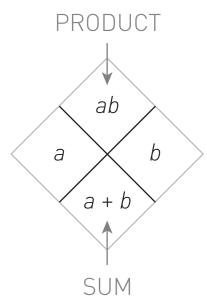
Diamond Problems

GUIDED SKILL SUPPORT

Overview

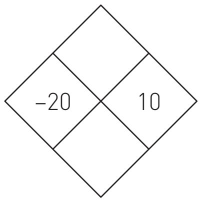
In every Diamond Problem, the product of the two numbers on the sides (left and right) is the top number, and their sum is the bottom number.



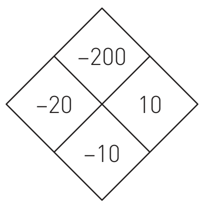
Diamond Problems are an excellent way to practice addition, subtraction, multiplication, and division of positive and negative integers, decimals, and fractions. They also have the benefit of preparing students for factoring binomials in algebra.

Examples

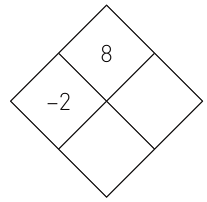
1.



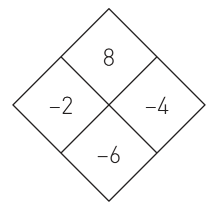
The top number is the product of −20 and 10, or −200. The bottom number is the sum of −20 and 10, or −20 + 10 = −10.



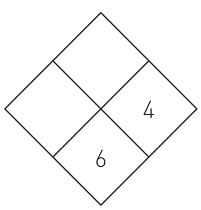
2.



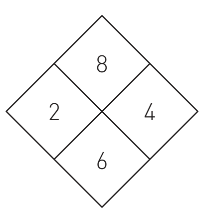
The product of the number on the right side and −2 is 8. Therefore, the right number will be the number you get when you divide 8 by −2, which is −4. The bottom number is the sum of −2 and −4, which is −6.



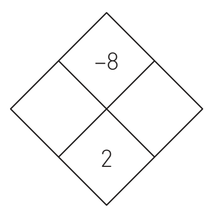
3.



To get the left number, subtract 4 from 6, 6 − 4 = 2. The product of 2 and 4 is 8, so 8 is the top number.

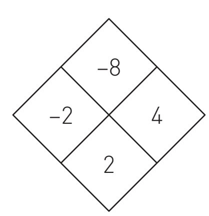


4.



The easiest way to determine the side numbers in a situation like this one is to look at all the pairs of factors of −8. They are −1 and 8, −2 and 4, −4 and 2, and −8 and 1.

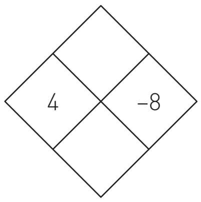
Only one of these pairs has a sum of 2: −2 and 4. Thus, the side numbers are −2 and 4. It does not matter which number is on which side.



Practice

Complete each of the following Diamond Problems.

1.



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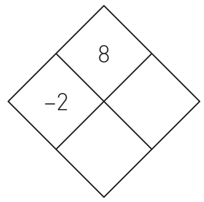
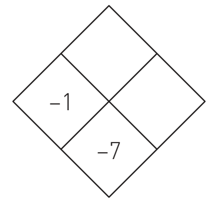


3.



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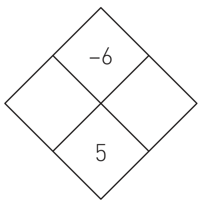
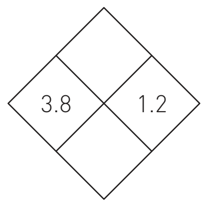
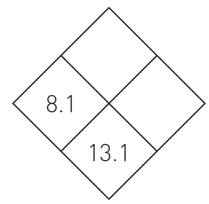


5.



6.



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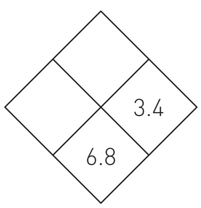

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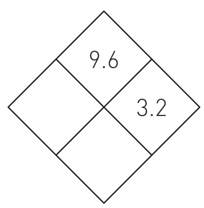
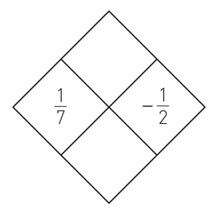
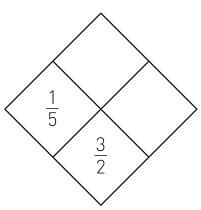


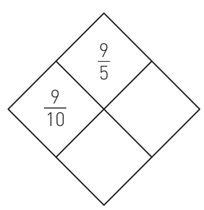
9.



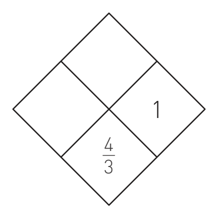
10.



11.



12.



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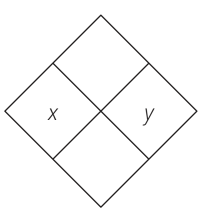
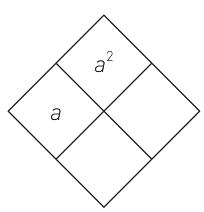
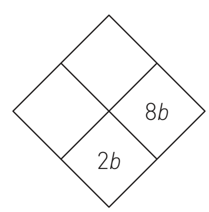


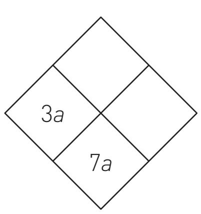
14.



15.



16.



Answers

1.

	-32	
4		-8
	-4	

2.

	8	
-2		-4
	-6	

3.

	6	
-1		-6
	-7	

4.

	-6	
6		-1
	5	

5.

	4.56	
3.8		1.2
	5	

6.

	40.5	
8.1		5
	13.1	

7.

	11.56	
3.4		3.4
	6.8	

8.

	9.6	
3		3.2
	6.2	

9.

	$-\frac{1}{14}$	
$\frac{1}{7}$		$-\frac{1}{2}$
	$-\frac{5}{14}$	

10.

	$\frac{13}{50}$	
$\frac{1}{5}$		$\frac{13}{10}$
	$\frac{3}{2}$	

11.

	$\frac{9}{5}$	
$\frac{9}{10}$		2
	$2\frac{9}{10}$	

12.

	$\frac{1}{3}$	
$\frac{1}{3}$		1
	$\frac{4}{3}$	

13.

	xy	
x		y
	$x + y$	

14.

	a^2	
a		a
	$2a$	

15.

	$-48b^2$	
$-6b$		$8b$
	$2b$	

16.

	$12a^2$	
$3a$		$4a$
	$7a$	

Describing and Extending Patterns

GUIDED SKILL SUPPORT

Overview

Students are asked to use their observation and pattern recognition skills to extend patterns and predict the number of dots that will be in a figure that is too large to draw. Later, variables will be used to describe the patterns.

Examples

1. Examine the dot pattern shown. Assume the pattern continues.

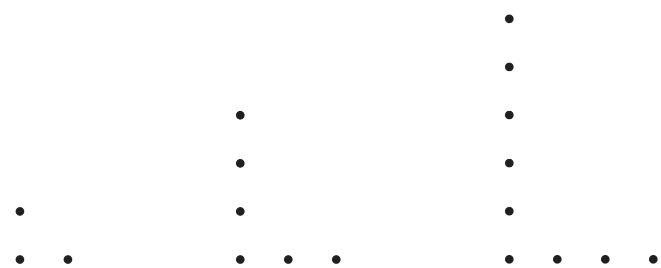


FIGURE 1 FIGURE 2 FIGURE 3

a. Draw Figure 4.

The number of horizontal dots is one more than the figure number and the number of vertical dots is twice the figure number.

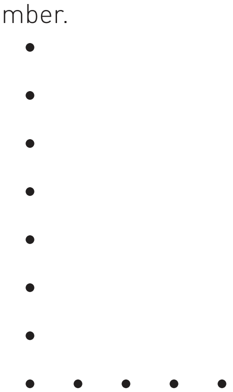


FIGURE 4

b. How many dots will be in Figure 10?

Figure 1 has three dots, Figure 2 has six dots, and Figure 3 has nine dots. The total number of dots is the figure number multiplied by 3.

Figure 10 has 30 dots.

Practice

For each dot pattern, draw the next figure and determine the number of dots in Figure 10.

1.

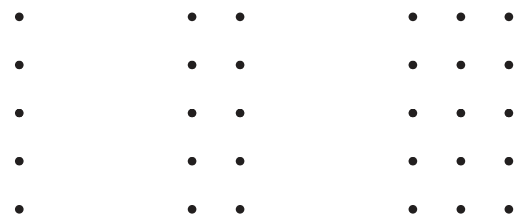


FIGURE 1 FIGURE 2 FIGURE 3

2.

FIGURE 1 FIGURE 2 FIGURE 3

3.

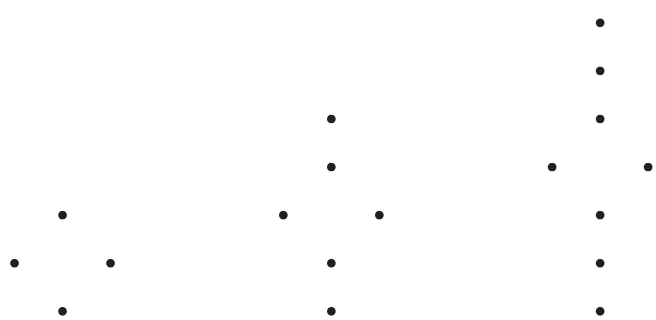


FIGURE 1 FIGURE 2 FIGURE 3

4.

FIGURE 1 FIGURE 2 FIGURE 3

5.

FIGURE 1 FIGURE 2 FIGURE 3

Answers

1. 50 dots

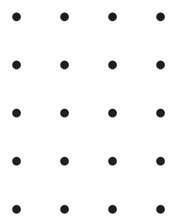


FIGURE 4

2. 110 dots

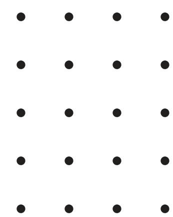


FIGURE 4

3. 22 dots

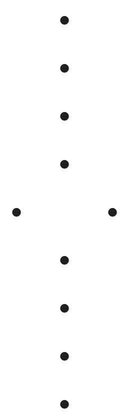


FIGURE 4

4. 40 dots

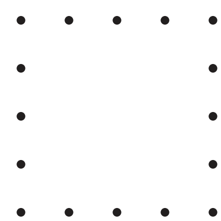


FIGURE 4

5. 140 dots

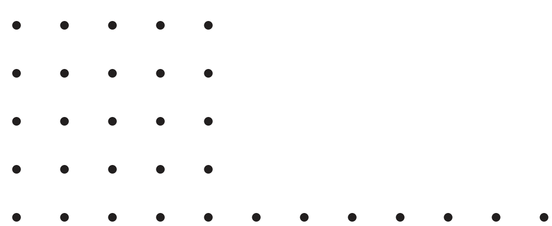


FIGURE 4

Graphing in Four Quadrants

GUIDED SKILL SUPPORT

Overview

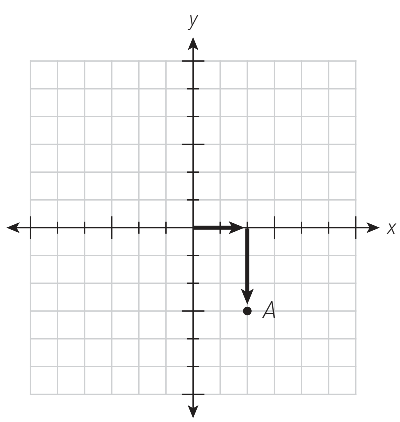
The graphing that began in earlier grades is now extended to include negative values, and students will graph algebraic equations with two variables.

Points on a coordinate graph are identified by two numbers in an ordered pair written as (x, y). The first number is the x-coordinate of the point and the second number is the y-coordinate. Taken together, the two coordinates name exactly one point on the graph. The following examples show how to place a point on an xy-coordinate graph.

Examples

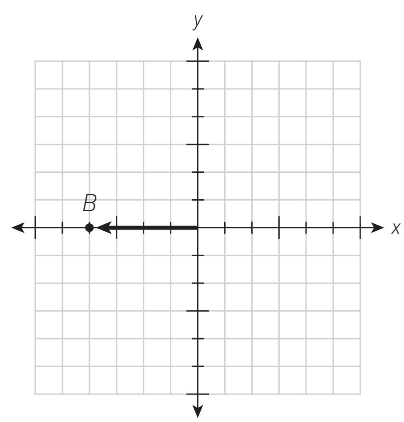
1. Graph point $A(2, -3)$.

Go right 2 units from the origin, (0, 0), then go down 3 units. Mark the point.



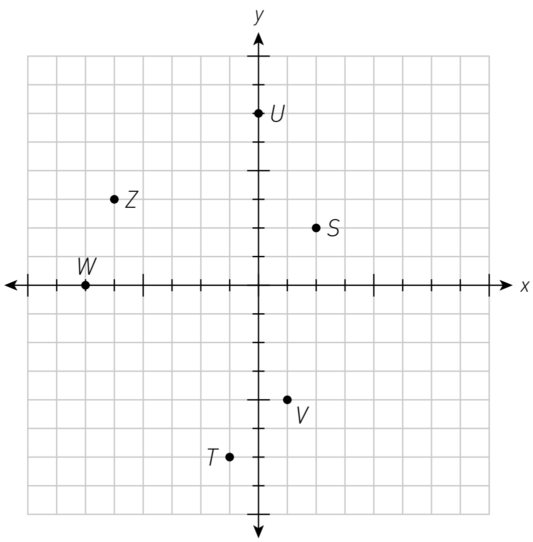
2. Plot the point $B(-4, 0)$ on a coordinate grid.

Go to the left from the origin 4 units, but do not go up or down. Mark the point.



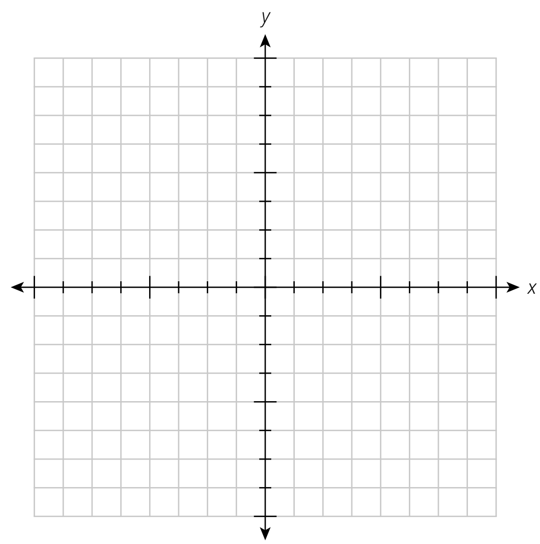
Practice

1. Name the coordinate pair for each point shown on the grid below.



2. Use the ordered pair to locate each point on a coordinate grid. Place a dot at each point and label it with its name.

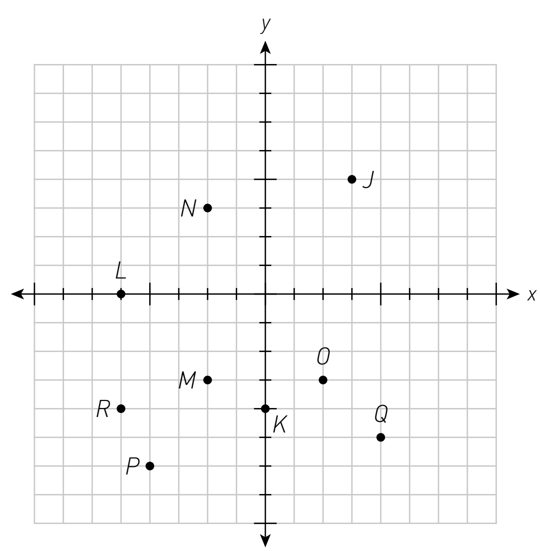
J(3, 4), *K*(0, −4), *L*(−5, 0), *M*(−2, −3), *N*(−2, 3), *O*(2, −3), *P*(−4, −6), *Q*(4, −5), *R*(−5, −4)



Answers

1. *S*(2, 2), *T*(−1, −6), *U*(0, 6), *V*(1, −4), *W*(−6, 0), *Q*(−5, 3)

2.



Writing Equations for Word Problems

GUIDED SKILL SUPPORT

Overview

At first, students used the 5-D Process to solve problems. However, solving complicated problems with the 5-D Process can be inefficient, and it may be difficult to determine the correct solution when it is not an integer. The patterns developed in the 5-D Process can be generalized by using a variable to write an equation. Once you have an equation for the problem, it is often more efficient to solve the equation than to continue to use the 5-D Process.

Examples

1. A box of fruit contains only grapefruits and nectarines. It has three times as many nectarines as grapefruits. Together, there are 36 pieces of fruit. How many pieces of each type of fruit are there?

DESCRIBE/ DRAW The number of nectarines is three times the number of grapefruits. The number of nectarines plus the number of grapefruits equals 36.

	DEFINE		DO	DECIDE
	Number of Grapefruits	**Number of Nectarines**	**Total Pieces of Fruit**	**36?**
Trial 1	11	33	44	too high
Trial 2	10	30	40	too high

After a few trials to establish a pattern in the problem, you can generalize it by using a variable. Since the number of grapefruits could be any number, use x to represent it. The pattern for the number of nectarines is three times the number of grapefruits, or $3x$. The total pieces of fruit is the sum of column one and column two, so the table becomes:

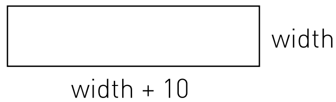
	DEFINE		DO	DECIDE
	Number of Grapefruits	**Number of Nectarines**	**Total Pieces of Fruit**	**36?**
Trial 1	11	33	44	too high
Trial 2	10	30	40	too high
Variable Expression	x	$3x$	$x + 3x$	36

Since the total needs to agree with the check, the equation is $x + 3x = 36$. Rewriting this yields $4x = 36$, so $x = 9$ (grapefruits) and then $3x = 27$ (nectarines).

DECLARE There are 9 grapefruits and 27 nectarines

2. The perimeter of a rectangle is 120 feet. If the length of the rectangle is 10 feet more than the width, what are the dimensions (length and width) of the rectangle?

DESCRIBE/ DRAW



	DEFINE		**DO**	**DECIDE**
	Width	**Length**	**Perimeter = (width + length) · 2**	**120?**
Trial 1	10	20	$(10 + 20) \cdot 2 = 60$	too low
Trial 2	20	30	$(20 + 30) \cdot 2 = 100$	too low

Again, since any width is possible, it is appropriate to use a variable such as x. The pattern for the second column is that it is 10 more than the first: $x + 10$. The perimeter is determined by multiplying the sum of the width and length by 2. The table now becomes:

	DEFINE		**DO**	**DECIDE**
	Width	**Length**	**Perimeter = (width + length) · 2**	**120?**
Trial 1	10	20	$(10 + 20) \cdot 2 = 60$	too low
Trial 2	20	30	$(20 + 30) \cdot 2 = 100$	too low
Variable Expression	x	$x + 10$	$(x + x + 10) \cdot 2$	120

Solving the equation:

$$\begin{aligned} (x + x + 10) \cdot 2 &= 120 \\ 2x + 2x + 20 &= 120 \\ 4x + 20 &= 120 \\ 4x &= 100 \\ x &= 25 \end{aligned}$$

So $x = 25$ (width) and $x + 10 = 35$ (length).

DECLARE The width is 25 feet and the length is 35 feet.

3. Jorge has some dimes and quarters. He has 10 more dimes than quarters and the collection of coins is worth \$2.40. How many dimes and quarters does Jorge have?

DESCRIBE/ DRAW The number of quarters plus 10 equals the number of dimes.
The total value of the coins is \$2.40.

	DEFINE				DO	DECIDE
	Number of Quarters	**Number of Dimes**	**Value of Quarters**	**Value of Dimes**	**Total Value**	**2.40?**
Trial 1	10	20	2.50	2.00	4.50	too high
Trial 2	8	18	2.00	1.80	3.80	too high
Variable Expression	x	$x + 10$	$0.25x$	$0.10(x + 10)$	$0.25x + 0.10(x + 10)$	2.40

Since you need to know both the number of coins and their value, the equation is more complicated. The number of quarters becomes x, but then in the table, the "Value of Quarters" column is $0.25x$. Similarly, the number of dimes is $x + 10$, but the value of dimes is $0.10(x + 10)$. Finally, to determine the numbers, the equation becomes $0.25x + 0.10(x + 10) = 2.40$.

Solving the equation:

$$\begin{aligned} 0.25x + 0.10x + 1.00 &= 2.40 \\ 0.35x + 1.00 &= 2.40 \\ 0.35x &= 1.40 \\ x &= 4.00 \end{aligned}$$

DECLARE There are 4 quarters, worth \$1.00 collectively, and 14 dimes, worth \$1.40 collectively, for a total value of \$2.40.

Practice

Start each problem using the 5-D Process. Then write and solve an equation.

1. A wooden board 100 centimeters long is cut into two pieces. One piece is 26 centimeters longer than the other. What are the lengths of the two pieces?

2. Thu is five years older than her brother Tuan. The sum of their ages is 51. What are their ages?

3. Tomás is thinking of a number. If he triples his number and subtracts 13, the result is 305. What number is Tomás thinking of?

4. Two consecutive numbers have a sum of 123. What are the two numbers?

5. Two consecutive even numbers have a sum of 246. What are the numbers?

6. Joe's age is three times Aaron's age, and Aaron is six years older than Christina. If the sum of all their ages is 149, what is Christina's age? Joe's age? Aaron's age?

7. Farmer Fran has 38 barnyard animals, consisting of only chickens and goats. If these animals have 116 legs, how many of each type of animal are there?

8. A scarf 156 centimeters long is split into three parts that will be made into new scarves. The two longer parts are the same length and are 15 centimeters longer than the shortest part. How long are the three parts?

9. Juan has 15 coins, all nickels and dimes. This collection of coins is worth 90¢. How many nickels and dimes are there? (Hint: Create separate columns for "Number of Nickels," "Value of Nickels," "Number of Dimes," and "Value of Dimes.")

10. Tickets to the school play are $5.00 for adults and $3.50 for students. If the total value of all the tickets sold was $2,517.50 and 100 more students bought tickets than adults, how many adults and students bought tickets?

11. A pipe 250 centimeters long is cut into five pieces: three short pieces of equal length and two pieces that are both 15 centimeters longer than the shorter ones. What are the lengths of the pipes?

12. Conrad has a collection of three types of coins: nickels, dimes, and quarters. There is an equal amount of nickels and quarters but three times as many dimes. If the entire collection is worth $9.60, how many nickels, dimes, and quarters are there?

Answers

Equations may vary.

1. $x + (x + 26) = 100$; The lengths of the boards are 37 cm and 63 cm.

2. $x + (x + 5) = 51$; Thu is 28 years old and her brother is 23 years old.

3. $3x - 13 = 305$; Tomás is thinking of the number 106.

4. $x + (x + 1) = 123$; The two consecutive numbers are 61 and 62.

5. $x + (x + 2) = 246$; The two consecutive even numbers are 122 and 124.

6. $x + (x + 6) + 3(x + 6) = 149$; Christine is 25, Aaron is 31, and Joe is 93 years old.

7. $2x + 4(38 - x) = 116$; Farmer Fran has 20 goats and 18 chickens.

8. $x + (x + 15) + (x + 15) = 156$; The lengths of the scarf pieces are 42, 57, and 57 cm.

9. $0.05x + 0.10(15 - x) = 0.90$; Juan has 12 nickels and 3 dimes.

10. $5x + 3.50(x + 100) = 2{,}517.50$; There were 255 adult and 355 student tickets purchased for the play.

11. $3x + 2(x + 15) = 250$; The lengths of the pipes are 44 and 59 cm.

12. $0.05x + 0.25x + 0.10(3x) = 9.60$; Conrad has 16 quarters, 16 nickels, and 48 dimes.

Box Plots

GUIDED SKILL SUPPORT

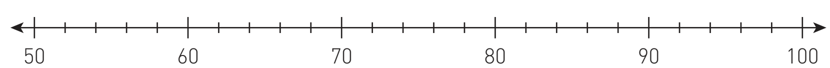
Overview

One way to display a distribution of one-variable numerical data is with a box plot. A box plot is the only display of data that clearly shows the median, quartiles, range, and outliers of a data set.

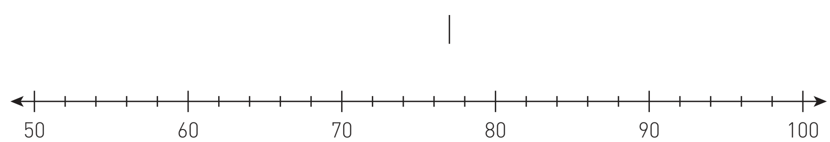
Examples

1. Display this data in a box plot: 51, 55, 55, 62, 65, 72, 76, 78, 79, 82, 83, 85, 91, and 93.

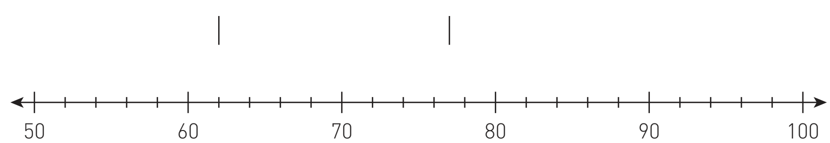
Since this data is already in order from least to greatest, the range is 93 – 51 = 42. A suitable number line could have equal intervals from 50 to 100.



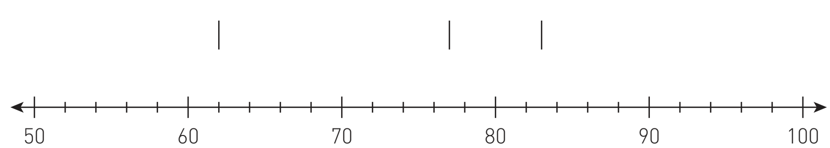
The median of the set of data is 77. Draw a vertical line segment at this value above the number line.



The median of the lower half of the data (the first quartile) is 62. Draw a vertical line segment at this value above the number line



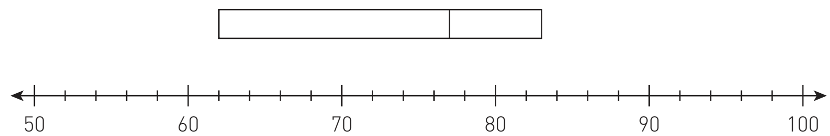
The median of the upper half of the data (the third quartile) is 83. Draw a vertical line segment at this value above the number line.



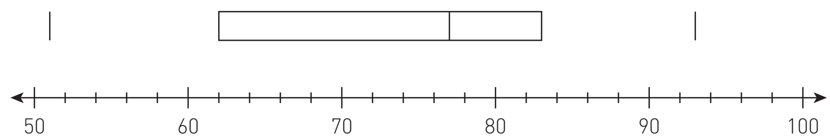
Solution continues on next page.

Solution continued from previous page.

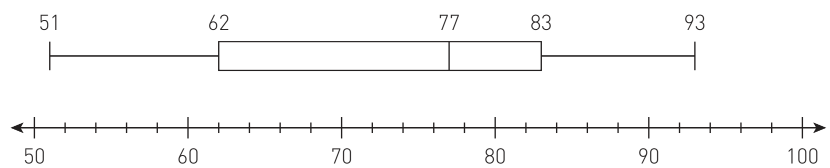
Draw horizontal lines connecting the first and third quartiles to form a box.



Place a vertical line segment at the minimum value (51) and one at the maximum value (93).



Use horizontal line segments to connect the minimum and maximum values to the box. Label values if needed.

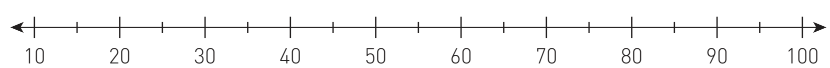


2. Display this data in a box plot: 62, 65, 93, 51, 12, 79, 85, 55, 72, 78, 83, 91, and 76.

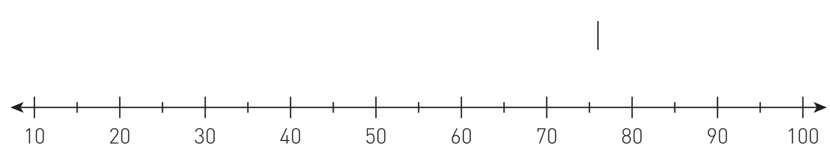
Place the data in order from least to greatest.

12, 51, 55, 62, 65, 72, 76, 78, 79, 83, 85, 91, 93

The range is 93 – 12 = 81. A suitable number line could have equal intervals from 10 to 100.



The median of the set of data is 76. Draw the line segment.



Solution continues on next page.

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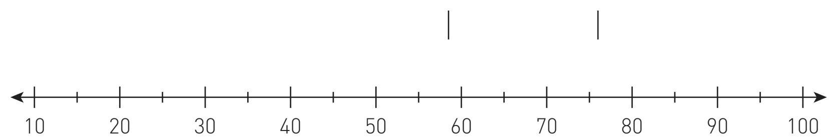
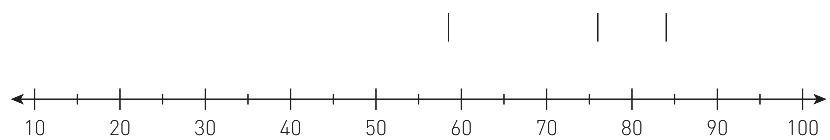
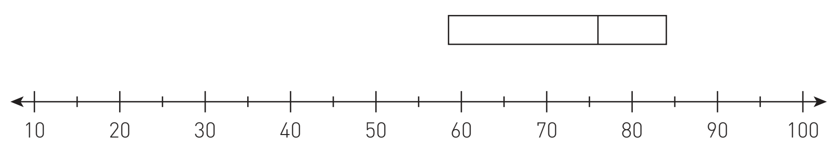
Since the lower half of the data has an even number of values, the first quartile is found by calculating the mean of the two middle values: 55 + 62 = 117; 117 ÷ 2 = 58.5. Draw the line segment.



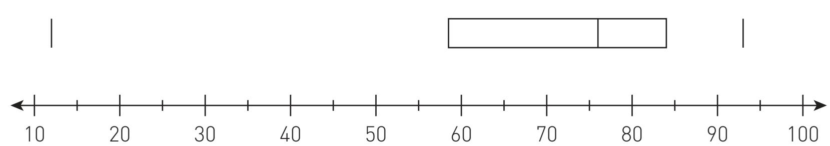
Since the upper half of the data also has an even number of values, the third quartile is found by calculating the mean of the two middle values: 83 + 85 = 168; 168 ÷ 2 = 84. Draw the line segment.



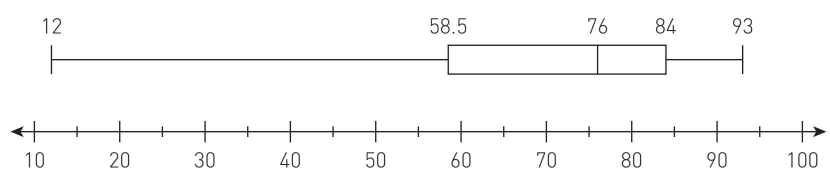
Draw horizontal lines connecting the first and third quartiles to form a box.



Place a vertical line segment at the minimum value (12) and one at the maximum value (93).



Use horizontal line segments to connect the minimum and maximum values to the box. Label values if needed.



Practice

Create a box plot for each set of data.

1. 45, 47, 52, 85, 46, 32, 83, 80, 75

2. 75, 62, 56, 80, 72, 55, 54, 80

3. 49, 54, 52, 58, 61, 72, 73, 78, 73, 82, 83, 73, 61, 67, 68

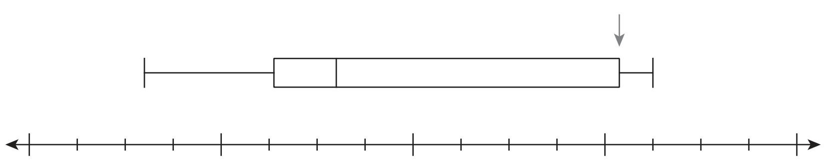
4. 65, 35, 48, 29, 57, 87, 94, 68, 86, 73, 58, 74, 85, 91, 88, 97

5. 265, 263, 269, 259, 267, 264, 253, 275, 264, 260, 273, 257, 291

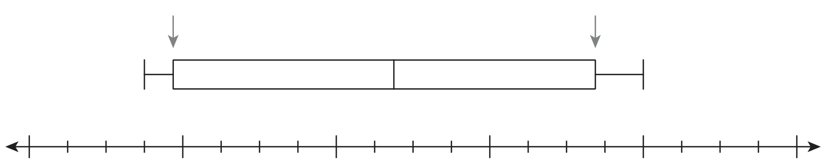
6. 48, 42, 37, 29, 49, 46, 38, 28, 45, 45, 35, 46.25, 34, 46, 46.5, 43, 46.5, 48, 41.25, 29, 47.75

Answers

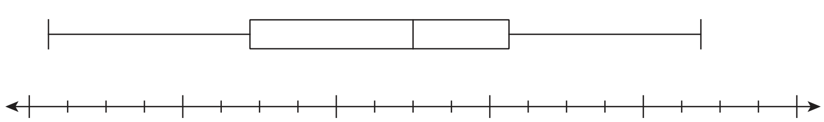
1.



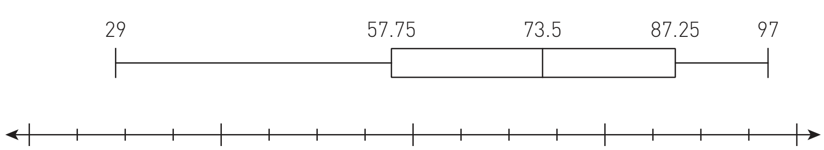
2.



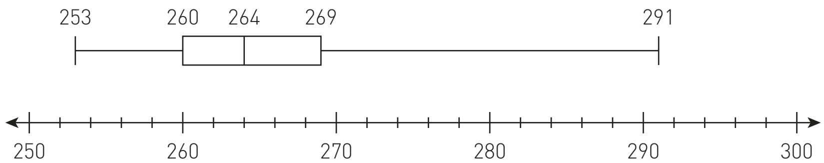
3.



4.



5.


253
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264
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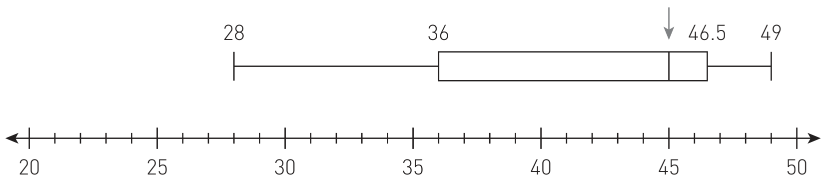
6.


28
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Proportional Relationships

GUIDED SKILL SUPPORT

Overview

A **proportion** is an equation stating the two ratios (fractions) are equal. Two values are in a proportional relationship if a proportion may be set up to relate the values.

For more information, see the Methods & Meanings boxes in Lessons 1.2.2 and 7.2.5 of the *Core Connections Course 3* text. For additional examples and practice, see the *Core Connections Course 3* Checkpoint 3 materials.

Examples

1. The average cost of a pair of designer jeans has increased by $15 in 4 years. What is the unit growth rate (dollars per year)?

The growth rate is $\frac{15\text{ dollars}}{4\text{ years}}$. To create a unit rate, we need a denominator of 1, so we write the equation $\frac{15\text{ dollars}}{4\text{ years}} = \frac{x\text{ dollars}}{1\text{ year}}$. Using a Giant One:

$$\frac{15\text{ dollars}}{4\text{ years}} = \frac{4}{4} \cdot \frac{x\text{ dollars}}{1\text{ year}} = 3.75\ \frac{\text{dollars}}{\text{year}}$$

2. Ryan's famous chili recipe uses 3 tablespoons of chili powder for 5 servings. How many tablespoons will he need to make enough for the family reunion, 40 servings?

The rate is $\frac{3\text{ tablespoons}}{5\text{ servings}}$, so the problem may be written as a proportion: $\frac{3}{5} = \frac{t}{40}$.

One method of solving the proportion is to use the Giant One:

$$\frac{3}{5} = \frac{t}{40} \Rightarrow \frac{3}{5} \cdot \frac{8}{8} = \frac{24}{40} \Rightarrow t = 24$$

Another method is to use cross multiplication:

$$\frac{3}{5} = \frac{t}{40}$$
$$5 \cdot t = 3 \cdot 40$$
$$5t = 120$$
$$t = 24$$

Finally, since the unit rate is $\frac{3}{5}$ tablespoon per serving, the equation $t = \frac{3}{5}s$ represents the general proportional situation, and the number of servings needed can be substituted into the equation: $t = \frac{3}{5} \cdot 40 = 24$. Using any method, the answer is 24 tablespoons.

3. Based on the table shown, what is the unit growth rate (meters per year)?

Height (m)	15	17	19	21
Time (years)	75	85	95	105

+ 2 + 2 + 2

Height (m)	15	17	19	21
Time (years)	75	85	95	105

+ 10 + 10 + 10

$$\frac{2\text{ meters}}{10\text{ years}} = \frac{2\text{ meters}}{10\text{ years}} \cdot \boxed{\frac{\frac{1}{10}}{\frac{1}{10}}} = \frac{\frac{1}{5}\text{ meter}}{1\text{ year}}$$

Practice

For problems 1 through 10, calculate the unit rate. For problems 11 through 25, solve each proportion.

1. Typing 731 words in 17 minutes (words per minute)

2. Reading 258 pages in 86 minutes (pages per minute)

3. Buying 15 boxes of cereal for $43.35 (cost per box)

4. Scoring 98 points in a 40-minute game (points per minute)

5. Buying $2\frac{1}{4}$ pounds of bananas for $1.89 (cost per pound)

6. Buying $\frac{2}{3}$ pounds of peanuts for $2.25 (cost per pound)

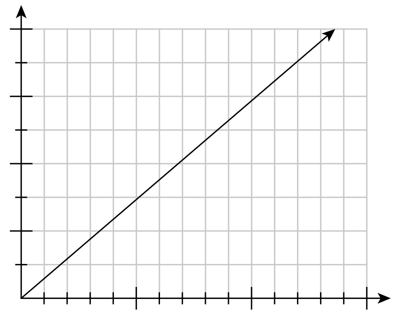
7. Mowing $1\frac{1}{2}$ acres of lawn in $\frac{3}{4}$ of a hour (acres per hour)

8. Paying $3.89 for 1.7 pounds of chicken (cost per pound)

9. What is the weight per centimeter?

Weight (g)	6	8	12	20
Length (cm)	15	20	30	50

10. For the graph shown, what is the rate in miles per hour?



11. If a box of 100 pencils costs \$4.75, how much should you expect to pay for 225 pencils?

12. When Amber does her math homework, she finishes 10 problems every 7 minutes. How long will it take for her to complete 35 problems?

13. Ben and his friends are having a TV marathon, and after 4 hours, they have watched 5 episodes of the show. About how long will it take to complete the season, which has 24 episodes?

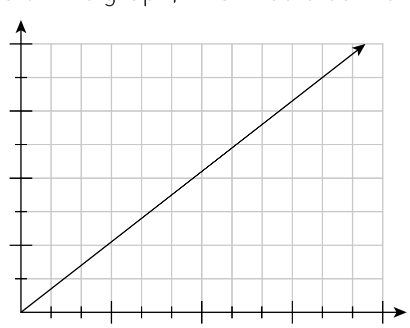
14. The tax on a \$600 vase is \$54. What should be the tax on a \$1,700 vase?

15. Use the table shown to determine how long it will take the Spirit club to wax 60 cars.

Number of Cars Waxed	8	16	32
Time (hours)	3	6	12

16. While baking, Elijah discovered a recipe that required $\frac{1}{2}$ cups of walnuts for every $2\frac{1}{4}$ cups of flour. How many cups of walnuts will he need for 4 cups of flour?

17. Based on the graph, what would be the cost to refill 50 bottles?



18. Salim grew $1\frac{3}{4}$ inches in $4\frac{1}{2}$ months. How much should he grow in one year?

19. On his afternoon jog, Chris took 42 minutes to run $3\frac{3}{4}$ miles. How many miles can he run in 60 minutes?

20. If Camila needs $1\frac{1}{3}$ cans of paint for each room in her house, how many cans of paint will she need to paint the entire 7-room house?

21. Stephen receives 20 minutes of video game time for every 45 minutes of dog walking he does. If he wants 90 minutes of game time, how many hours will he need to work?

22. Sarah's grape vine grew 15 inches in 6 weeks. Write an equation to represent its growth after t weeks.

23. On average, Mingze makes 45 out of 60 shots with the basketball. Write an equation to represent the average number of shots made out of x attempts.

24. Write an equation to represent the situation in problem 14.

25. Write an equation to represent the situation in problem 17.

Answers

1. $43\,\frac{\text{words}}{\text{minute}}$	**2.** $3\,\frac{\text{pages}}{\text{minute}}$	**3.** $2.89\,\frac{\text{dollars}}{\text{box}}$
4. $2.45\,\frac{\text{points}}{\text{minute}}$	**5.** $0.84\,\frac{\text{dollars}}{\text{pound}}$	**6.** $3.38\,\frac{\text{dollars}}{\text{pound}}$
7. $2\,\frac{\text{acres}}{\text{hour}}$	**8.** $2.29\,\frac{\text{dollars}}{\text{pound}}$	**9.** $\frac{2\text{ grams}}{5\text{ centimeter}}$
10. $\approx 27\,\frac{\text{miles}}{\text{hour}}$	**11.** \$10.69	**12.** 24.5 minutes
13. 19.2 hours	**14.** \$153	**15.** 22.5 hours
16. $\frac{8}{9}$ cups	**17.** \$175	**18.** $4\frac{2}{3}$ inches
19. $\approx$ 5.36 miles	**20.** $9\frac{1}{3}$ cans	**21.** $3\frac{3}{8}$ hours
22. $g = \frac{5}{2}t$	**23.** $s = \frac{3}{4}x$	**24.** $t = 0.09c$
25. $C = 3.5b$		