Rigid Transformations

GUIDED SKILL SUPPORT

Overview

Studying transformations of geometric shapes builds a foundation for a key idea in geometry: congruence. In this introduction to transformations, the students explore three rigid motions: translations, reflections, and rotations. A translation slides a figure horizontally, vertically, or both. A reflection flips a figure across a fixed line (for example, the x-axis). A rotation turns an object about a point (for example, (0, 0)).

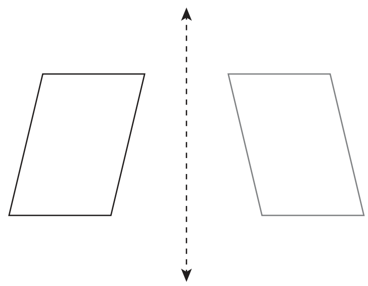
This exploration is done with simple tools that can be found at home (tracing paper) and/or computer software. Students change the position and/or orientation of a shape by applying one or more of these motions to the original figure to create its image in a new position without changing its size or shape. Transformations also lead directly to studying symmetry in shapes. These ideas will help with describing and classifying geometric shapes later in the course.

For additional information, see the Methods & Meanings box in Lesson 6.1.3 of the *Core Connections Course 3* text.

Examples

1. Decide which transformation was used on each pair of shapes. Some may be a combination of transformations.

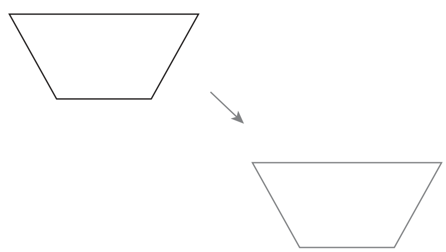
a.



The parallelogram is reflected (flipped) across an invisible vertical line. Imagine a mirror running vertically and equally distanced between the two figures. One figure would be the reflection of the other.

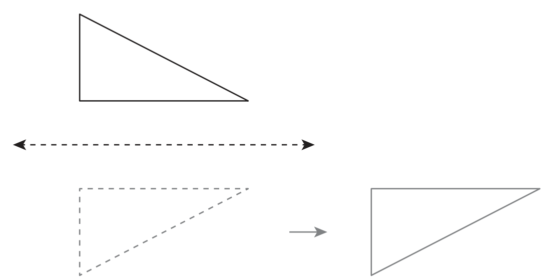
Reflecting a shape once changes its orientation, that is, how its parts point in different directions on the flat surface. The two sides of the figure on the left slant upwards to the right, whereas in its reflection, they slant upwards to the left. Likewise, the angles in the figure at left "switch positions" in the figure at right.

b.



The shape is translated (or slid) to the right and down. The orientation is the same.

c.

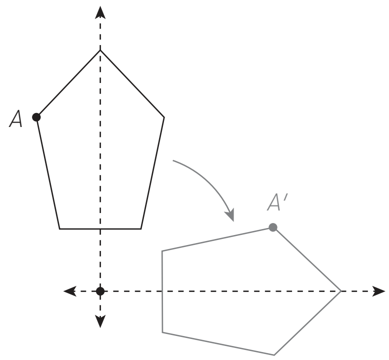


This is a combination of transformations. First, the triangle is reflected (flipped) across an invisible horizontal line. Then, it is translated (slid) to the right.

Example continues on next page.

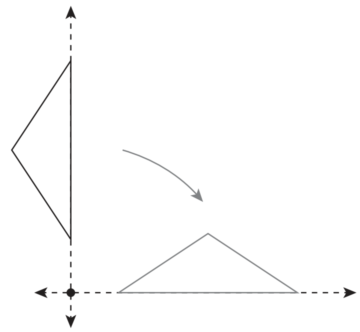
Example continued from previous page.

d.



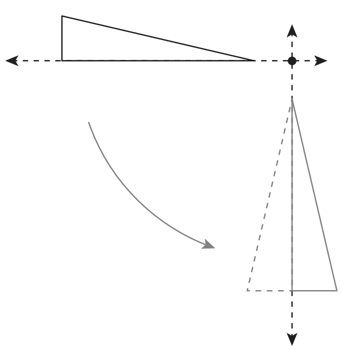
The pentagon has been rotated (turned) clockwise to create the second figure. Imagine tracing the first figure on tracing paper, then holding the tracing paper with a pin at one point below the first pentagon, then turning the paper clockwise 90°. The second pentagon would be the result. Some students might see this as a reflection across a diagonal line. The pentagon itself could be, but with the labeling of point *A*, the entire shape cannot be a reflection. If it had been reflected, point *A* would have to be on the corner below the one shown in the rotated figure.

e.



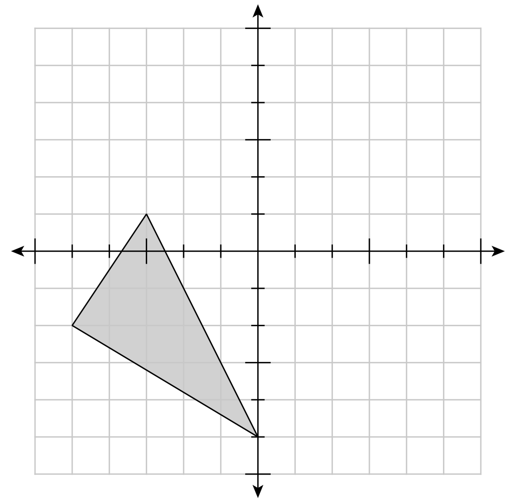
The triangle with the vertical side is rotated 90° clockwise, resulting in the triangle with the horizontal side.

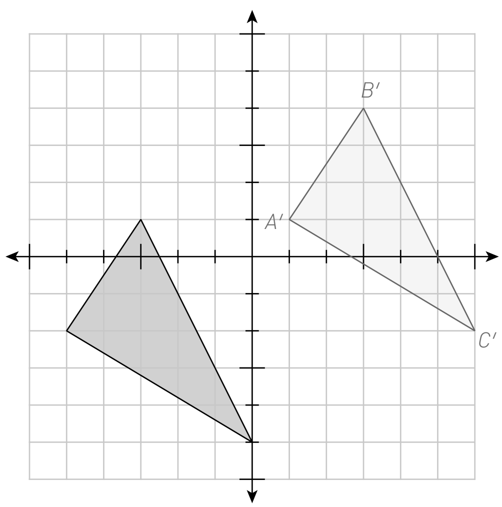
f.



The triangle is rotated (the horizontal side becomes vertical) but also reflected since the orientation is different from the first figure.

2. Translate (slide) ΔABC right six units and up three units. Give the coordinates of the vertices of the new triangle.



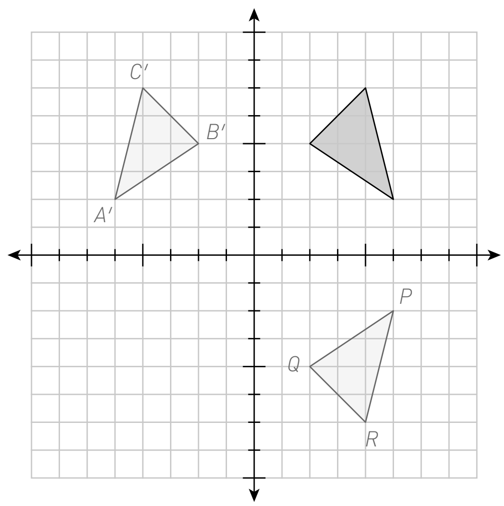


The original vertices are $A(-5, -2)$, $B(-3, 1)$, and $C(0, -5)$. The new vertices are $A'(1, 1)$, $B'(3, 4)$, and $C'(6, -2)$. Notice that the change to each original point (x, y) can be represented by $(x + 6, y + 3)$.

3. Reflect (flip) ΔABC with coordinates $A(5, 2)$, $B(2, 4)$, and $C(4, 6)$ across the y-axis to get $\Delta A'B'C'$. Then reflect ΔABC across the x-axis.

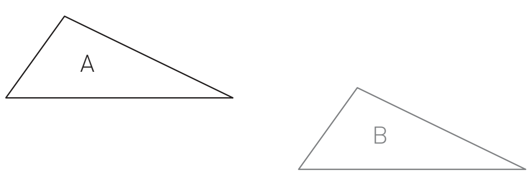
A reflection is always the same distance from the line of reflection as the original figure is. The new points after reflecting ΔABC are $A'(-5, 2)$, $B'(-2, 4)$, and $C'(-4, 6)$. Notice that in reflecting across the y-axis, the change to each original point (x, y) can be represented by $(-x, y)$.

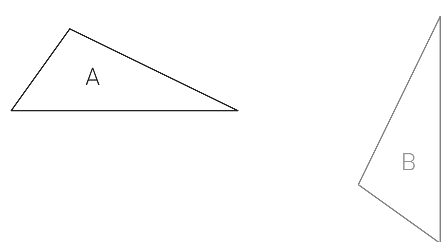
If you reflect ΔABC across the x-axis to get ΔPQR, then the new points are $P(5, -2)$, $Q(2, -4)$, and $R(4, -6)$. In this case, reflecting across the x-axis, the change to each original point (x, y) can be represented by $(x, -y)$.



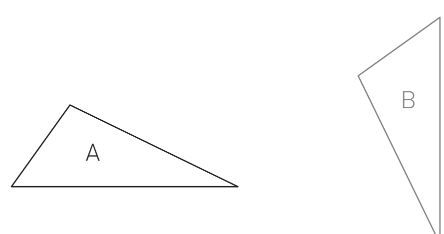
Practice

1. For each pair of triangles, describe the transformation or series of transformations that moves triangle A to the location of triangle B.

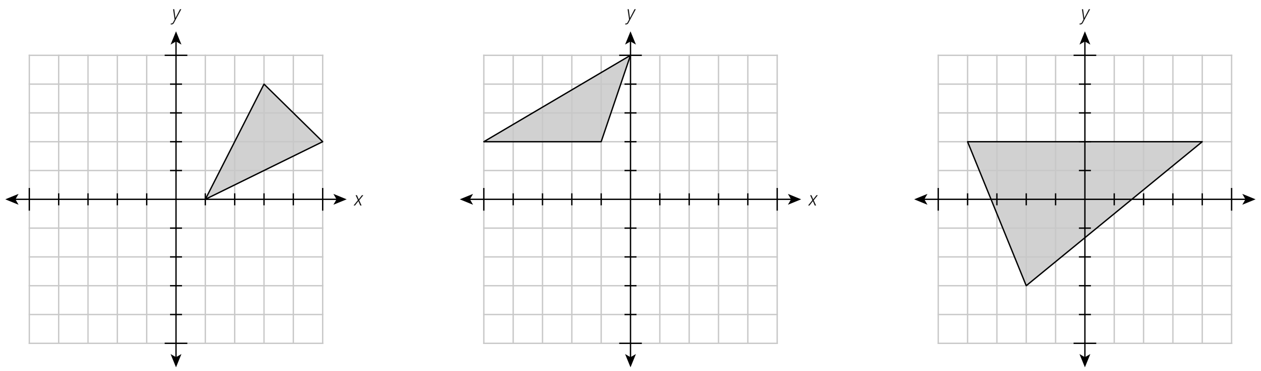
a.


b.


c.


d.


2. For parts (a) through (p), state the new coordinates after each transformation. Refer to the figures shown:



a. Slide shape A left 2 units and down 3 units.

b. Slide shape B right 3 units and down 5 units.

c. Slide shape C left 1 unit and up 2 units.

d. Flip shape A across the *x*-axis.

e. Flip shape B across the *x*-axis.

f. Flip shape C across the *x*-axis.

g. Flip shape A across the *y*-axis.

h. Flip shape B across the *y*-axis.

i. Flip shape C across the *y*-axis.

j. Rotate shape A 90° counterclockwise about the origin.

k. Rotate shape B 90° counterclockwise about the origin.

l. Rotate shape C 90° counterclockwise about the origin.

m. Rotate shape A 180° counterclockwise about the origin.

n. Rotate shape C 180° counterclockwise about the origin.

o. Rotate shape B 270° counterclockwise about the origin.

p. Rotate shape C 90° clockwise about the origin.

Answers

1. Responses may vary.

a. translation

b. rotation and translation

c. reflection

d. rotation and reflection

2. Coordinates are given in the order X', Y', Z'.

a. (−1, −3), (1, 1), (3, −1)

b. (−2, −3), (2, −3), (3, 0)

c. (−5, 4), (3, 4), (−3, −1)

d. (1, 0), (3, −4), (5, −2)

e. (−5, −2), (−1, −2), (0, −5)

f. (−4, −2), (4, −2), (−2, 3)

g. (−1, 0), (−3, 4), (−5, 2)

h. (5, 2), (1, 2), (0, 5)

i. (4, 2), (−4, 2), (2, −3)

j. (0, 1), (−4, 3), (−2, 5)

k. (−2, −5), (−2, −1), (−5, 0)

l. (−2, −4), (−2, 4), (3, −2)

m. (−1, 0), (−3, −4), (−5, −2)

n. (4, −2), (−4, −2), (2, 3)

o. (2, 5), (2, 1), (5, 0)

p. (−2, 4), (2, −4), (−3, 2)

Similar Figures

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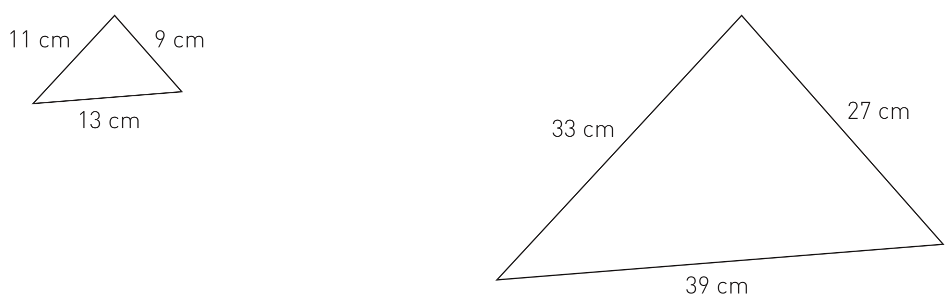
Overview

Two figures that have the same shape but not necessarily the same size are similar. In similar figures, the measures of the corresponding angles are equal and the ratios of the corresponding sides are proportional. This ratio is called the scale factor.

For information about corresponding sides and angles of similar figures, see the Methods & Meanings box in Lesson 6.2.2 of the *Core Connections Course 3* text. For information about scale factor and similarity, see the Methods & Meanings box in Lesson 6.2.6 of the *Core Connections Course 3* text.

Examples

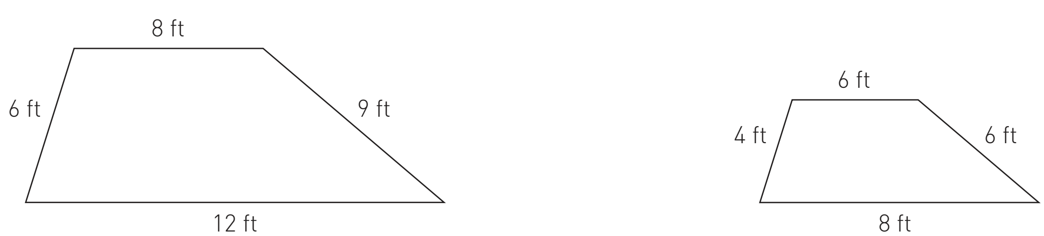
1. Determine whether the figures are similar. If so, what is the scale factor?



$\frac{39}{13} = \frac{33}{11} = \frac{27}{9} = \frac{3}{1}$ or 3

The ratios of corresponding sides are equal, so the figures are similar. The scale factor to enlarge the small figure to the large one is 3. The scale factor that reduces the large figure to the small figure is $\frac{1}{3}$.

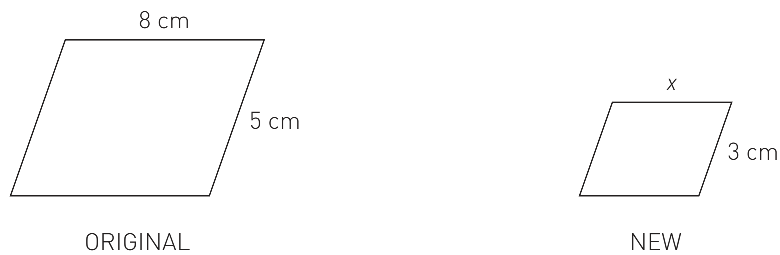
2. Are the figures similar? If so, state the scale factor.



$\frac{6}{4} = \frac{12}{8} = \frac{9}{6}$ and all of these equal $\frac{3}{2}$.

$\frac{8}{6} = \frac{4}{3}$ so the shapes are not similar.

3. What is the scale factor for the pair of similar figures? Use the scale factor to calculate the length of the side labeled with a variable.



scale factor = $\frac{3}{5}$ original $\cdot \frac{3}{5} \Rightarrow$ new $8 \cdot \frac{3}{5} = x$

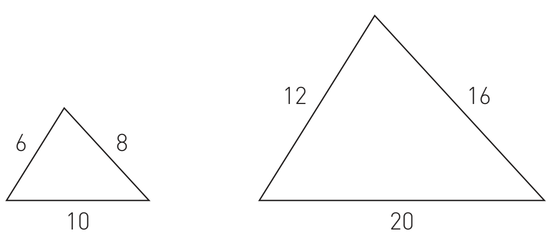
$x = \frac{24}{5}$

$x = 4.8$ cm

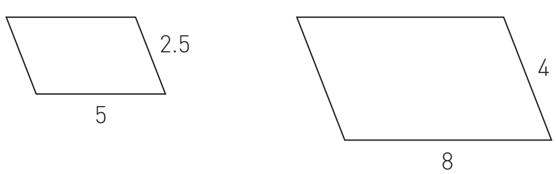
Practice

Determine whether the figures are similar. If so, state the scale factor of the first to the second.

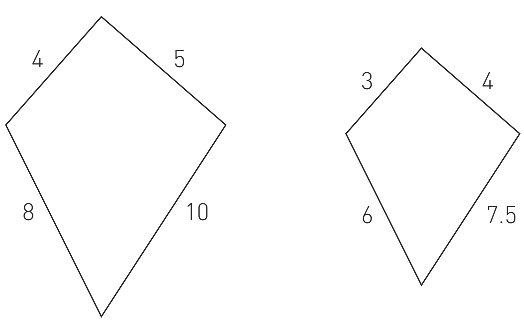
1.



2.

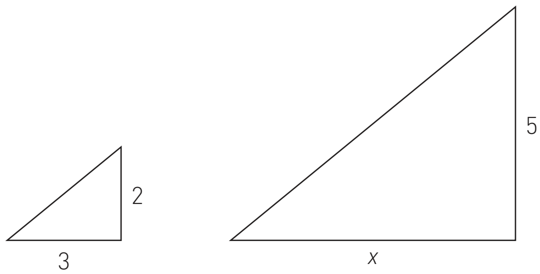


3.

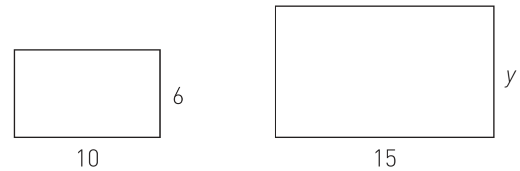


What is the scale factor for each pair of similar figures? Use the scale factor to calculate the length of the side labeled with a variable.

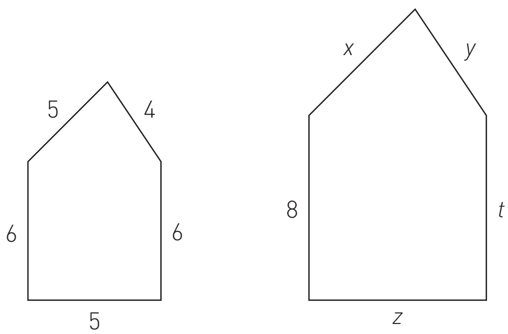
4.



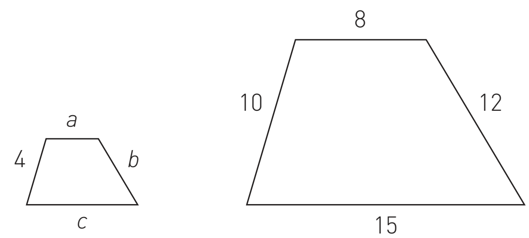
5.



6.



7.



Answers

1. similar; 2

2. similar; $\frac{8}{5} = 1.6$

3. not similar

4. $\frac{5}{2}$; $x = 7.5$

5. $\frac{3}{2}$; $y = 9$

6. $\frac{4}{3}$; $x = \frac{20}{3} = 6\frac{2}{3}$, $y = \frac{16}{3} = 5\frac{1}{3}$; $t = 8$, $z = \frac{20}{3} = 6\frac{2}{3}$

7. $\frac{5}{2}$; $a = \frac{16}{5} = 3.2$, $b = \frac{24}{5} = 4.8$, $c = 6$

Scaling to Solve Problems

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Overview

Students used scale factors (multipliers) to enlarge and reduce figures as well as increasing and decreasing quantities. All of the original quantities or lengths were multiplied by the scale factor to get the new quantities or lengths. To reverse this process and scale from the new situation back to the original, we divide by the scale factor. Division by a scale factor is the same as multiplying by its reciprocal. This same concept is useful in solving equations with fractional coefficients.

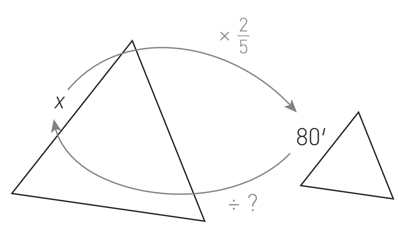
To remove a fractional coefficient, you may divide each term in the equation by the coefficient or multiply each term by the reciprocal of the coefficient. Recall that a reciprocal is the multiplicative inverse of a number, that is, the product of the two numbers is 1.

For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. The reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$. The reciprocal of 5 is $\frac{1}{5}$.

Scaling may also be used with percentage problems where a quantity is increased or decreased by a certain percent. Scaling by a factor of 1 does not change the quantity. Increasing by a certain percent may be found by converting the percent to a decimal and then multiplying by (1 + the decimal), while decreasing by a certain percent may be found by converting the percent to a decimal and then multiplying by (1 – the decimal).

Examples

1. The large triangle shown was reduced by a scale factor of $\frac{2}{5}$ to create a similar triangle. If the side labeled *x* now has a length of 80′ in the new figure, what was the original length?



To undo the reduction, multiply 80′ by the reciprocal of $\frac{2}{5}$, which is $\frac{5}{2}$, or divide 80′ by $\frac{2}{5}$.

$80 \div \frac{2}{5}$ is the same as $80 \cdot \frac{5}{2}$, so $x = 200'$.

2. Solve: $\frac{3}{4}x = 27$

METHOD 1

Use division and the common denominator method.

$$\frac{3}{4}x = 27$$

$$\frac{\frac{3}{4}x}{\frac{3}{4}} = \frac{27}{\frac{3}{4}}$$

$$x = \frac{27}{\frac{3}{4}}$$

$$x = 27 \div \frac{3}{4}$$

$$x = \frac{108}{4} \div \frac{3}{4}$$

$$x = 108 \div 3$$

$$x = 36$$

METHOD 2

Use reciprocals.

$$\frac{3}{4}x = 27$$

$$\left(\frac{4}{3}\right)\left(\frac{3}{4}x\right) = (27)\left(\frac{4}{3}\right)$$

$$x = \frac{108}{3}$$

$$x = 36$$

3. Samantha wants to leave a 15% tip on her lunch bill of $12.50. What scale factor should be used, and how much money should she leave?

Since tipping increases the total, the scale factor is (1 + 15%) = 1.15. She should leave (1.15)(12.50) = $14.38 or about $14.50.

4. Carlos sees that all DVDs are on sale at 40% off. If the regular price of a DVD is $24.95, what is the scale factor, and how much is the sale price??

If items are reduced 40%, the scale factor is (1 – 40%) = 0.60. The sale price is (0.60)(24.95) = $14.97.

Practice

1. A rectangle was enlarged by a scale factor of $\frac{5}{2}$, and the new width is 40 cm. What was the original width?

2. The length of the side of a triangle was reduced by a scale factor of $\frac{2}{3}$. If the new side is now 18 inches, what was the original side?

3. The scale factor used to create the design for a backyard is 2 inches for every 75 feet ($\frac{2}{75}$). If, on the design, the fire pit is 0.5 inches away from the house, how far from the house, in feet, should the fire pit be dug?

4. After a very successful year, Cheap Rentals raised the salaries of its employees by a scale factor of $\frac{11}{10}$. If Luan now makes $14.30 per hour, what did she earn before?

5. Solve: $\frac{3}{4}x = 60$

6. Solve: $\frac{2}{5}x = 42$

7. Solve: $\frac{3}{5}y = 40$

8. Solve: $-\frac{3}{4}m = 6$

9. What is the total cost of a $39.50 family dinner after you add a 20% tip?

10. If the current cost to attend Magicland Park is now $29.50 per person, what will be the cost after an 8% increase?

11. Winter coats are on clearance at 60% off. If the regular price is $79, what is the sale price?

12. The company president has offered to reduce her salary 10% to cut expenses. If she now earns $175,000, what will be her new salary?

Answers

1.	16 cm	**2.**	27 inches	**3.**	$18\frac{3}{4}$ feet	**4.**	$13.00
5.	80	**6.**	105	**7.**	$66\frac{2}{3}$	**8.**	−8
9.	$47.40	**10.**	$31.86	**11.**	$31.60	**12.**	$157,500