Solving Equations With Multiple Variables

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Overview

Solving an equation with more than one variable uses the same process as solving an equation with one variable. The only difference is that, instead of the answer always being a number, it may be an expression that includes numbers and variables. The same steps can be used: removing parentheses, rewriting by combining like terms, removing the same thing from both sides of the equation, moving the desired variables to one side of the equation and the rest of the variables to the other side, and division or multiplication.

Examples

1. Solve $3x - 2y = 6$ for y.

$3x - 2y = 6$

$-2y = -3x + 6$ Subtract $3x$.

$y = \frac{-3x + 6}{-2}$ Divide by -2.

$y = \frac{3}{2}x - 3$ Rewrite with fewer terms.

2. Solve $7 + 2(x + y) = 11$ for y.

$7 + 2(x + y) = 11$

$2(x + y) = 4$ Subtract 7.

$2x + 2y = 4$ Distribute the 2.

$2y = -2x + 4$ Subtract $2x$.

$y = \frac{-2x + 4}{2}$ Divide by 2.

$y = -x + 2$ Rewrite with fewer terms.

3. Solve $y = 3x - 4$ for x.

$y = 3x - 4$

$y + 4 = 3x$ Add 4.

$\frac{y + 4}{3} = x$ Divide by 3.

4. Solve $I = prt$ for t.

$I = prt$

$\frac{I}{pr} = t$ Divide by pr.

Practice

Solve each equation for the specified variable.

1. y in $5x + 3y = 15$

2. x in $5x + 3y = 15$

3. w in $2l + 2w = P$

4. m in $4n = 3m - 1$

5. a in $2a + b = c$

6. a in $b - 2a = c$

7. p in $6 - 2(q - 3p) = 4p$

8. x in $y = \frac{1}{4}x - 1$

9. r in $4(r - 3s) = r - 5s$

Answers

Other equivalent forms are possible.

1. $y = -\frac{5}{3}x + 5$

2. $x = -\frac{3}{5}y + 3$

3. $w = -l + \frac{P}{2}$

4. $m = \frac{4n + 1}{3}$

5. $a = \frac{c - b}{2}$

6. $a = \frac{c - b}{-2}$ or $\frac{b - c}{2}$

7. $p = q - 3$

8. $x = 4y - 4$

9. $r = \frac{7s}{3}$

Solving Equations With Fractional Coefficients

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Overview

Students used scale factors (multipliers) to enlarge and reduce figures as well as to increase and decrease quantities. To convert from original quantities or lengths to new ones, all of the original quantities or lengths must be multiplied by the scale factor. To reverse this process and scale from the new situation back to the original, the new values are divided by the scale factor. Division by a scale factor is the same as multiplying by a reciprocal, and this concept is also useful in solving one-step equations with fractional coefficients. To remove a fractional coefficient, students divide each term in the equation by the coefficient or multiply each term by the reciprocal of the coefficient.

To remove fractions in more complicated equations, students use "Fraction Busters." Multiplying all of the terms of an equation by a common denominator will remove all of the fractions from the equation. Then the equation can be solved in the usual way.

For additional information, see the Methods & Meanings box in Lesson 5.2.1 of the *Core Connections Course 3* text. For additional examples and practice, see the *Core Connections Course 3* Checkpoint 7 materials.

Examples

1. Solve: $\frac{2}{3}x = 12$

METHOD 1

Use division and common denominators.

$$\frac{2}{3}x = 12$$

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{12}{\frac{2}{3}}$$

$$x = \frac{12}{\frac{2}{3}}$$

$$x = 12 \div \frac{2}{3}$$

$$x = \frac{36}{3} \div \frac{2}{3}$$

$$x = \frac{36}{2}$$

$$x = 18$$

METHOD 2

Use reciprocals.

$$\frac{2}{3}x = 12$$

$$\left(\frac{3}{2}\right)\left(\frac{2}{3}x\right) = (12)\left(\frac{3}{2}\right)$$

$$x = 18$$

2. Solve: $\frac{x}{2}+\frac{x}{5}=6$

Multiplying by 10 (the common denominator) will eliminate the fractions.

$$\frac{x}{2}+\frac{x}{5}=6$$

$$(10)\left(\frac{x}{2}+\frac{x}{5}\right)=(6)(10)$$

$$(10)\left(\frac{x}{2}\right)+(10)\left(\frac{x}{5}\right)=(6)(10)$$

$$5x+2x=60$$

$$7x=60$$

$$x=\frac{60}{7}$$

$$x=8.57$$

Practice

Solve each equation.

1. $\frac{3}{4}x=60$

2. $\frac{2}{5}x=42$

3. $\frac{3}{5}y=40$

4. $-\frac{8}{3}m=6$

5. $\frac{3x+1}{2}=5$

6. $\frac{x}{3}-\frac{x}{5}=3$

7. $\frac{y+7}{3}=\frac{y}{5}$

8. $\frac{m}{3}-\frac{2m}{5}=\frac{1}{5}$

9. $-\frac{3}{5}x=\frac{2}{3}$

10. $\frac{x}{2}+\frac{x-3}{5}=3$

11. $\frac{1}{3}x+\frac{1}{4}x=4$

12. $\frac{2x}{5}+\frac{x-1}{3}=4$

Answers

1. $x=80$

2. $x=105$

3. $y=66\frac{2}{3}$

4. $m=-\frac{9}{4}$

5. $y=3$

6. $x=22.5$

7. $y=-17\frac{1}{2}$

8. $m=-3$

9. $x=-\frac{10}{9}$

10. $x=\frac{36}{7}$

11. $x=\frac{48}{7}$

12. $x=\frac{65}{11}$

Systems of Linear Equations

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Overview

When two lines are considered together, they form a system of linear equations. They intersect at one point unless they are parallel or their equations are different forms of the same line. The point of intersection is the only pair of (x, y) values that will make *both* equations true. One way to locate the point of intersection is to graph the two lines. However, graphing is both time-consuming and, in many cases, not exact.

When two linear equations are written equal to y (in general, in the form $y = mx + b$), it is easy to take advantage of the fact that both y values are the same (equal) at the point of intersection. For example, if two lines are described by the equations $y = -2x + 5$ and $y = x - 1$, then the fact that both y values are equal at the point of intersection means the other two sides of the equations must also be equal to each other at that point. The right sides of these equations are said to have "equal values" at the point of intersection, so they can be set equal to each other: $-2x + 5 = x - 1$. This looks like the work done with Equation Mats.

This equation can be solved in the usual way, with the result $x = 2$. Now the x-coordinate of the point of intersection is known. Since this value will be the same in both of the original equations at the point of intersection, $x = 2$ can be substituted in either equation to solve for y: $y = -2(2) + 5$, so $y = 1$, or $y = 2 - 1$ and $y = 1$. The two lines in this example intersect at (2, 1).

For additional information, see the Methods & Meanings boxes in Lessons 5.2.2, 5.2.3, and 5.2.4 of *Core Connections Course 3*.

Examples

1. Determine the point of intersection for $y = 5x + 1$ and $y = -3x - 15$.

Substitute the equal parts of the equations. Solve for x.

$y = \underbrace{5x + 1}$ $\qquad$ $y = \underbrace{-3x - 15}$

Replace x with -2 in either original equation and solve for y.

$x = -2$	$x = -2$
$y = 5x + 1$	$y = -3x - 15$
$y = 5(-2) + 1$	$y = -3(-2) - 15$
$y = -10 + 1$	$y = 6 - 15$
$y = -9$	$y = -9$

The two lines intersect at $(-2, -9)$.

2. The Mathematical Amusement Park is different from other amusement parks. Visitors encounter their first decision involving math when they pay their entrance fee. They have a choice between two plans. With Plan 1, they pay \$5 to enter the park and \$3 for each ride. With Plan 2, they pay \$12 to enter the park and \$2 for each ride. For what number of rides will the plans cost the same amount?

The first step in the solution is to write an equation that describes the total cost of each plan. In this example, let x equal the number of rides and y be the total cost. Then the equation to represent Plan 1 for x rides is $y = 5 + 3x$. Similarly, the equation representing Plan 2 for x rides is $y = 12 + 2x$.

We know that if the two plans cost the same, then the y-values of $y = 5 + 3x$ and $y = 12 + 2x$ must be the same. The next step is to write one equation using x, then solve for x.

First, substitute the equal parts of the equations. Solve for x.

$$y = \underbrace{5 + 3x} \qquad\qquad y = \underbrace{12 + 2x}$$

Then use the value of x to find y.

$$x = 7 \qquad\qquad\qquad x = 7$$

$$\begin{aligned} y &= 5 + 3x & \qquad y &= 12 + 2x \\ y &= 5 + 3(7) & \qquad y &= 12 + 2(7) \\ y &= 5 + 21 & \qquad y &= 12 + 14 \\ y &= 26 & \qquad y &= 26 \end{aligned}$$

This means that if you go on 7 rides, both plans will have the same cost of \$26.

Practice

Determine the point of intersection (x, y) for each system of linear equations.

1. $y = x - 6$
$y = 12 - x$

2. $y = 3x - 5$
$y = x + 3$

3. $y = 2x + 16$
$y = 5x + 4$

4. $y = 3x - 5$
$y = 2x + 14$

5. $y = x + 7$
$y = 4x - 5$

6. $y = 7 - 3x$
$y = 2x - 8$

Write a system of linear equations for each problem and use them to determine a solution.

7. Jacques will wash the windows of a house for $15.00 plus $1.00 per window. Ray will wash them for $5.00 plus $2.00 per window. Let *x* be the number of windows and *y* be the total charge for washing them. Write an equation that represents how much each person charges to wash windows. Solve the system of equations and explain what the solution means and when it would be most economical to hire each window washer.

8. Elle has moved to Hawksbluff for one year and wants to join a rock climbing gym. She has narrowed her choices to two places: Adventure Climb and Bouldering Place. Adventure Climb charges a fee of $95 to join and an additional $15 per month. Bouldering Place charges a fee of $125 to join and a monthly fee of $12. Write an equation to represent each gym's charge. What do your variables represent? Solve the system of equations and state when the costs will be the same. Elle will only live there for one year, so which gym will be less expensive for her?

9. Ginny is raising pumpkins to enter a contest to see who can grow the heaviest pumpkin. Her best pumpkin weighs 22 pounds and is growing at the rate of 2.5 pounds per week. Martha planted her pumpkins late. Her best pumpkin weighs 10 pounds, but she expects it to grow 4 pounds per week. Assuming that their pumpkins grow at these rates, in how many weeks will their pumpkins weigh the same? How much will they weigh? If the contest ends in seven weeks, who will have the heavier pumpkin at that time?

10. Larry and Betty are siblings who are saving money to buy their own laptop computers. Larry has $215 and can save $35 each week. Betty has $380 and can save $20 each week. When will Larry and Betty have the same amount of money?

Answers

1. (9, 3)

2. (4, 7)

3. (4, 24)

4. (19, 52)

5. (4, 11)

6. (3, −2)

7. Let x = number of windows, y = cost. Jacques: $y = 15 + 1x$; Roy: $y = 5 + 2x$. The solution is (10, 25), which means that the cost to wash 10 windows is \$25. For fewer than 10 windows, hire Roy; for more than 10 windows, hire Jacques.

8. Let x = weeks, y = total charges. Adventure Climb: $y = 95 + 15x$; Bouldering Place: $y = 125 + 12x$. The solution is (10, 245). At 10 months, the cost at either club is \$245. For 12 months, the Bouldering Place is less expensive.

9. Let x = weeks and y = weight of the pumpkin. Ginny: $y = 2.5x + 22$; Martha: $y = 4x + 10$. The solution is (8, 42), so their pumpkins will weigh 42 pounds in 8 weeks. Ginny would win with 39.5 pounds, to 38 pounds for Martha.

10. Let x = weeks, y = total money saved. Larry: $y = 35x + 215$; Betty: $y = 20x + 380$. The solution is (11, 600). They will both have \$600 in 11 weeks.