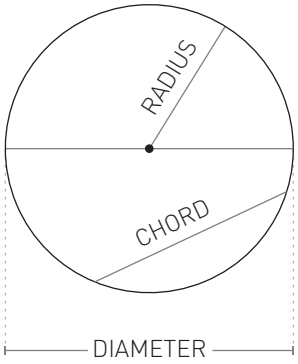
Circumference and Area of Circles

GUIDED SKILL SUPPORT

Overview

The radius of a circle is a line segment from its center to any point on the circle. The diameter of a circle is a line segment that passes through its center and connects two points on the circle. The length of a diameter is twice the length of a radius, that is, $d = 2r$ or $r = \frac{d}{2}$.



A chord of a circle is a line segment joining any two points on a circle.

The circumference of a circle is similar to the perimeter of a polygon. The circumference is the length around a circle. It would tell you how much string would be needed to go around a circle once.

The ratio between the circumference and the diameter of any circle is the number π (pi). That is, $\pi = \frac{C}{d}$, where C is the circumference of a circle and d is its diameter. The number π can be approximated with the fraction $\frac{22}{7}$, the decimal 3.14, or the π button on a calculator.

The formula for the circumference of a circle is $C = 2\pi r$, where r is the radius of the circle (half the diameter). The formula can also be written as $C = \pi d$, where d represents the diameter of the circle, since the diameter is equal to twice the radius of a circle. The units of the circumference of the circle should match the units of the radius and diameter.

The formula for the area of a circle is $A = \pi r^2$, where r is the radius of the circle. Area is measured in square units.

To read more about the in-class exploration of the area of a circle, see Lesson 9.1.2 in the *Core Connections Course 2* text. For additional information, see the Methods & Meanings box in Lesson 9.1.2 of the *Core Connections Course 2* text.

Examples

1. Calculate the circumference and area of a circle with a radius of 17 feet. Approximate the number π with the fraction $\frac{22}{7}$, the decimal 3.14, or the π button on a calculator.

To determine the circumference, use $C = 2\pi r$ and substitute 17 for r.

$r = 17$

$C = 2\pi r$

$= 2\pi(17)$

$= 34\pi$

≈ 106.81 feet

To determine the area of the circle, use $A = \pi r^2$ and substitute 17 for r.

$r = 17$

$A = \pi r^2$

$= \pi(17)^2$

$= \pi \cdot 289$

≈ 907.92 square feet

2. Calculate the circumference and area of a circle with a diameter of 84 cm. Approximate the number π with the fraction $\frac{22}{7}$, the decimal 3.14, or the π button on a calculator.

To calculate the circumference when the diameter is given, use $C = \pi d$ and substitute 84 for d.

$$\begin{aligned} d &= 84 \\ &\downarrow \\ C &= \pi d \\ &= \pi(84) \\ &= 84\pi \\ &\approx 263.89 \text{ cm} \end{aligned}$$

To determine the area of the circle, use the formula $A = \pi r^2$. First, calculate the radius of the circle, which is half of the diameter: 84 ÷ 2 = 42 cm. Then substitute 42 for r.

$$\begin{aligned} r &= 42 \\ &\downarrow \\ A &= \pi r^2 \\ &= \pi(42)^2 \\ &= \pi \cdot 1{,}764 \\ &\approx 5{,}541.77 \text{ square cm} \end{aligned}$$

3. Determine the radius and diameter of a circle with a circumference of 25.12 cm. Approximate the number π with the fraction $\frac{22}{7}$, the decimal 3.14, or the π button on a calculator.

Since circumference is given, use the formula $C = \pi d$ to calculate the diameter.

$$\begin{aligned} C &= 25.12 \\ &\downarrow \\ C &= \pi d \\ 25.12 &= \pi d \\ \frac{25.12}{\pi} &= \frac{\pi d}{\pi} \\ 8 \text{ cm} &\approx d \end{aligned}$$

Since the radius is half of the diameter, $\frac{d}{2} = r$.

$$\begin{aligned} d &\approx 8 \\ &\downarrow \\ \frac{d}{2} &= r \\ \frac{8}{2} &\approx r \\ 4 \text{ cm} &\approx r \end{aligned}$$

Practice

Calculate the circumference and the area of the circles with the following radii or diameter lengths. Approximate the number π with the fraction $\frac{22}{7}$, the decimal 3.14, or the π button on a calculator. Round to the nearest hundredth.

1. $r = 6$ cm **2.** $r = 3.2$ in **3.** $d = 16$ ft **4.** $r = \frac{1}{2}$ m

5. $d = \frac{4}{5}$ cm **6.** $r = 5$ in **7.** $r = 3.6$ cm **8.** $r = 2\frac{1}{4}$ in

9. $d = 14.5$ ft **10.** $r = 12.02$ m

Determine the radius and diameter of each circle given the circumference. Approximate the number π with the fraction $\frac{22}{7}$, the decimal 3.14, or the π button on a calculator. Round your answer to the nearest hundredth.

11. $C = 48.36$ yd **12.** $C = 35.6$ ft **13.** $C = 194.68$ mm

Answers

Responses may vary depending on which approximation for π was used in the calculation. 3.14 was used to calculate the following answers.

1. $C = 37.68$ cm, $A = 113.04$ cm^2

2. $C = 20.10$ in, $A = 32.15$ in^2

3. $C = 50.24$ ft, $A = 200.96$ ft^2

4. $C = 3.14$ m, $A = 0.79$ m^2

5. $C = 5.02$ cm, $A = 2.51$ cm^2

6. $C = 31.4$ in, $A = 78.5$ in^2

7. $C = 22.61$ cm, $A = 40.69$ cm^2

8. $C = 14.13$ in, $A = 15.90$ in^2

9. $C = 45.53$ ft, $A = 165.05$ ft^2

10. $C = 75.49$ m, $A = 453.67$ m^2

11. $r = 7.70$ yd, $d = 15.40$ yd

12. $r = 5.67$ ft, $d = 11.34$ ft

13. $r = 31$ mm, $d = 62$ mm

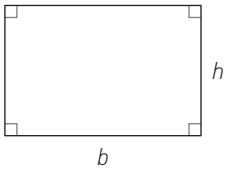
Area of Composite Shapes

GUIDED SKILL SUPPORT

Overview

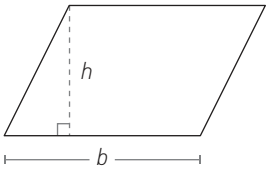
Knowledge of the areas of polygons (rectangles, parallelograms, triangles, trapezoids) can be used to determine the areas of more complicated figures. One way to do this is to use subproblems (that is, solving smaller problems in order to solve a larger problem).

Area is the number of square units in a flat region. The formulas to calculate the area of several kinds of polygons are shown.



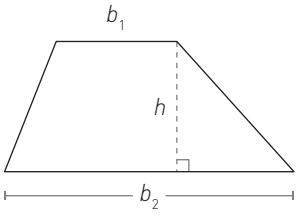
RECTANGLE

$A = bh$



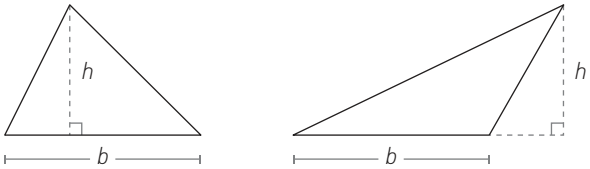
PARALLELOGRAM

$A = bh$



TRAPEZOID

$A = \frac{1}{2}(b_1 + b_2)h$



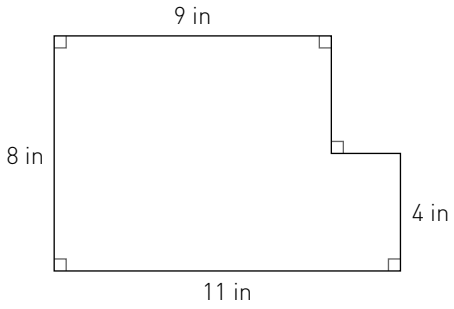
TRIANGLE

$A = \frac{1}{2}bh$

For a review of calculating the area of polygons, see the *Core Connections Course 1* Chapter 5 Guided Skill Support "Area of Polygons." For additional general information, see the Methods & Meanings box in Lesson 1.1.2 of the *Core Connections Course 2* text. For additional examples and practice, see the *Core Connections Course 2* Checkpoint 1 materials.

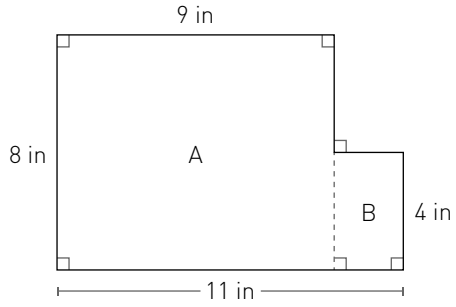
Examples

1. Determine the area of the figure.



METHOD 1

Divide the figure into rectangles and calculate the area of each rectangle.



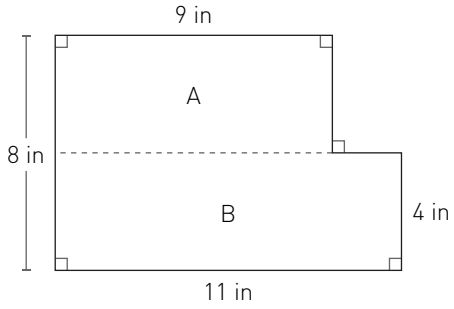
Rectangle A: $8 \cdot 9 = 72$ square inches

Rectangle B:
$4 \cdot (11 - 9) = 4 \cdot 2 = 8$ square inches

Then add the area of rectangle A to the area of rectangle B: $72 + 8 = 80$ square inches

METHOD 2

Divide the figure into rectangles and calculate the area of each rectangle.



Rectangle A:
$9 \cdot (8 - 4) = 9 \cdot 4 = 36$ square inches

Rectangle B: $11 \cdot 4 = 44$ square inches

Then add the area of rectangle A to the area of rectangle B: $36 + 44 = 80$ square inches

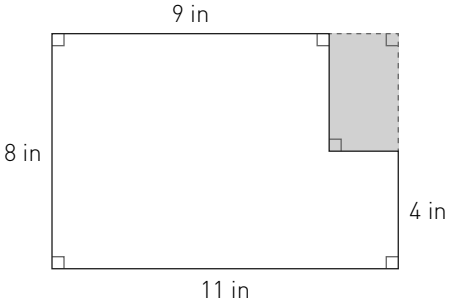
METHOD 3

Make a large rectangle by enclosing the upper right corner.

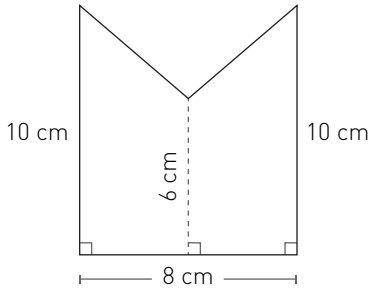
Area of the new, larger rectangle:
$8 \cdot 11 = 88$ square inches

Area of shaded rectangle:
$(8 - 4) \cdot (11 - 9) = 4 \cdot 2 = 8$ square inches

Then subtract the area of the shaded rectangle from the area of the larger rectangle: $88 - 8 = 80$ square inches

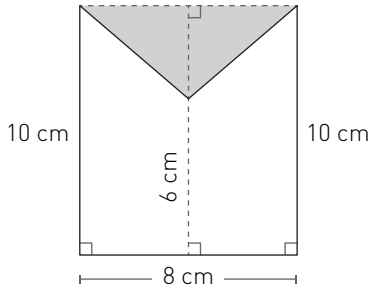


2. Determine the area of the figure.



One method is shown, but there are many other ways to approach this problem.

Make a rectangle out of the figure by enclosing the top.



Area of entire rectangle: $8 \cdot 10 = 80$ square cm

Area of shaded triangle: $b = 8$ and $h = 10 - 6 = 4$, so $A = \frac{1}{2}(8 \cdot 4) = \frac{1}{2}(32) = 16$ square cm

Subtract the area of the triangle from the area of the rectangle: $80 - 16 = 64$ square cm.

Practice

Determine the area of each figure.

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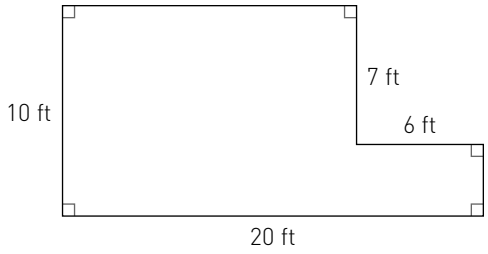

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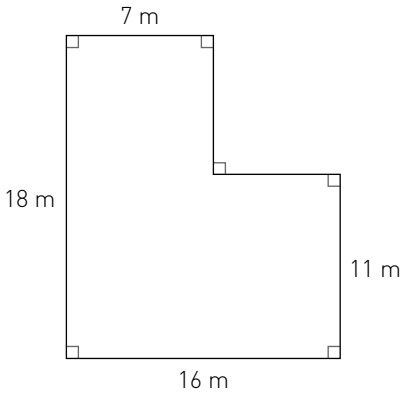

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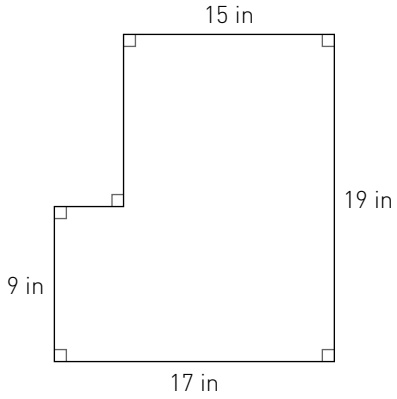

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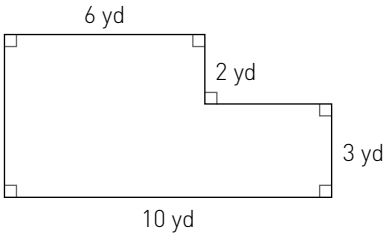

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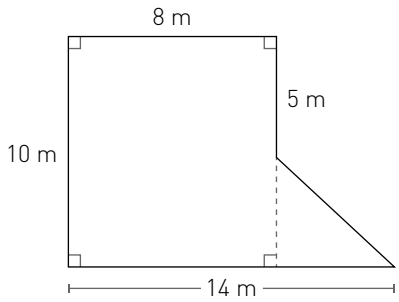

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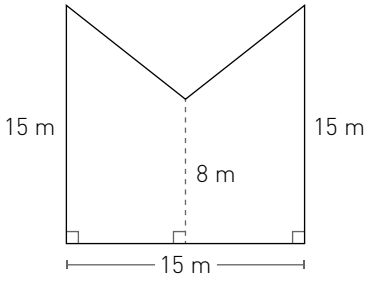

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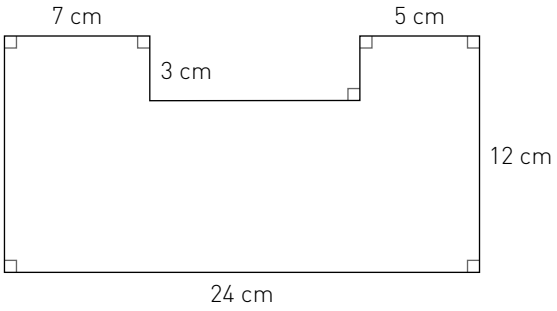

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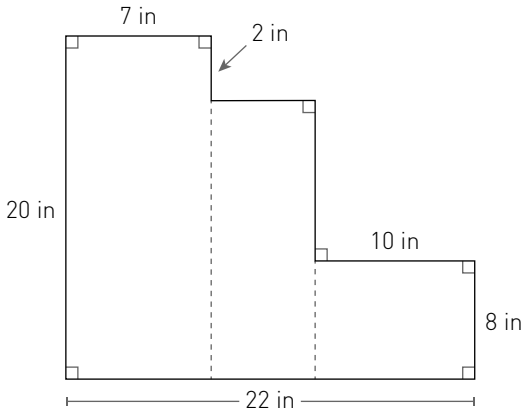

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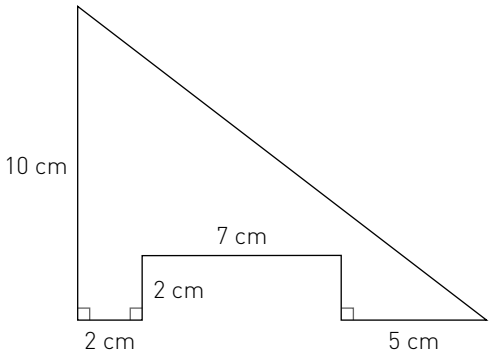

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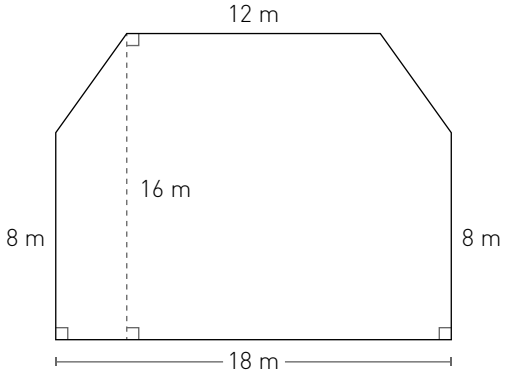
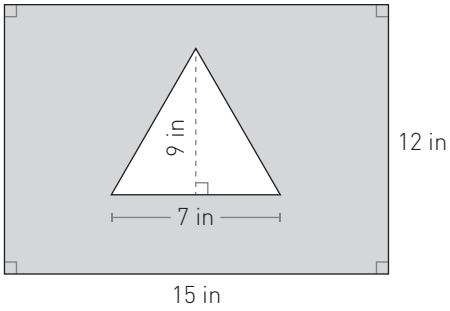
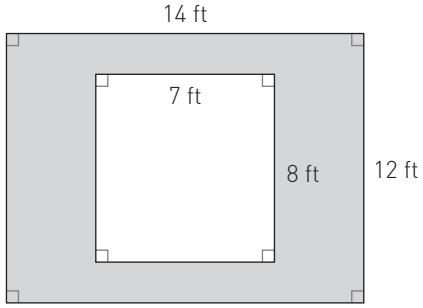


11. Determine the area of the shaded region.



12. Determine the area of the shaded region.



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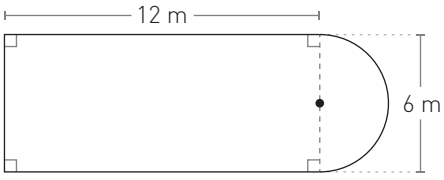
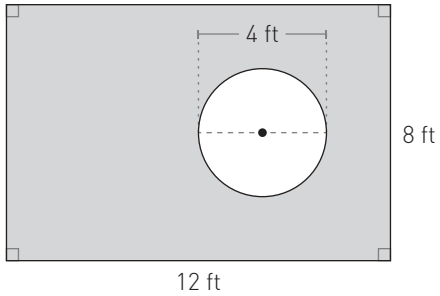


14. Determine the area of the shaded region.



Answers

1.	158 ft^2	**2.**	225 m^2	**3.**	303 in^2	**4.**	42 yd^2	**5.**	95 m^2
6.	172.5 m^2	**7.**	252 cm^2	**8.**	310 ft^2	**9.**	56 cm^2	**10.**	264 m^2
11.	148.5 in^2	**12.**	112 ft^2	**13.**	86.1 m^2	**14.**	83.4 ft^2		

Volume and Surface Area of Prisms

GUIDED SKILL SUPPORT

Overview

Volume is a three-dimensional concept. It measures the amount of interior space of a three-dimensional figure based on a cubic unit, that is, the number of 1 by 1 by 1 cubes that will fit inside a figure. The volume of a prism is the area of the base, B, multiplied by the height, h, of the prism. Note that either base of a prism will have the same area, so you can use the area of either one.

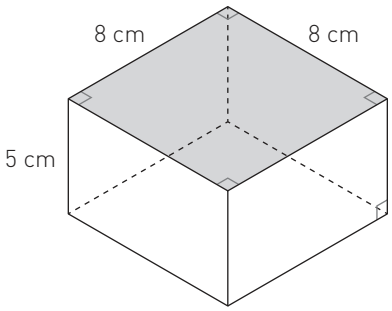
$$\text{Volume} = (\text{Area of base}) \cdot (\text{height}) \text{ or } V = Bh$$

The surface area of a prism is the sum of the areas of all of the faces, including the bases. Surface area is expressed in square units.

For additional information, see the Methods & Meanings box in Lesson 9.2.3 of the *Core Connections Course 2* text.

Examples

1. Calculate the volume and surface area of the square prism.



VOLUME

Calculate the area of one of the bases, which is a square.

$$B = s^2$$
$$= 8^2$$
$$= 64 \text{ cm}^2$$

Calculate the volume of the prism by multiplying the area of either base, B, by the height, h, of the prism.

$$V = Bh$$
$$= 64 \cdot 5$$
$$= 320 \text{ cm}^3$$

SURFACE AREA

Calculate the area of the two bases (top and bottom). There are two square bases, and the area of one of the bases is 64 cm^2 (from volume calculations).

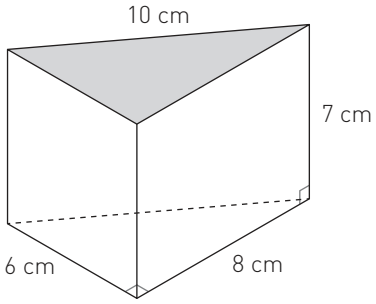
Calculate the area of each lateral face. There are four lateral faces, and they are all the same rectangle.

$$8 \cdot 5 = 40 \text{ cm}^2$$

Calculate the surface area by adding together the areas of the bases and the lateral faces.

$$64 + 64 + 40 + 40 + 40 + 40 = 288 \text{ cm}^2$$

2. Calculate the volume and surface area of the triangular prism.



VOLUME

Calculate the area of one of the bases, which is a triangle.

$$B = \frac{1}{2}bh$$
$$= \frac{1}{2}(8)(6)$$
$$= 24 \text{ cm}^2$$

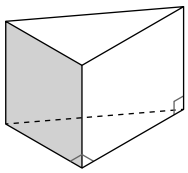
Calculate the volume of a prism by multiplying the area of either base, B, by the height, h, of the prism.

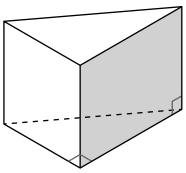
$$V = Bh$$
$$= 24 \cdot 7$$
$$= 168 \text{ cm}^3$$

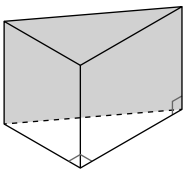
SURFACE AREA

Calculate the area of the two bases (top and bottom). There are two triangle bases, and the area of one of the bases is 24 cm^2 (from volume calculations).

Calculate the area of each lateral face. There are three lateral faces, and they are all rectangles.


FACE 1


FACE 2


FACE 3

Area of face 1: $6 \cdot 7 = 42 \text{ cm}^2$

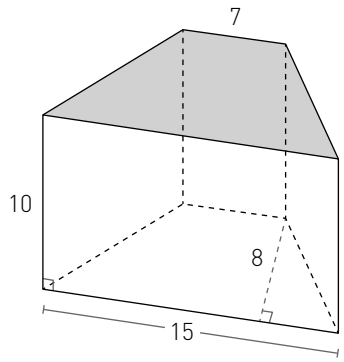
Area of face 2: $8 \cdot 7 = 56 \text{ cm}^2$

Area of face 3: $10 \cdot 7 = 70 \text{ cm}^2$

Calculate the surface area by adding together the areas of the bases and the lateral faces.

$$24 + 24 + 42 + 56 + 70 = 216 \text{ cm}^2$$

3. Calculate the volume of the trapezoidal prism.



Calculate the area of one of the bases, which is a trapezoid.

$$B = \frac{1}{2}(b_1 + b_2)h$$
$$= \frac{1}{2}(7 + 15) \cdot 8$$
$$= 88 \text{ units}^2$$

Calculate the volume by multiplying the area of the base, B, by the height, h.

$$V = Bh$$
$$= 88 \cdot 10$$
$$= 880 \text{ units}^2$$

4. Calculate the height of a right prism with a volume of 132.5 cm^3 and base area of 25 cm^2.

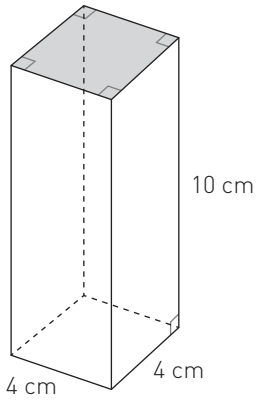
The volume of a prism is calculated by multiplying the base area by the height of the prism. When given the volume, inverse operations (division in this case) can be used to calculate the missing dimension of the prism.

$$V = Bh$$
$$132.5 = 25(h)$$
$$\frac{132.5}{25} = h$$
$$5.3 \text{ cm} = h$$

Practice

Calculate the volume and surface area of each prism. The base of each figure is shaded.

1.



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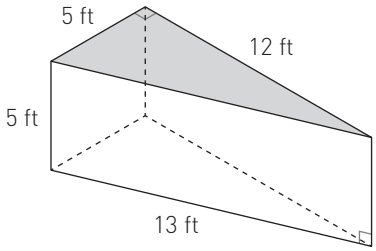

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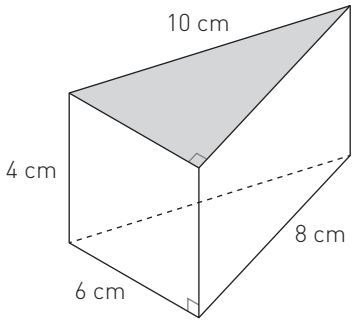

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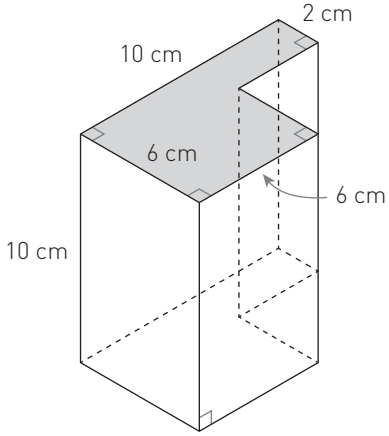
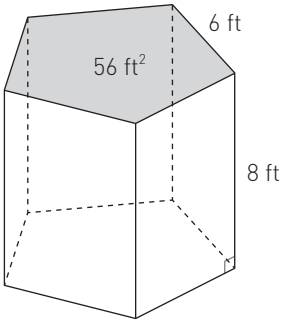


5. All sides of the base pentagon have the same length of 6 feet.



6. Determine the volume of a rectangular prism with base area 32 cm^2 and height 1.5 cm.

7. Determine the height of a rectangular prism with base area 32 cm^2 and volume 176 cm^3.

8. Determine the base area of a triangular prism with volume 47.01 cm^3 and height 3.2 cm.

Answers

1. surface area = 192 cm^2, volume = 160 cm^3

2. surface area = 210 ft^2, volume = 150 ft^3

3. surface area = 144 cm^2, volume = 96 cm^3

4. surface area = 408 cm^2, volume = 440 cm^3

5. surface area = 352 ft^2, volume = 448 ft^3

6. 48 cm^3

7. 5.5 cm

8. 14.69 cm^2