

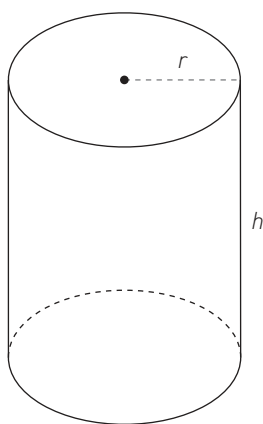
Volume and Surface Area of a Cylinder

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Volume of a Cylinder

Overview

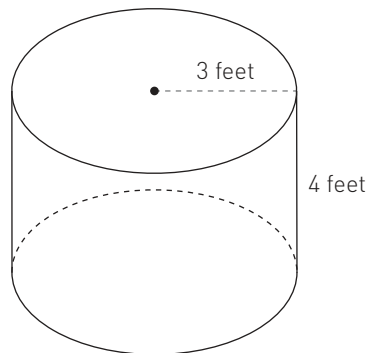
The volume of a cylinder is the area of its base multiplied by the length of its height: $V = B \cdot h$. Since the base of a cylinder is a circle of area $A = r^2\pi$, we can write $V = r^2\pi h$.



For additional information, see the Methods & Meanings box in Lesson 10.1.2 of the *Core Connections Course 3* text.

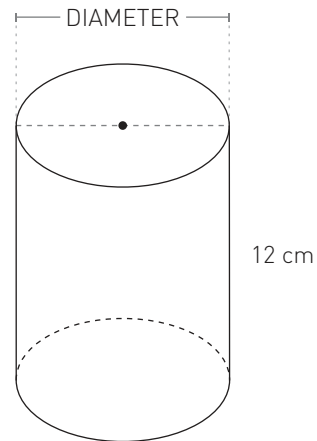
Examples

1. Calculate the volume of the cylinder shown. Use a calculator for the value of π .



$$\begin{aligned}
 r &= 3 & h &= 4 \\
 V &= r^2 \pi h \\
 &= (3)^2 \pi (4) \\
 &= 36\pi \\
 &\approx 113.10 \text{ ft}^3
 \end{aligned}$$

2. The cylinder shown has a volume of 355 cm^3 and a height of 12 cm. What is its diameter? Use a calculator for the value of π .



$$\begin{aligned}
 V &= 355 & h &= 12 \\
 V &= r^2 \pi h \\
 355 &= r^2 \pi (12) \\
 \frac{355}{12\pi} &= r^2 \\
 9.42 &= r^2 \\
 r &\approx 3.07 \\
 d &= 2r \\
 d &\approx 2(3.07) \\
 d &\approx 6.14 \text{ cm}
 \end{aligned}$$

Practice

Calculate the volume of each cylinder.

1. $r = 5 \text{ cm}$, $h = 10 \text{ cm}$
2. $r = 7.5 \text{ inches}$, $h = 8.1 \text{ inches}$
3. diameter = 10 cm , $h = 5 \text{ cm}$
4. base area = 50 cm^2 , $h = 4 \text{ cm}$
5. $r = 17 \text{ cm}$, $h = 10 \text{ cm}$
6. $d = 29 \text{ cm}$, $h = 13 \text{ cm}$

Calculate the missing part of each cylinder.

7. If the volume is $5,175 \text{ ft}^3$ and the height is 23 ft , calculate the diameter.
8. If the volume is $26,101.07 \text{ inches}^3$ and the radius is 17.23 inches , calculate the height.
9. If the circumference is 126 cm and the height is 15 cm , calculate the volume.

Answers

- | | | |
|--------------------------|----------------------------|-----------------------------|
| 1. 785.40 cm^3 | 2. $1,431.39 \text{ in}^3$ | 3. 392.70 cm^3 |
| 4. 200 cm^3 | 5. $9,079.20 \text{ cm}^3$ | 6. $8,586.76 \text{ cm}^3$ |
| 7. 16.93 ft | 8. 28 inches | 9. $18,951.68 \text{ cm}^3$ |

Surface Area of a Cylinder

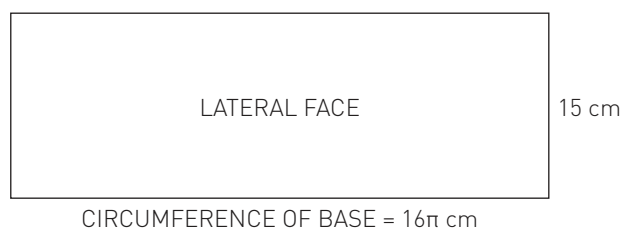
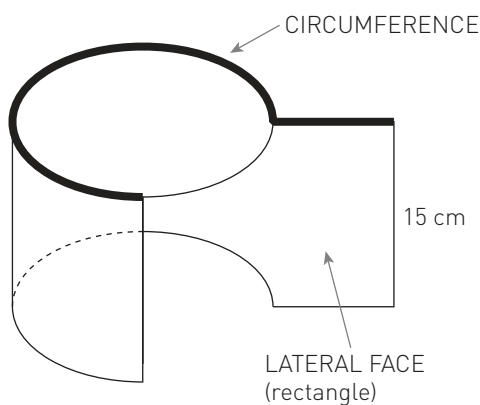
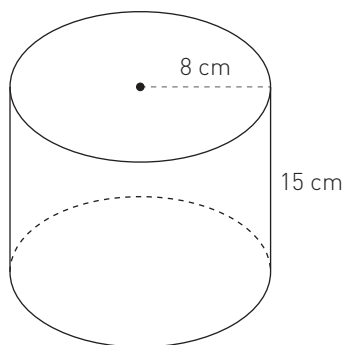
Overview

The surface area of a cylinder is the sum of the two base areas and the lateral surface area. The formula for the surface area is $SA = 2r^2\pi + \pi dh$ or $SA = 2r^2\pi + 2\pi rh$, where r = radius, d = diameter, and h = height of the cylinder.

For additional information, see the Methods & Meanings box in Lesson 10.1.3 of the *Core Connections Course 3* text.

Examples

1. Calculate the volume of the cylinder shown. Use a calculator for the value of π .

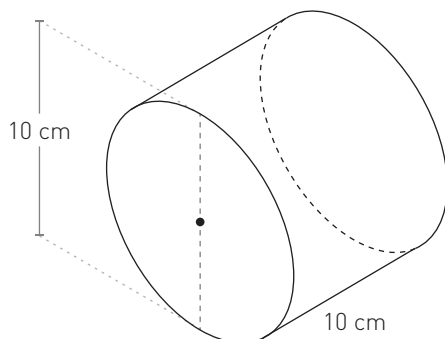


Area of the two circular bases: $2(8)^2\pi = 128\pi \text{ cm}^2$

Area of the lateral face: $\pi(16)15 = 240\pi \text{ cm}^2$

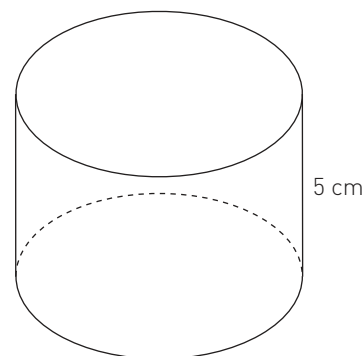
Surface area of the cylinder: $128\pi + 240\pi = 368\pi \text{ cm}^2 \approx 1,156.11 \text{ cm}^2$

2. Calculate the surface area of the cylinder. Use a calculator for the value of π .



$$\begin{aligned}
 r &= 5 & h &= 10 \\
 SA &= 2r^2\pi + 2\pi rh \\
 &= 2(5)^2\pi + 2\pi(5)(10) \\
 &= 50\pi + 100\pi \\
 &= 150\pi \\
 &\approx 471.24 \text{ cm}^2
 \end{aligned}$$

3. If the volume of the cylinder shown is $500\pi \text{ cm}^3$, what is the surface area?



$$\begin{aligned}
 V &= 500\pi & h &= 5 \\
 V &= \pi r^2 h \\
 500\pi &= \pi r^2(5) \\
 \frac{500\pi}{5\pi} &= r^2 \\
 100 &= r^2 \\
 10 &= r \\
 SA &= 2r^2\pi + 2\pi rh \\
 &= 2(10)^2\pi + 2\pi(10)(5) \\
 &= 200\pi + 100\pi \\
 &= 300\pi \\
 &\approx 942.48 \text{ cm}^2
 \end{aligned}$$

Practice

Calculate the surface area of each cylinder.

- $r = 6 \text{ cm}$, $h = 10 \text{ cm}$
- $r = 3.5 \text{ in}$, $h = 25 \text{ in}$
- $d = 9 \text{ in}$, $h = 8.5 \text{ in}$
- $d = 15 \text{ cm}$, $h = 10 \text{ cm}$
- base area = 25, height = 8
- volume = $1,000 \text{ cm}^3$, height = 25 cm

Answers

1. 603.19 cm^2

2. 626.75 in^2

3. 367.57 in^2

4. 824.69 cm^2

5. 191.80 units^2

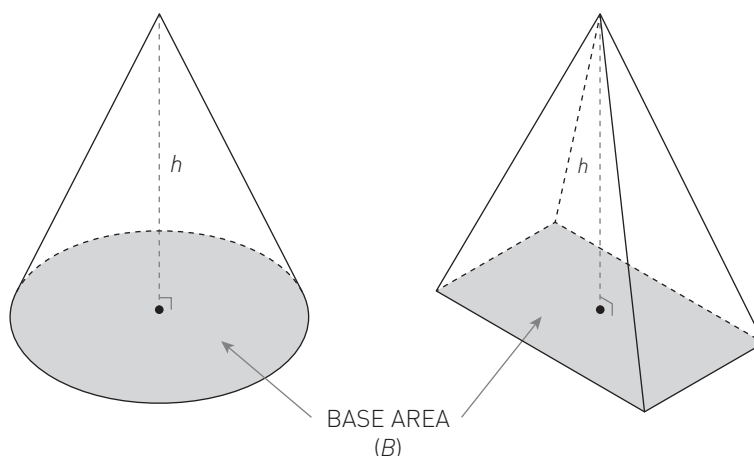
6. 641.51 cm^2

Volume of Pyramids and Cones

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Overview

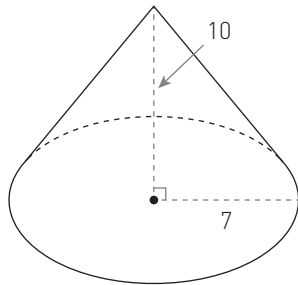
The volume of a pyramid is one-third the volume of the prism with the same base and height, and the volume of a cone is one-third the volume of the cylinder with the same base and height. The formula for the volume of the pyramid or cone with an area of the base B and height h is $V = \frac{1}{3}Bh$. For the cone, since the base is a circle, the formula may also be written $V = \frac{1}{3}r^2\pi h$.



For additional information, see the Methods & Meanings box in Lesson 10.1.4 of the *Core Connections Course 3* text.

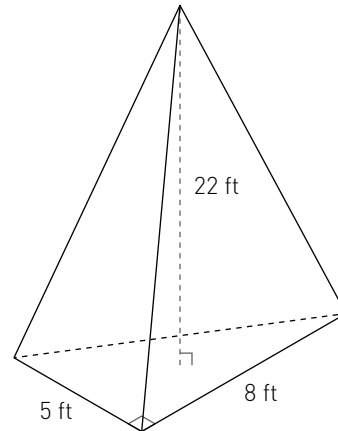
Examples

1. Calculate the volume of the cone shown.



$$\begin{aligned}
 r &= 7 & h &= 10 \\
 V &= \frac{1}{3}r^2\pi h \\
 &= \frac{1}{3}(7)^2\pi(10) \\
 &= \frac{490\pi}{3} \\
 &\approx 513.13 \text{ units}^3
 \end{aligned}$$

2. Calculate the volume of the pyramid.



$$\begin{aligned}
 l &= 5 & w &= 8 \\
 B &= \frac{1}{2}lw \\
 B &= \frac{1}{2}(5)(8) \\
 B &= 20 & h &= 22 \\
 V &= \frac{1}{3}Bh \\
 V &= \frac{1}{3}(20)(22) \\
 V &\approx 146.67 \text{ ft}^3
 \end{aligned}$$

3. If the volume of a cone is $4,325.87 \text{ cm}^3$ and its radius is 9 cm, calculate its height.

$$\begin{aligned}
 V &= 4,325.87 & r &= 9 \\
 V &= \frac{1}{3}r^2\pi h \\
 4,325.87 &= \frac{1}{3}(9)^2\pi h \\
 12,977.61 &= 81\pi h \\
 \frac{12,977.61}{81\pi} &= h \\
 51 \text{ cm} &\approx h
 \end{aligned}$$

Practice

Calculate the volume of each cone.

1. $r = 4$ cm, $h = 10$ cm
2. $r = 2.5$ inches, $h = 10.4$ inches
3. $d = 12$ inches, $h = 6$ inches
4. $d = 9$ cm, $h = 10$ cm
5. $r = 6\frac{1}{3}$ ft, $h = 12\frac{1}{2}$ ft
6. $r = 3\frac{1}{4}$ ft, $h = 6$ ft

Calculate the volume of each pyramid.

7. The base is a square with side 8 cm, and $h = 12$ cm.
8. The base is a right triangle with legs 4 ft and 6 ft, and $h = 10\frac{1}{2}$ ft.
9. The base is a rectangle with width 6 in and length 8 in, and $h = 5$ in.

Calculate the missing part of each cone described.

10. If $V = 1,000$ cm³ and $r = 10$ cm, calculate h .
11. If $V = 2,000$ cm³ and $h = 15$ cm, calculate r .
12. If the circumference of the base = 126 cm and $h = 10$ cm, calculate the volume.

Answers

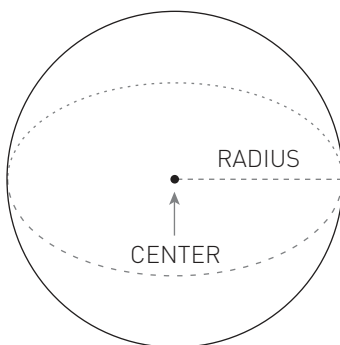
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|---------------------------|---------------------------|------------------------------|
| 1. 167.55 cm ³ | 2. 68.07 in ³ | 3. 226.19 in ³ |
| 4. 212.06 cm ³ | 5. 525.05 ft ³ | 6. 66.37 ft ³ |
| 7. 256 cm ³ | 8. 42 ft ³ | 9. 80 in ³ |
| 10. 9.54 cm | 11. 11.28 cm | 12. 4,211.24 cm ³ |

Volume of Spheres

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Overview

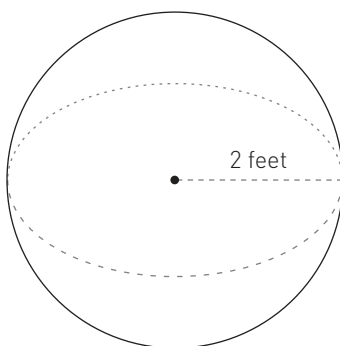
For a sphere with radius r , the volume is found using $V = \frac{4}{3}\pi r^3$.



For more information, see the Methods & Meanings box in Lesson 10.1.5 of the *Core Connections Course 3* text.

Examples

1. Calculate the volume of the sphere.



$$\begin{aligned}
 r &= 2 \\
 V &= \frac{4}{3}\pi r^3 \\
 &= \frac{4}{3}\pi \cdot 2^3 \\
 &= \frac{32\pi}{3} \longleftarrow 3.14 \approx \pi \\
 &\approx \frac{32(3.14)}{3} \\
 &\approx 33.49 \text{ ft}^3
 \end{aligned}$$

2. A sphere has a volume of 972π units³. Calculate the radius.

$$V = \frac{4}{3}\pi r^3$$

Original formula.

$$972\pi = \frac{4}{3}\pi r^3$$

Substitution.

$$2,916\pi = 4\pi r^3$$

Multiply by 3 to remove the fraction.

$$\frac{2,916\pi}{4\pi} = r^3$$

Divide by 4π to isolate r .

$$729 = r^3$$

$$\sqrt[3]{729} = r$$

Take the cube root to undo cubing.

$$9 = r$$

Practice

Use the given information to calculate the exact and approximate volume of each sphere.

- radius = 10 cm
- radius = 4 ft
- diameter = 10 cm
- diameter = 3 miles
- circumference of great circle = 12π units
- circumference of great circle = 3π units

Use the given information to answer each question related to spheres.

- If the radius is 7 cm, calculate the volume.
- If the diameter is 10 inches, calculate the volume.
- If the volume of the sphere is 36π units³, calculate the radius.
- If the volume of the sphere is $\frac{256\pi}{3}$ units³, calculate the radius.

Answers

1. $\frac{4,000\pi}{3} \approx 4,186.67 \text{ cm}^3$

2. $\frac{256\pi}{3} \approx 267.94 \text{ ft}^3$

3. $\frac{500\pi}{3} \approx 523.33 \text{ cm}^3$

4. $\frac{9\pi}{2} \approx 14.13 \text{ mi}^3$

5. $288\pi \approx 904.32 \text{ units}^3$

6. $\frac{9\pi}{2} \approx 14.13 \text{ mi}^3$

7. $\frac{1,372\pi}{3} \approx 1,436.75 \text{ cm}^3$

8. $\frac{500\pi}{3} \approx 523.60 \text{ in}^3$

9. $r = 3 \text{ units}$

10. $r = 4 \text{ units}$