or line segments. Vertical angles always have equal measures.



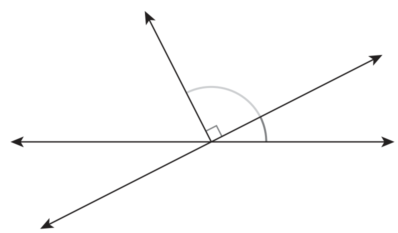
If two angles have measures that add up to 180°, they are called **supplementary** angles. The two angles indicated are supplementary.



If two angles have measures that add up to 90°, they are called **complementary** angles. The angles indicated are complementary because together, they form a right angle.



Adjacent angles are angles that are next to each other. They must have a common vertex, share a common side, and have no interior points in common.

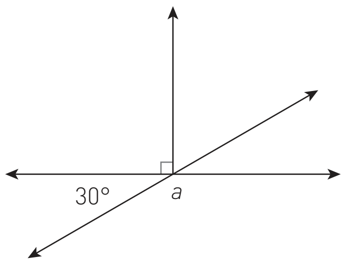


For more information see the Methods & Meanings box in Lesson 8.3.1 and Lesson 8.3.2 of the *Core Connections Course 2* text.

Examples

1. Determine the unknown angle measurement. Lines that appear to be straight are straight.

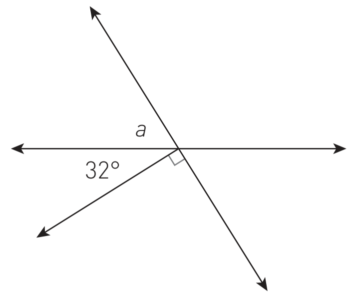
a.



The two angles are supplementary, so their measures sum to 180°.

$$30° + a = 180°$$
$$a = 150°$$

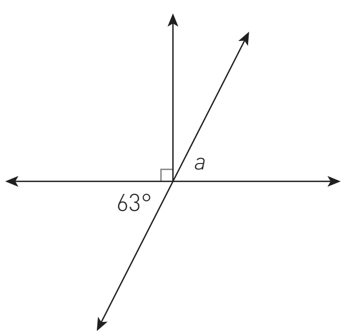
b.



The 32° angle and the angle labeled *a* are complementary, so their measures sum to 90°.

$$32° + a = 90°$$
$$a = 58°$$

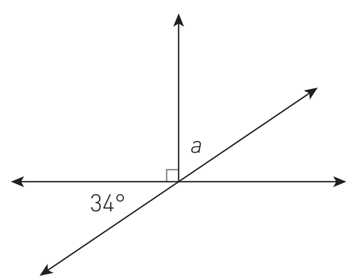
c.



The two angles are vertical angles, so their measures are equal.

$$a = 63°$$

d.

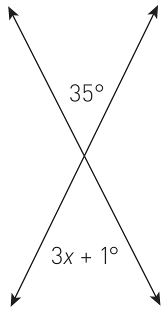


The two angles together with the right angle have measures that total 180°. 180° – 90° = 90°, so the two angles are complementary.

$$34° + a = 90°$$
$$a = 56°$$

2. Use what you know about angle relationships to write an equation and then solve for *x*. Lines that appear straight are straight. If necessary, round your answer to the nearest hundredth.

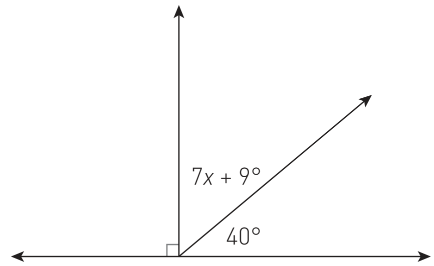
a.



The two angles are vertical angles, so their measures are equal.

$$\begin{aligned} 3x + 1 &= 35 \\ -1 \quad &\;\; -1 \\ 3x &= 34 \\ \div 3 \quad &\;\; \div 3 \\ x &= \tfrac{34}{3} \\ x &\approx 11.33^\circ \end{aligned}$$

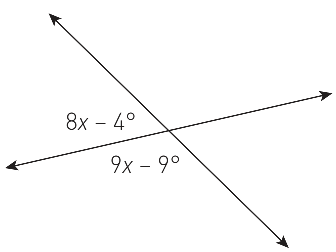
b.



The two angles are complementary, so their measures sum to 90°.

$$\begin{aligned} (7x - 9) + (40) &= 90 \\ 7x + 31 &= 90 \\ -31 \quad &\;\; -31 \\ 7x &= 59 \\ \div 7 \quad &\;\; \div 7 \\ x &= \tfrac{59}{7} \\ x &\approx 8.49^\circ \end{aligned}$$

c.



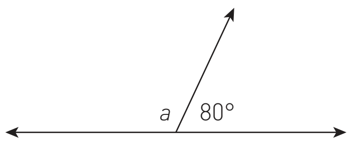
The two angles are supplementary, so their measures sum to 180°.

$$\begin{aligned} (8x - 4) + (9x - 9) &= 180 \\ 8x + 9x - 4 - 9 &= 180 \\ 17x - 13 &= 180 \\ +13 \quad &\;\; +13 \\ 17x &= 193 \\ \div 17 \quad &\;\; \div 17 \\ x &= \tfrac{193}{17} \\ x &\approx 11.35^\circ \end{aligned}$$

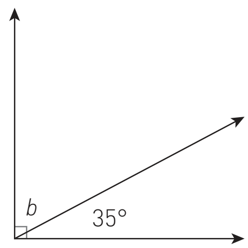
Practice

Determine the value of each variable using angle relationships.

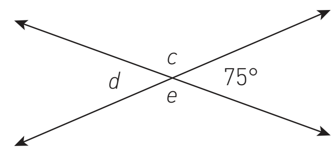
1.



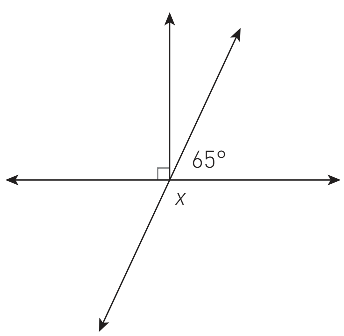
2.



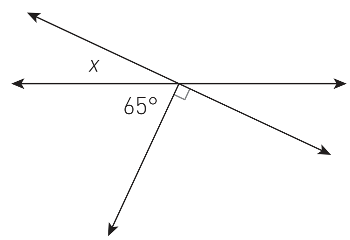
3.



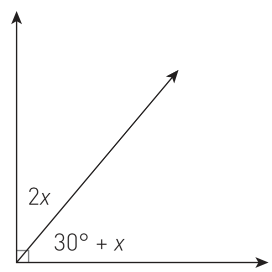
4.



5.



6.



Answers

1. $a = 100°$

2. $b = 55°$

3. $c = 105°$, $d = 75°$, $e = 105°$

4. $x = 115°$

5. $x = 25°$

6. $x = 20°$

Triangle Properties

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Overview

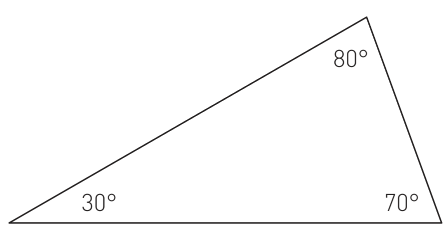
A triangle is a polygon with three sides, three angles, and three vertices. For three side lengths to form a triangle, the sum of the lengths of any two sides must be greater than the length of the third side. Three side lengths will either form one unique triangle or no triangle. Three angles do not determine a unique triangle.

For more information, see the Methods & Meanings box in Lesson 8.3.1 and Lesson 8.3.4 of the *Core Connections Course 2* text.

Examples

1. Draw a triangle with angles measuring 70°, 30°, and 80°. Is your triangle unique? Justify your answer.

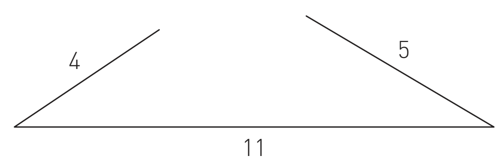
Three angles do not define a unique triangle, since many other triangles can be constructed with the same angles by varying the side lengths. One example triangle with angles 70°, 30°, and 80° is shown.



2. Can a triangle be formed with the following side lengths?

a. 5 cm, 11 cm, 4 cm

No triangle can be formed because $4 + 5 \not> 11$.



b. 8 cm, 10 cm, 7 cm

A triangle can be formed because the sum of any two lengths is greater than the third length: $8 + 7 > 10$, $8 + 10 > 7$, and $7 + 10 > 8$.

Practice

Determine whether a triangle can be created with the given conditions. If a triangle can be made, determine whether it is unique.

1. side lengths: 12 in, 8 in, 9 in

2. side lengths: 4 cm, 10 cm, 5 cm

3. side lengths: 3 cm, 7 cm, 6 cm

4. side lengths: 25 ft, 22 ft, 12 ft

5. side lengths: 18 in, 12 in, 5 in

6. side lengths: 12 in, 6 in, 5 in

7. side lengths: 6 ft, 8 ft, angle: 45°

8. side length: 4 cm, angles: 60°, 60°, 60°

9. angles: 90°, 60°, 30°

Answers

1. yes, unique

2. no triangle

3. yes, unique

4. yes, unique

5. no triangle

6. no triangle

7. yes, not unique

8. yes, unique

9. yes, not unique