Exponents and Scientific Notation

GUIDED SKILL SUPPORT

Exponents

Overview

In the expression 5^2, 5 is the base and 2 is the exponent. For x^a, x is the base and a is the exponent. 5^2 means $5 \cdot 5$ and 5^3 means $5 \cdot 5 \cdot 5$, so you can write $\frac{5^5}{5^2}$ (which means $5^5 \div 5^2$) as $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5}$.

You can look for the Giant Ones to rewrite this. There are two Giant Ones, namely, $\frac{5}{5}$ twice, so $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5} = 5^3$ or 125. Writing 5^3 is usually sufficient.

When there is a variable, it is treated the same way. $\frac{x^7}{x^3}$ means $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x}$. The Giant One here is $\frac{x}{x}$ (three of them), if $x \neq 0$. The answer is x^4.

$5^2 \cdot 5^3$ means $(5 \cdot 5)(5 \cdot 5 \cdot 5)$, which is 5^5. Additionally, $(5^2)^3$ means $(5^2)(5^2)(5^2)$ or $(5 \cdot 5)(5 \cdot 5)(5 \cdot 5)$, which is 5^6.

When multiplying two exponent expressions with the same variable as their base, such as $x^4 \cdot x^5$, you only need to add the exponents. The answer is x^9. When taking an exponent of an exponent expression, such as $(x^4)^5$ (that is, x^4 to the fifth power), you multiply the exponents, in this case, $x^4 \cdot x^4 \cdot x^4 \cdot x^4 \cdot x^4$ or $x^{4 \cdot 5}$. The answer is x^{20}.

If the problem is $\frac{x^{10}}{x^4}$, you subtract the bottom exponent from the top exponent $(10 - 4)$. The answer is x^6. It is the same for problems like $\frac{x^{10}}{x^{-4}}$. Subtract: $10 - (-4) = 14$, and the answer is x^{14}.

Be sure the bases are the same to use these laws. An expression such as $x^5 \cdot y^6$ cannot be further simplified.

In general, the laws of exponents are:

$$x^a \cdot x^b = x^{(a+b)} \qquad (x^a)^b = x^{ab} \qquad \frac{x^a}{x^b} = x^{(a-b)}$$

$$x^0 = 1 \qquad x^{-n} = \frac{1}{x^n} \qquad (x^a y^b)^c = x^{ac} y^{bc}$$

These rules hold if $x \neq 0$ and $y \neq 0$.

For additional information, see the Methods & Meanings box in Lesson 8.2.4 of the *Core Connections Course 3* text.

Examples

1. $x^8 \cdot x^7$

$x^8 \cdot x^7 = x^{15}$

2. $\frac{x^{19}}{x^{13}}$

$\frac{x^{19}}{x^{13}} = x^6$

3. $(z^8)^3$

$(z^8)^3 = z^{24}$

4. $(x^2y^3)^4$

$(x^2y^3)^4 = x^8y^{12}$

5. $\frac{x^4}{x^{-3}}$

$\frac{x^4}{x^{-3}} = x^7$

6. $(2x^2y^3)^2$

$(2x^2y^3)^2 = 4x^4y^6$

7. $(3x^2y^{-2})^3$

$(3x^2y^{-2})^3 = 27x^6y^{-6}$ or $\frac{27x^6}{y}$

8. $\frac{x^8y^5z^2}{x^3y^6z^{-2}}$

$\frac{x^8y^5z^2}{x^3y^6z^{-2}} = \frac{x^5z^4}{y}$ or $x^5y^{-1}z^4$

9. 2^{-3}

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

10. $5^2 \cdot 5^{-4}$

$5^2 \cdot 5^{-4} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Practice

Rewrite each expression.

1.	$5^2 \cdot 5^4$	**2.**	$x^3 \cdot x^4$	**3.**	$\frac{5^{16}}{5^{14}}$	**4.**	$\frac{x^{10}}{x^6}$
5.	$(5^3)^3$	**6.**	$(x^4)^3$	**7.**	$(4x^2y^3)^4$	**8.**	$\frac{5^2}{5^{-3}}$
9.	$5^5 \cdot 5^{-2}$	**10.**	$(y^2)^{-3}$	**11.**	$(4a^2b^{-2})^3$	**12.**	$\frac{x^5y^4z^2}{x^4y^3z^2}$
13.	$\frac{x^6y^2z^3}{x^{-2}y^3z^{-1}}$	**14.**	$4x^2 \cdot 2x^3$	**15.**	4^{-2}	**16.**	3^{-3}
17.	$6^3 \cdot 6^{-2}$	**18.**	$(3^{-1})^2$				

Answers

1.	5^6	**2.**	x^7	**3.**	5^2	**4.**	x^4
5.	5^9	**6.**	x^{12}	**7.**	$256x^8y^{12}$	**8.**	5^5
9.	5^3	**10.**	y^{-6} or $\frac{1}{y^6}$	**11.**	$64a^6b^{-6}$ or $\frac{64a^6}{b^6}$	**12.**	xy
13.	$\frac{x^8z^4}{y}$	**14.**	$8x^5$	**15.**	$\frac{1}{16}$	**16.**	$\frac{1}{27}$
17.	6	**18.**	$\frac{1}{9}$				

Scientific Notation

Overview

Scientific notation is a way of writing very large and very small numbers compactly. A number is said to be in scientific notation when:

- It is written as the product of two factors.
- The first factor is less than 10 and greater than or equal to 1.
- The second factor has a base of 10 and an integer exponent (so, the factor is a power of 10).
- The factors are separated by a multiplication sign.

A positive exponent indicates a number whose absolute value is greater than 1. A negative exponent indicates a number whose absolute value is less than 1.

Scientific Notation	Standard Form
5.32×10^{11}	532,000,000,000
2.61×10^{-15}	0.00000000000000261

It is important to note that the exponent in scientific notation is not necessarily equal to the number of zeros used in standard form.

The number 5.32×10^{11} means $5.32 \times 100{,}000{,}000{,}000$. Standard form, in this case, is 532,000,000,000. In this example, you move the decimal point to the right 11 places to convert the number to standard form.

The number 2.61×10^{-15} means $2.61 \times 0.000000000000001$. You move the decimal point to the left 15 places to convert the number to standard form. The standard form is 0.00000000000000261.

Note: On your scientific calculator, displays like 4.357e12 and 3.65e–3 are numbers expressed in scientific notation. The first number means 4.357×10^{12} and the second means 3.65×10^{-3}. The calculator does this because there is not enough room on its display window to show the entire number.

For additional information, see the Methods & Meanings box in Lesson 8.2.3 of the *Core Connections Course 3* text.

Examples

1. Write each number in standard form.

a. 7.84×10^{8}

784,000,000

b. 3.72×10^{-3}

0.00372

2. Write each number in scientific notation.

a. 52,050,000

5.205×10^{7}

b. 0.000372

3.72×10^{-4}

Practice

Write each number in standard form.

1. 7.85×10^{11} **2.** 1.235×10^{9} **3.** 1.2305×10^{3} **4.** 3.89×10^{-7}

5. 5.28×10^{-4}

Write each number in scientific notation.

6. 391,000,000,000 **7.** 0.0000842 **8.** 123,056.7 **9.** 0.000000502

10. 25.7 **11.** 0.035 **12.** 5,600,000 **13.** 1,346.8

14. 0.000000000006 **15.** 634,700,000,000,000

Answers

1. 785,000,000,000 **2.** 1,235,000,000 **3.** 1,230.5 **4.** 0.000000389

5. 0.000528 **6.** 3.91×10^{11} **7.** 8.42×10^{-5} **8.** 1.230567×10^{5}

9. 5.02×10^{-7} **10.** 2.57×10^{1} **11.** 3.5×10^{-2} **12.** 5.6×10^{6}

13. 1.3468×10^{3} **14.** 6.0×10^{-12} **15.** 6.347×10^{14}

