Combining Like Terms

GUIDED SKILL SUPPORT

Overview

Algebraic expressions can also be simplified by taking terms that have the same variable(s) raised to the same powers and combining them (by adding or subtracting) into one term. The skill of combining like terms is necessary for solving equations. For additional information, see the Methods & Meanings box in Lesson 2.1.3 of the *Core Connections Course 3* text.

Examples

1. Combine like terms to rewrite the expression $3x + 5x + 7x$ in the fewest terms possible.

All these terms have x as the variable, so they are combined into one term, $15x$.

2. Rewrite the expression $3x + 12 + 7x + 5$ in the fewest terms possible.

The terms with x can be combined. The terms without variables (the constants) can also be combined.

$$\begin{aligned} &3x + 12 + 7x + 5 \\ &= 3x + 7x + 12 + 5 \\ &= 10x + 17 \end{aligned}$$

Note that in the rewritten form, the term with the variable is listed before the constant term.

3. Rewrite the expression $5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$ in the fewest terms possible.

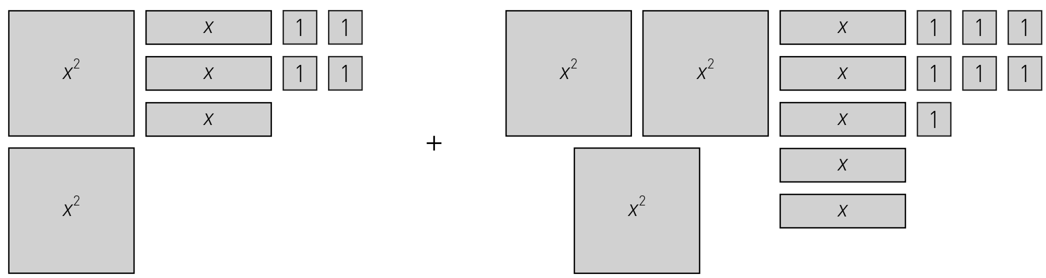
$$\begin{aligned} &5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1 \\ &= 4x^2 + 2x^2 + 5x + 2x + x + 10 - 6 - 1 \\ &= 6x^2 + 8x + 3 \end{aligned}$$

Note that terms with the same variable but with different exponents are not combined and are listed in order of decreasing power of the variable, and the constant term is last.

4. Rewrite the expression $(2x^2 + 3x + 4) + (3x^2 + 5x + 7)$ in the fewest terms possible.

The algebra tiles are used to model how to combine like terms.

The large square represents x^2, the rectangle represents x, and the small square represents 1. We can only combine tiles that are alike: large squares with large squares, rectangles with rectangles, and small squares with small squares. If we want to combine $2x^2 + 3x + 4$ and $3x^2 + 5x + 7$, visualize the tiles to help combine the like terms:



$2x^2 + 3x + 4$ $3x^2 + 5x + 7$

The combination of the two sets of tiles, written algebraically, is $5x^2 + 8x + 11$.

5. Rewrite the expression $(2x^2 - 5x + 6) + (3x^2 + 4x - 9)$ in the fewest terms possible.

Sometimes it is helpful to take an expression that is written horizontally, circle the terms with their signs, and rewrite like terms in vertical columns before you combine them.

$$\boxed{2x^2}\boxed{-5x}\boxed{+6}\boxed{+3x^2}\boxed{+4x}\boxed{-9} \Longrightarrow \begin{array}{r} 2x^2 - 5x + 6 \\ +\ 3x^2 + 4x - 9 \\ \hline 5x^2 \quad - x - 3 \end{array}$$

Practice

Write the following expressions with the fewest terms possible.

1. $(2x^2 + 6x + 10) + (4x^2 + 2x + 3)$

2. $(3x^2 + x + 4) + (x^2 + 4x + 7)$

3. $(8x^2 + 3) + (4x^2 + 5x + 4)$

4. $(4x^2 + 6x + 5) - (3x^2 + 2x + 4)$

5. $(4x^2 - 7x + 3) + (2x^2 - 2x - 5)$

6. $(3x^2 - 7x) - (x^2 + 3x - 9)$

7. $(5x + 6) + (-5x^2 + 6x - 2)$

8. $2x^2 + 3x + x^2 + 4x - 3x^2 + 2$

9. $3c^2 + 4c + 7x - 12 + (-4c^2) + 9 - 6x$

10. $2a^2 + 3a^3 - 4a^2 + 6a + 12 - 4a + 2$

Answers

1. $6x^2 + 8x + 13$

2. $4x^2 + 5x + 11$

3. $12x^2 + 5x + 7$

4. $x^2 + 4x + 1$

5. $6x^2 - 9x - 2$

6. $2x^2 - 10x + 9$

7. $-5x^2 + 11x + 4$

8. $7x + 2$

9. $-c^2 + 4c + x - 3$

10. $3a^3 - 2a^2 + 2a + 14$

Rewriting Expressions

GUIDED SKILL SUPPORT

Overview

An Expression Mat is an organizational tool that is used to represent algebraic expressions. Pairs of Expression Mats can be modified to make an Equation Mat. The upper half of an Expression Mat is the positive region and the lower half is the negative region. A matching pair of tiles with one tile positive and the other one negative represents zero (0).

Tiles may be removed from or moved on an Expression Mat in one of three ways:

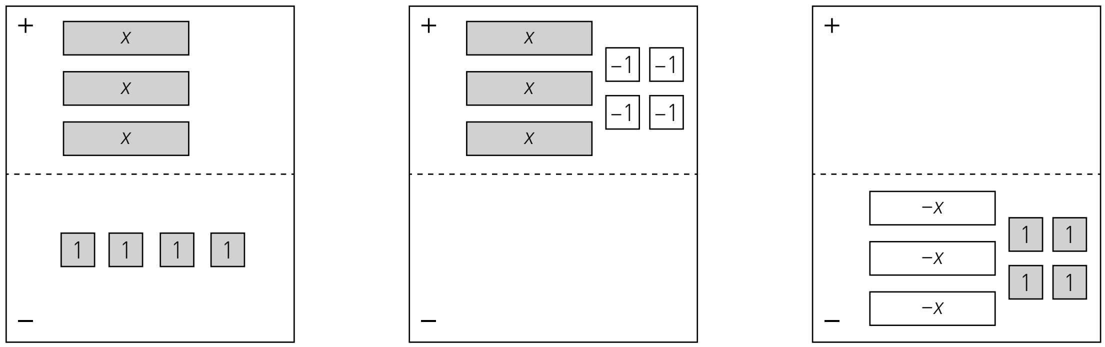
- removing the same number of opposite tiles in the same region;
- flipping a tile from one region to another. Such moves create "opposites" of the original tile, so a positive tile in one region becomes negative in the other region, and an negative tile in one region becomes positive in the other; and
- removing an equal number of identical tiles from both the "+" and "−" regions.

See the Methods & Meanings box in Lesson 2.1.4 of the *Core Connections Course 3* text.

Examples

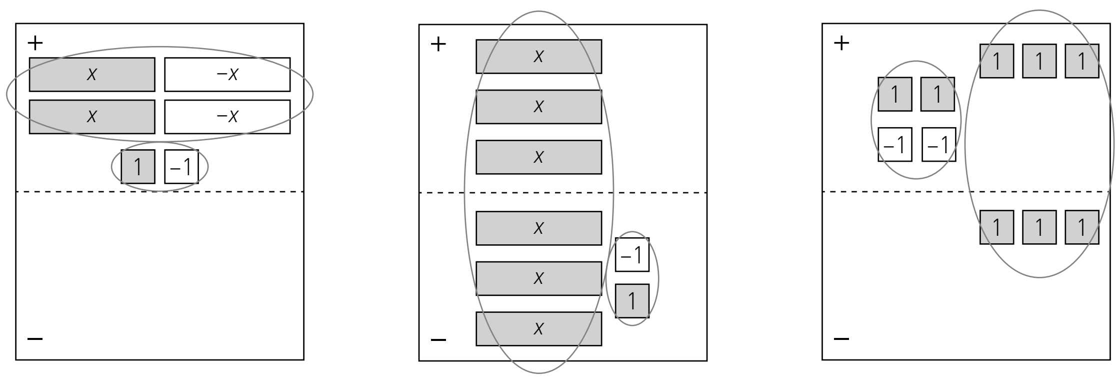
1. Represent the expression $3x - 4$ on an Expression Mat.

$3x - 4$ can be represented in various ways.



2. Represent the value 0 on an Expression Mat.

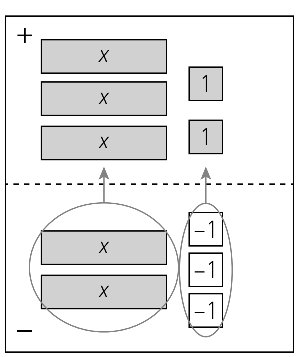
The following Expression Mats all represent 0.



3. $3x + 2 - (2x - 3)$

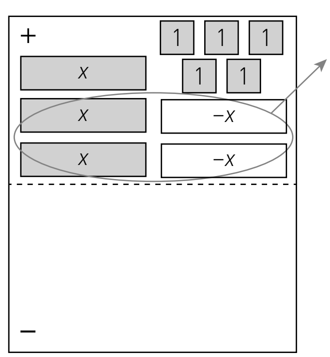
Rewrite expressions by flipping tiles and removing zeros.

STEP 1



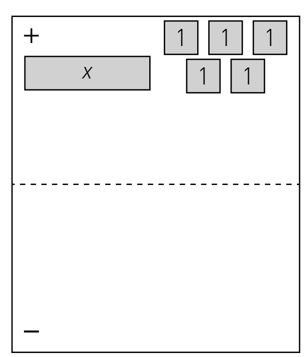
$3x + 2 - (2x - 3)$

STEP 2



$3x + 2 - 2x + 3$

STEP 3

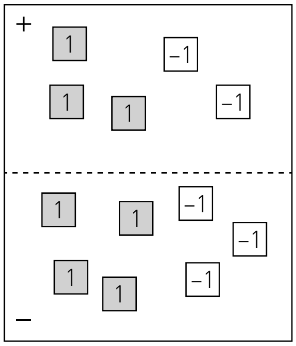


$x + 5$

Practice

Use valid moves for each expression.

1.



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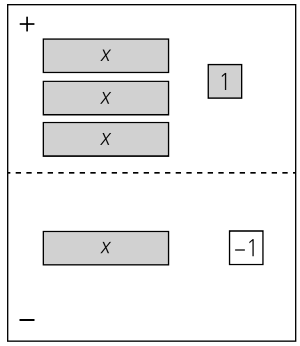

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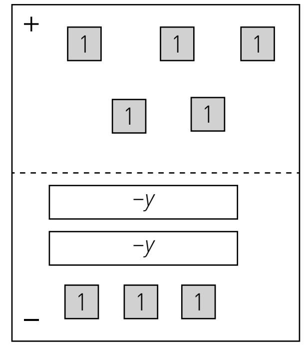
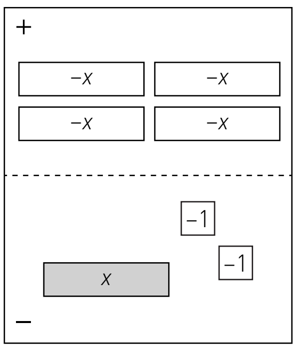


4.



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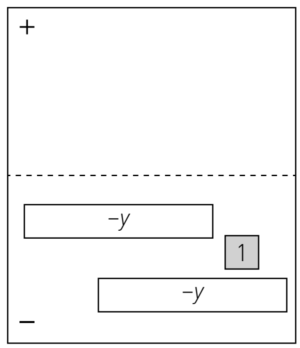
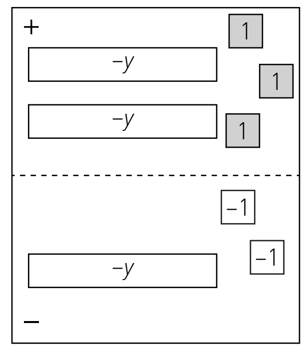


6.



7. $3 + 5x - 4 - 7x$

8. $-x - 4x - 7$

9. $-(-x + 3)$

10. $4x - (x + 2)$

11. $5x - (-3x + 2)$

12. $x - 5 - (2 - x)$

13. $1 - 2y - 2y$

14. $-3x + 5 + 5x - 1$

15. $3 - (y + 5)$

16. $-(x + y) + 4x + 2y$

17. $3x - 7 - (3x - 7)$

18. $-(x + 2y + 3) - 3x + y$

Answers

1. 0

2. $2x + 2$

3. $2y + 2$

4. $-5x + 2$

5. $2y - 1$

6. $-y + 5$

7. $-2x - 1$

8. $-5x - 7$

9. $x - 3$

10. $3x - 2$

11. $8x - 2$

12. $2x - 7$

13. $-4y + 1$

14. $2x + 4$

15. $-y - 2$

16. $3x + y$

17. 0

18. $-4x - y - 3$

Comparing Quantities

GUIDED SKILL SUPPORT

Overview

Combining two Expression Mats into an Expression Comparison Mat creates a concrete model for rewriting (and later solving) inequalities and equations.

Tiles may be removed or moved on the mat in the following ways:

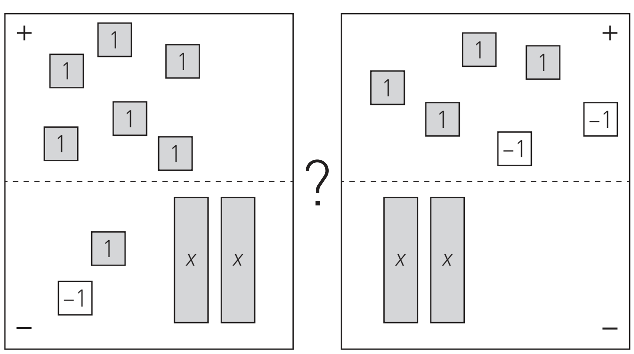
- removing the same number of opposite tiles (zeros) on the same side;
- removing an equal number of identical tiles (balanced set) from both the left and right sides;
- adding the same number of opposite tiles (zeros) on the same side; and
- adding an equal number of identical tiles (balanced set) to both the left and right sides.

These strategies are called "valid moves."

After adding and/or removing tiles on the Expression Comparison Mat, students are asked to tell which side is greater. Sometimes it is only possible to tell which side is greater if you know possible values of the variable.

Examples

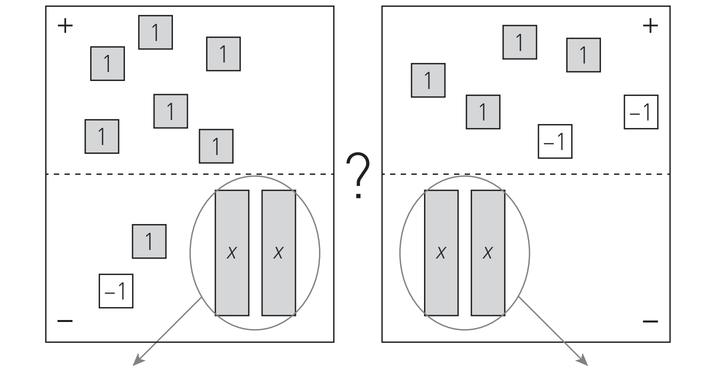
1. Determine which side is greater by using valid moves.



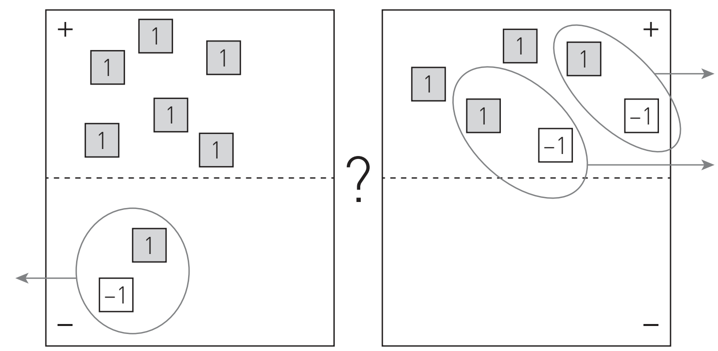
Solution shown on next page.

Example continued from previous page.

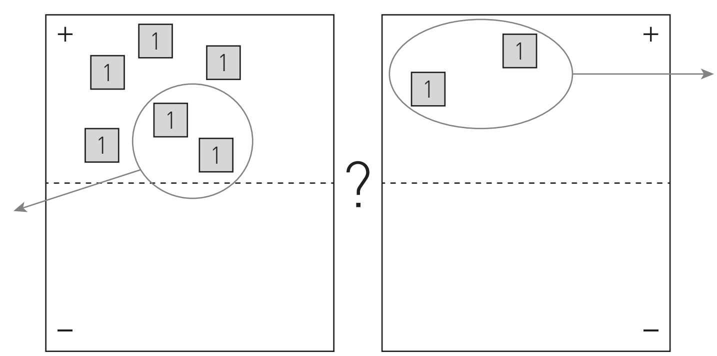
STEP 1: Remove balanced set.



STEP 2: Remove zeros.



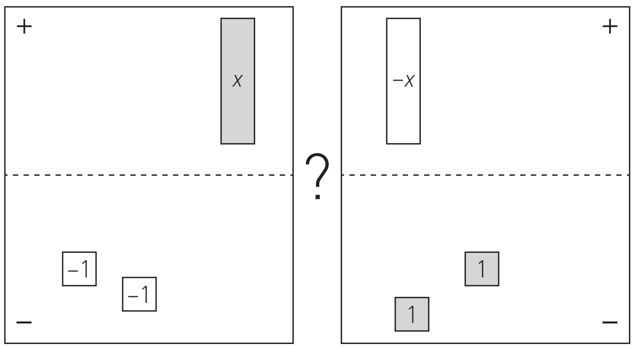
STEP 3: Remove balanced set.



The left side is greater because, after Step 3, this is $4 > 0$.

Note that this example shows only one of several possible strategies.

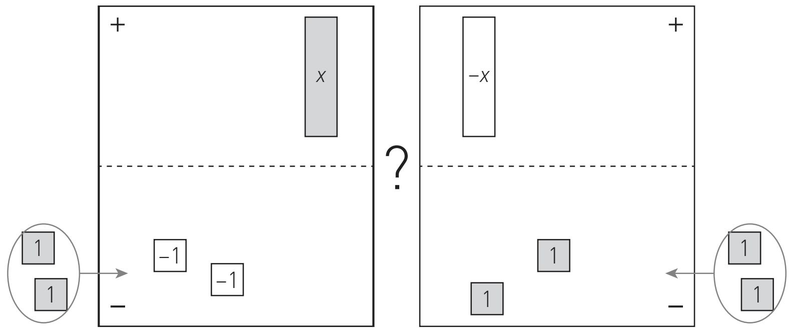
2. Determine which side is greater by using valid moves so that all the x-variables are on one side and all the unit tiles are on the other.



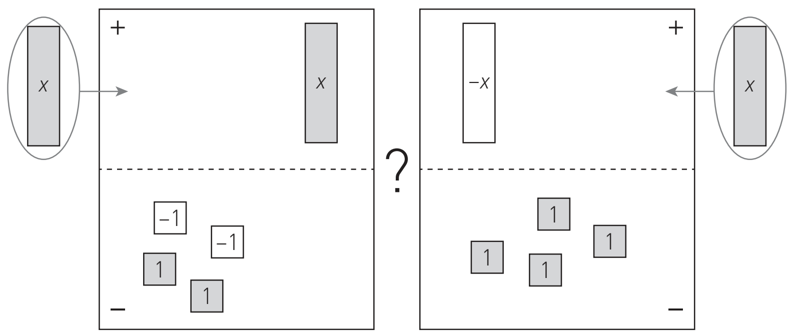
Solution shown on next page.

Example continued from previous page.

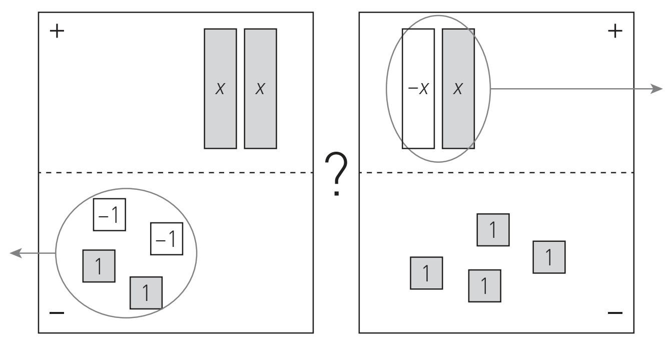
STEP 1: Add balanced set.



STEP 2: Add balanced set.



STEP 3: Remove zeros.



What remains is $2x$ on the left and –(4) on the right. There are other possible arrangements. Whatever the arrangement, it is not possible to tell which side is greater because we do not know the value of "x." Students are expected to record the results algebraically as directed by the teacher. One possible recording is shown.

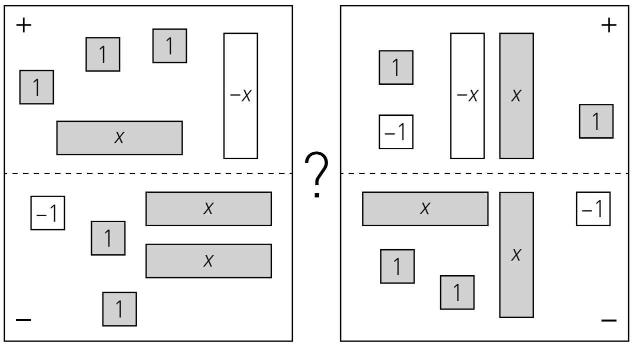
LEFT EXPRESSION RIGHT EXPRESSION

$$x + x - (-2 + 2) \ \textbf{?} \ -x + x - (2 + 2)$$
$$2x \ \textbf{?} \ 4$$

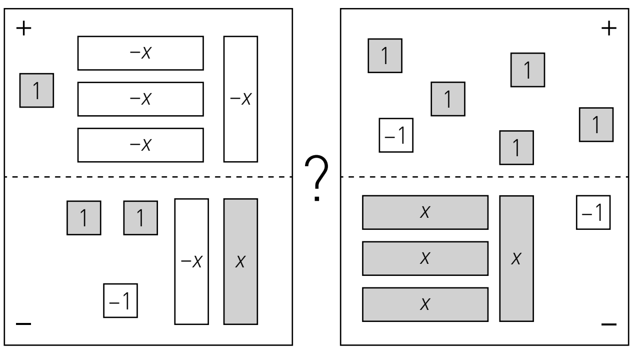
Practice

For each of the following problems, use the strategies of removing zeros or removing balanced sets to determine which side is greater, if possible. Record your steps.

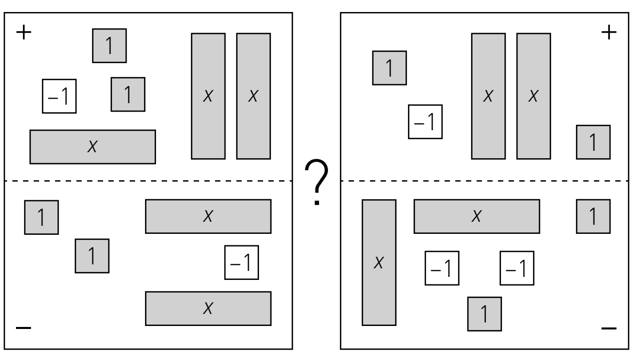
1.



2.



3.



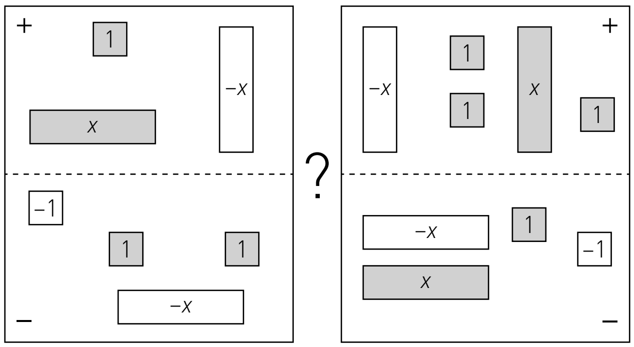
4. Left: 5 + (−8)
Right: −7 + 6

5. Left: $2(x + 3) - 2$
Right: $4x - 2 - x + 4$

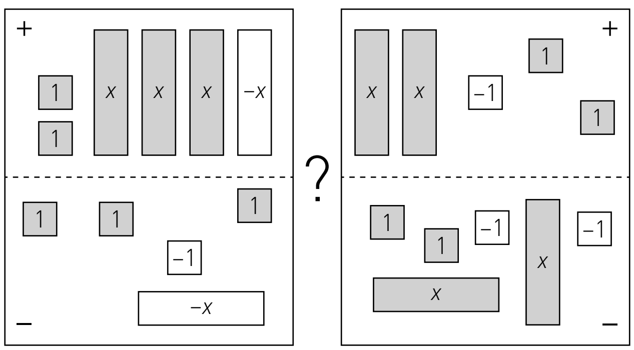
6. Left: $4 + (-2x) + 4x$
Right: $x^2 + 2x + 3 - x^2$

For each of the following problems, use the strategies of removing zeros or adding/removing balanced sets so that all the x-variables are on one side and the unit tiles are on the other. Record your steps.

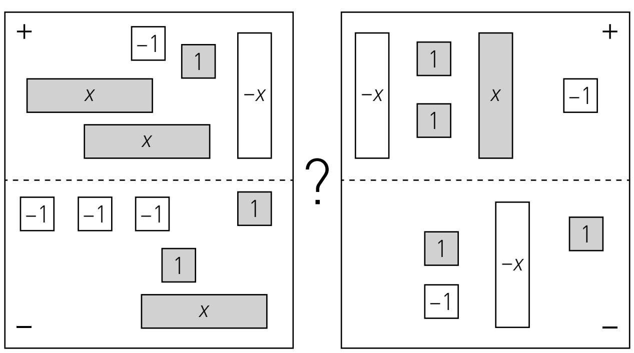
7.



8.



9.



10. Left: $3x - 2$
Right: $2x + 1$

11. Left: $4x + 2 + (-5)$
Right: $2x + 3 + (-8)$

12. Left: $2x + 3 + (-8)$
Right: $-x - 3$

Answers

Answers to problems 7 through 12 may vary, but the comparison cannot be determined without the value of *x*.

1. left

2. right

3. The comparison depends on the value of *x*.

4. right

5. The comparison depends on the value of *x*.

6. left

7. left: x; right: 3

8. left: $3x$; right: 1

9. left: 1; right: x

10. left: x; right: 3

11. left: $2x$; right: -2

12. left: $3x$; right: -6

Solving Equations

GUIDED SKILL SUPPORT

Overview

Combining two Expression Mats into an Equation Mat creates a concrete model for solving equations. Practicing solving equations using the model will help students transition to solving equations abstractly with better accuracy and understanding.

In general, and as shown in the first example, a negative sign in front of a set of parentheses causes everything inside the parentheses to "flip" from the top to the bottom or the bottom to the top of an Expression Mat. That is, all terms in the expression change signs. After representing the parentheses, use valid moves to have the fewest number of tiles on each Expression Mat. Next, isolate the variables on one side of the Equation Mat and the non-variables on the other side by adding balanced sets and removing matching tiles from both sides. Then determine the value of the variable. Students should be able to explain their steps. See the Methods & Meanings boxes in Lessons 2.1.7 and 3.2.3 of the *Core Connections Course 3* text. For additional examples and practice, see the *Core Connections Course 3* Checkpoint 5 materials.

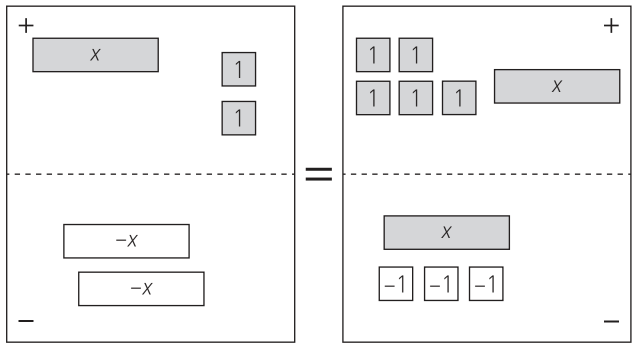
Once students understand how to solve equations using an Equation Mat, they may use the visual experience of moving tiles to solve equations with variables and numbers. The procedures for moving variables and numbers in the solving process follow the same rules.

Note: When the process of solving an equation ends with different numbers on each side of the equal sign (for example, $2 = 4$), there is no solution to the problem. When the result is the same expression or number on each side of the equation (for example, $x + 2 = x + 2$), it means that all numbers are solutions. See the Methods & Meanings box in Lesson 3.2.4 of the *Core Connections Course 3* text.

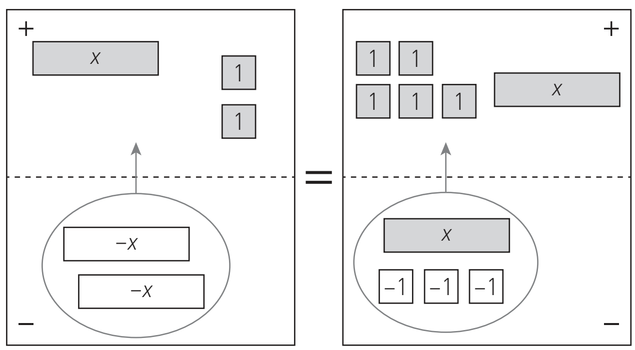
Examples

1. Solve $x + 2 - (-2x) = x + 5 - (x - 3)$.

First, build the equation on the Equation Mat.



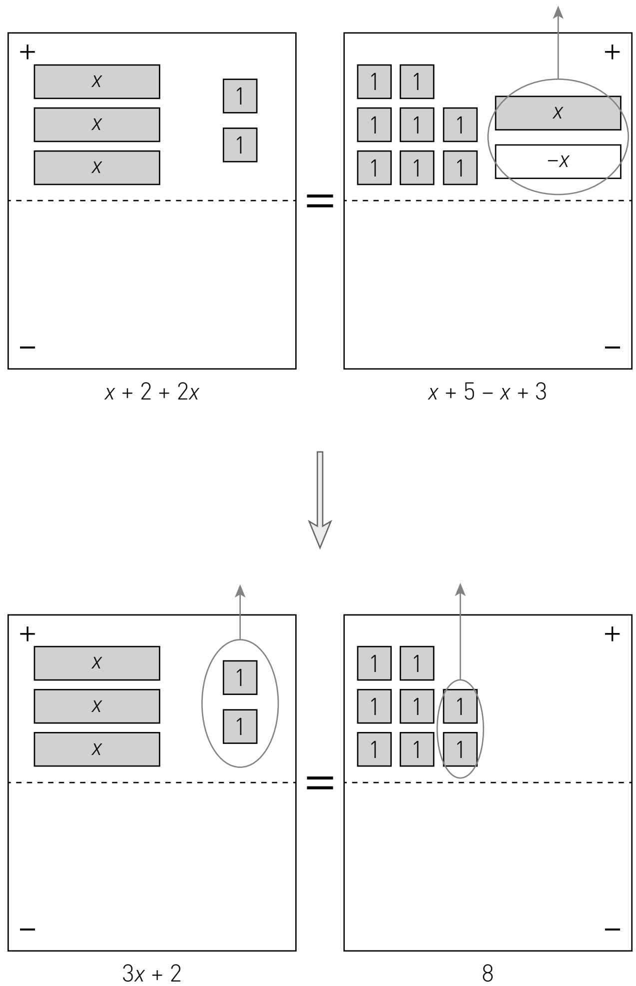
Second, flip the tiles from the negative region to the positive region.



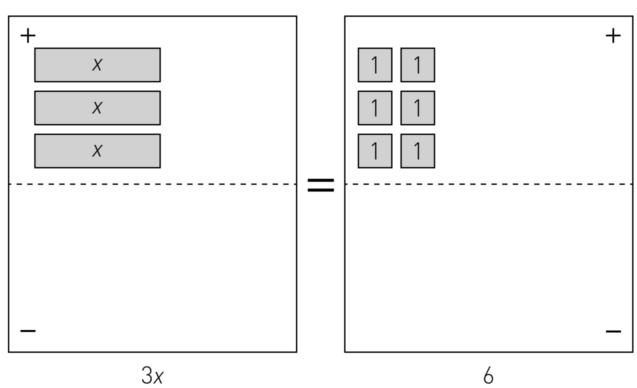
Solution continues on next page.

Solution continued from previous page.

Third, use valid moves on each side of the Equation Mat.



Isolate x-terms on one side and other terms on the other side by removing matching tiles from both sides of the Equation Mat.



Finally, since both sides of the equation are equal, determine the value of x.

$$3x = 6$$
$$x = 2$$

2. $3x + 3x - 1 = 4x + 9$

$3x + 3x - 1 = 4x + 9$	
$6x - 1 = 4x + 9$	rewrite in fewer terms
$2x = 10$	add 1, subtract $4x$ on each side
$x = 5$	divide

3. $-2x + 1 - (-3x + 3) = -4 + (-x - 2)$

$-2x + 1 - (-3x + 3) = -4 + (-x - 2)$	
$-2x + 1 + 3x - 3 = -4 - x - 2$	remove parentheses (flip)
$x - 2 = -x - 6$	rewrite in fewer terms
$2x = -4$	add x, add 2 to each side
$x = -2$	divide

Practice

Solve each equation.

1. $2x - 3 = -x + 3$

2. $1 + 3x - x = x - 4 + 2x$

3. $4 - 3x = 2x - 6$

4. $3 + 3x - (x - 2) = 3x + 4$

5. $-(x + 3) = 2x - 6$

6. $-4 + 3x - 1 = 2x + 1 + 2x$

7. $-x + 3 = 10$

8. $5x - 3 + 2x = x + 7 + 6x$

9. $4y - 8 - 2y = 4$

10. $9 - (1 - 3y) = 4 + y - (3 - y)$

11. $2x - 7 = -x - 1$

12. $-2 - 3x = x - 2 - 4x$

13. $-3x + 7 = x - 1$

14. $1 + 2x - 4 = -3 - (-x)$

15. $2x - 1 - 1 = x - 3 - (-5 + x)$

16. $-4x - 3 = x - 1 - 5x$

17. $10 = x + 6 + 2x$

18. $-(x - 2) = x - 5 - 3x$

19. $6 - x - 3 = 4x - 8$

20. $0.5x - (-x + 3) = x - 5$

Answers

1. $x = 2$

2. $x = 5$

3. $x = 2$

4. $x = 1$

5. $x = 1$

6. $x = -6$

7. $x = -7$

8. no solution

9. $y = 6$

10. $y = -7$

11. $x = 2$

12. all numbers

13. $x = 2$

14. $x = 0$

15. $x = 2$

16. no solution

17. $x = 1\frac{1}{3}$

18. $x = -7$

19. $x = 2\frac{1}{5}$

20. $x = -4$