Properties of Angles, Lines, and Triangles

GUIDED SKILL SUPPORT

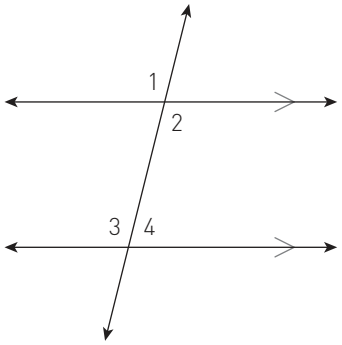
Overview

An angle is formed when two lines or line segments intersect. In this course, angles are measured in degrees (°). Angle relationships can be seen when two parallel lines are intersected by a transversal, as well as in triangles.

PARALLEL LINES

The diagram shown shows the following angle relationships:

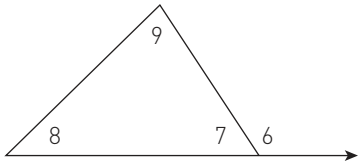
- corresponding angles have equal measure: $m\angle 1 = m\angle 3$
- alternate interior angles have equal measure: $m\angle 2 = m\angle 3$
- same-side interior angles are supplementary: $m\angle 2 + m\angle 4 = 180°$
- vertical angles have equal measure: $m\angle 1 = m\angle 2$
- straight angles measure 180°: $m\angle 3 + m\angle 4 = 180°$



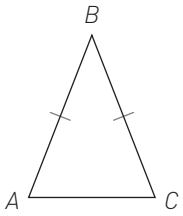
TRIANGLES

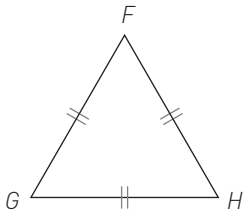
The diagram shown contains the following angle relationships:

- interior angles are supplementary: $m\angle 7 + m\angle 8 + m\angle 9 = 180°$
- the exterior angle equals the sum of the two remote interior angles: $m\angle 6 = m\angle 8 + m\angle 9$



Isosceles triangles and equilateral triangles have additional angle relationships. In the isosceles triangle ΔABC shown, $BA = BC$ and $m\angle A = m\angle C$. In the equilateral triangle ΔGFH shown, $GF = FH = HG$ and $m\angle G = m\angle F = m\angle H = 60°$.

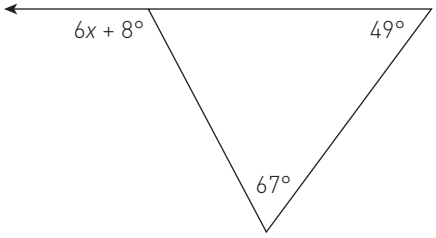




For more information, see the Methods & Meanings boxes in Lessons 9.1.2, 9.1.3, and 9.1.4 of the *Core Connections Course 3* text.

Examples

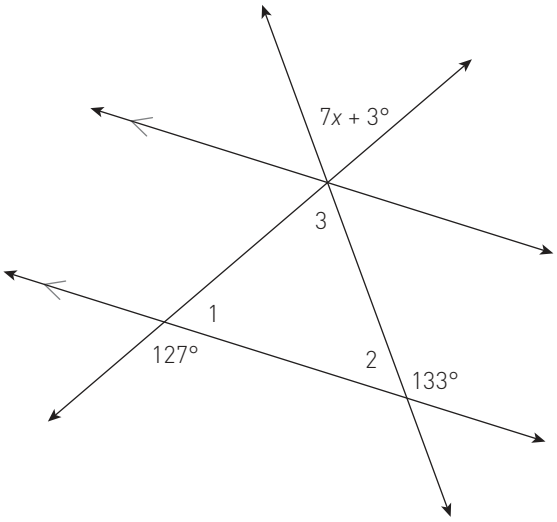
1. Solve for x.



Use the Exterior Angle Theorem:

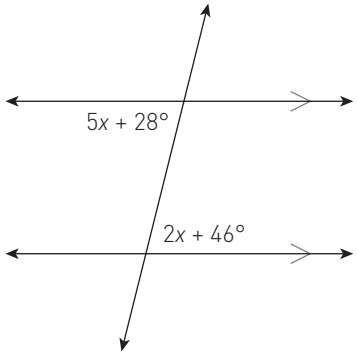
$$6x + 8° = 49° + 67°$$
$$6x + 8° = 116°$$
$$6x = 108°$$
$$x = \frac{108°}{6}$$
$$x = 18°$$

2. Solve for x.



There are a number of relationships in this diagram. First, $\angle 1$ and the 127° angle are supplementary, so $m\angle 1 + 127° = 180°$, $m\angle 1 = 53°$. Using the same idea, $m\angle 2 = 47°$. Next, $m\angle 3 + 53° + 47° = 180°$, so $m\angle 3 = 80°$. Because angle 3 forms a vertical pair with the angle marked $7x + 3°$, $80° = 7x + 3°$, so $x = 11°$.

3. Determine the measure of the acute alternate interior angles.



Parallel lines mean that alternate interior angles are equal, so:

$$5x + 28° = 2x + 46°$$
$$3x = 18°$$
$$x = 6°$$

Using either algebraic expression, calculate the angle measure:

$x = 6°$

$5x + 28°$
$= 5(6°) + 28°$
$= 30° + 28°$
$= 58°$

$x = 6°$

$2x + 46°$
$= 2(6°) + 46°$
$= 12° + 46°$
$= 58°$

Practice

Use the geometric properties you have learned to solve for x in each diagram and write the property you use in each case.

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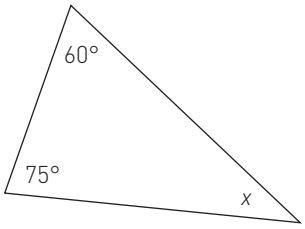

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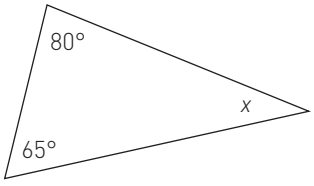

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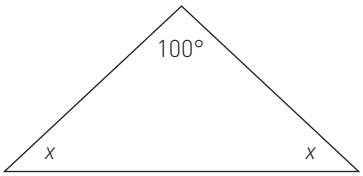

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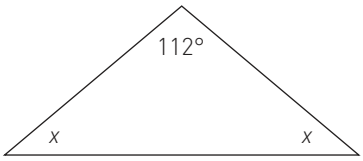

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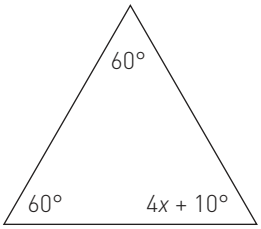

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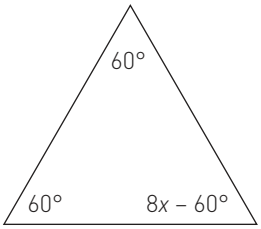

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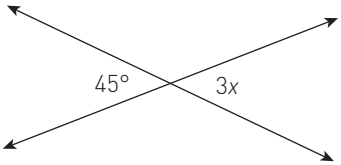

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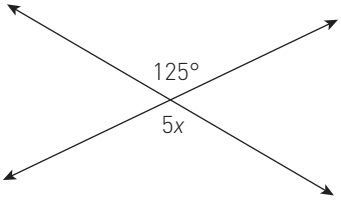

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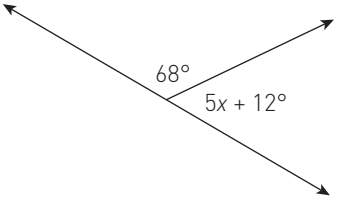

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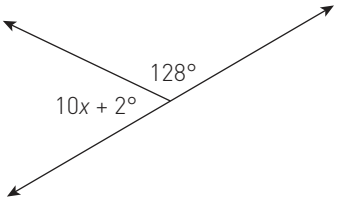

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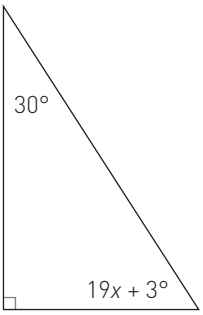

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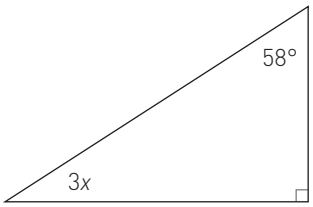

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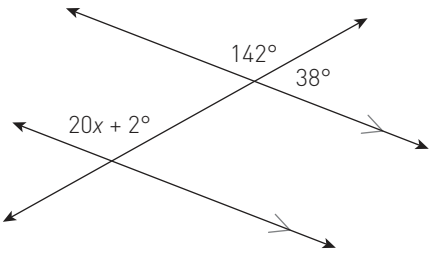

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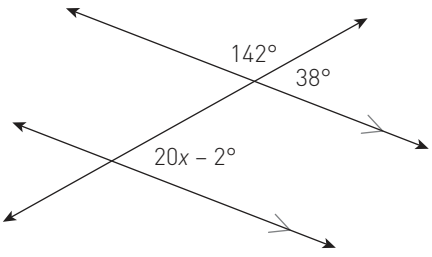

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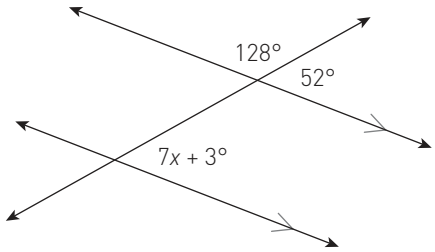

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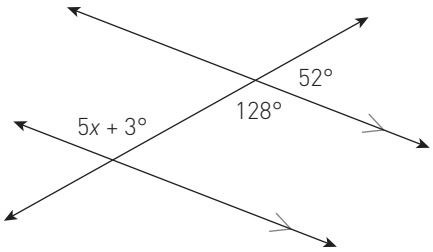

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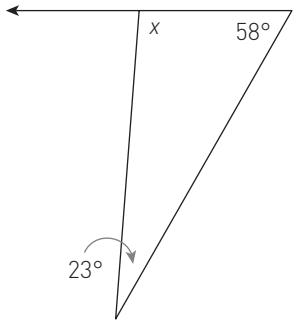

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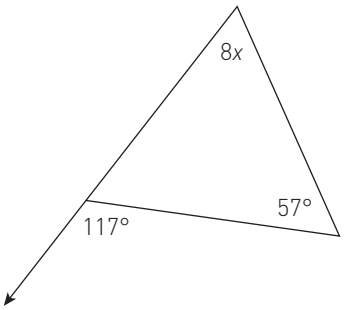

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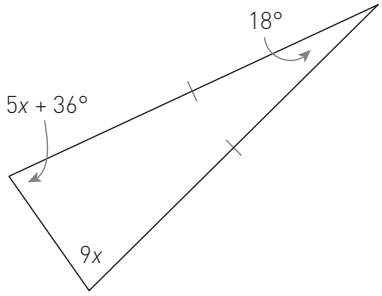

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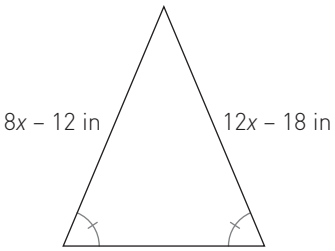

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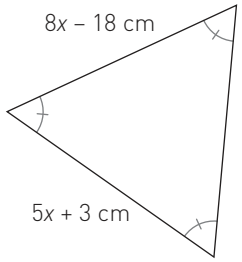

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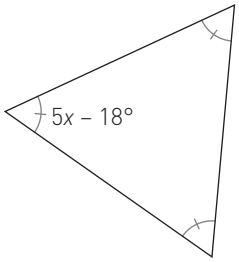

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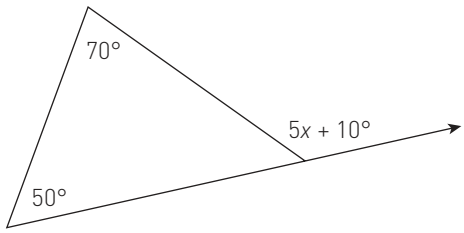

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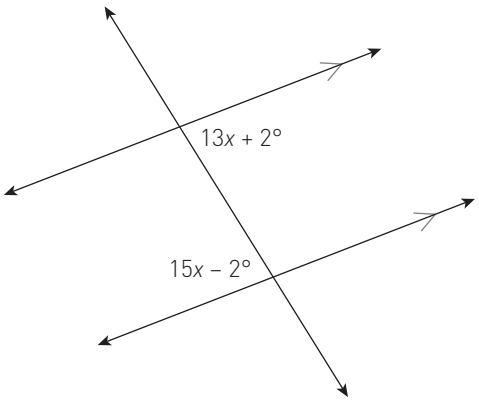

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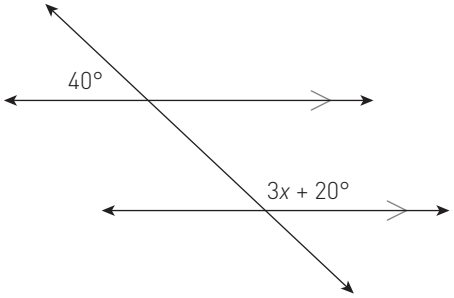

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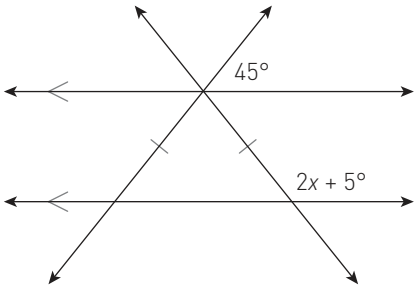

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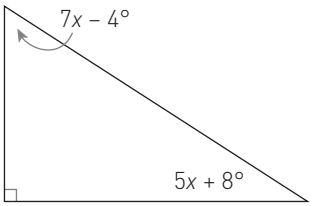

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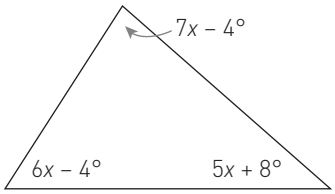


Answers

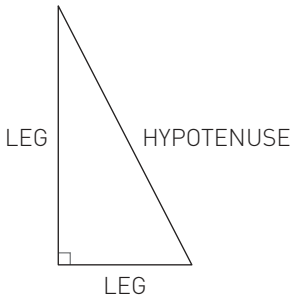
1.	45°	**2.**	35°	**3.**	40°	**4.**	34°
5.	12.5°	**6.**	15°	**7.**	15°	**8.**	25°
9.	20°	**10.**	5°	**11.**	3°	**12.**	$\frac{32}{3}° = 10\frac{2}{3}°$
13.	7°	**14.**	2°	**15.**	7°	**16.**	25°
17.	99°	**18.**	7.5°	**19.**	9°	**20.**	1.5°
21.	7°	**22.**	15.6°	**23.**	22°	**24.**	2°
25.	40°	**26.**	65°	**27.**	$7\frac{1}{6}°$	**28.**	10°

Right Triangle Theorem

GUIDED SKILL SUPPORT

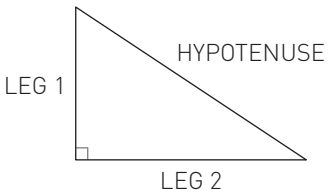
Overview

A right triangle is a triangle in which the two shorter sides form a right angle. The shorter sides are called "legs," and the third and longest side is called the "hypotenuse."



The Right Triangle Theorem states that for any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$(\text{leg } 1)^2 + (\text{leg } 2)^2 = (\text{hypotenuse})^2$$

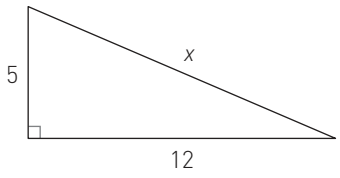


For additional information, see the Methods & Meanings box in Lesson 9.2.3 of the *Core Connections Course 3* text.

Examples

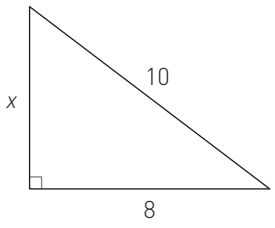
1. Use the Right Triangle Theorem to calculate the value of x.

a.



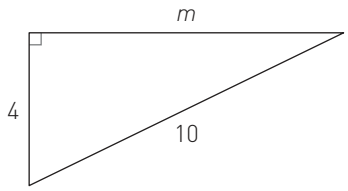
$$5^2 + 12^2 = x^2$$
$$25 + 144 = x^2$$
$$169 = x^2$$
$$13 = x$$

b.



$$x^2 + 8^2 = 10^2$$
$$x^2 + 64 = 100$$
$$x^2 = 36$$
$$x = 6$$

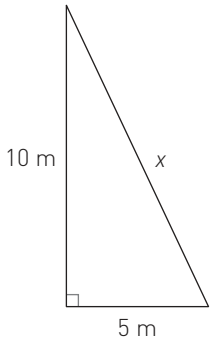
2. Not all problems will have rational answers. Use square root notation for the exact answer and then use your calculator to approximate the value of m.



$4^2 + m^2 = 10^2$
$16 + m^2 = 100$
$m^2 = 84$
$m = \sqrt{84}$
$m \approx 9.17$

3. A guy wire is needed to support a tower. The wire is attached to the ground 5 meters from the base of the tower. How long is the wire if the tower is 10 meters tall?

First, draw a diagram to model the problem, then write an equation using the Right Triangle Theorem and solve it.



$x^2 = 10^2 + 5^2$
$x^2 = 100 + 25$
$x^2 = 125$
$x = \sqrt{125}$
$x \approx 11.18$ m

Practice

For problems 1 through 6, write an equation and solve it to calculate the length of the unknown side. For problems 7 through 12, draw a diagram, write an equation, and solve it. Round answers to the nearest hundredth.

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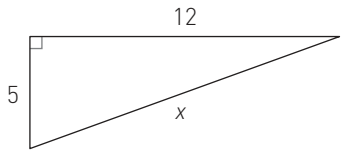

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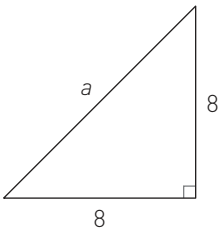

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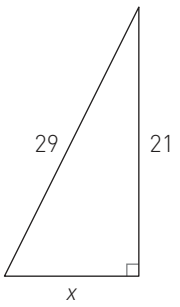

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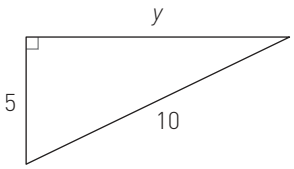

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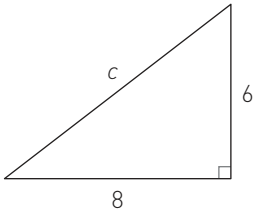

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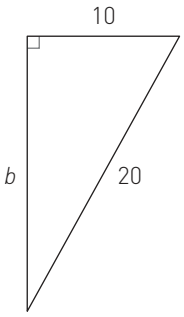

7. Calculate the diagonal of a television screen 30 inches tall by 35 inches wide.

8. A 9-meter ladder is 1 meter from the base of a building. How high up the building will the ladder reach?

9. Sam drove 8 miles south and then 5 miles west. How far is he from his starting point?

10. The length of the hypotenuse of a right triangle is 6 centimeters. If one leg is 4 centimeters, how long is the other leg?

11. Calculate the length of a path that runs diagonally across a 55-yard by 100-yard field.

12. How long could an umbrella be and still fit in the bottom of a suitcase measuring 1.5 feet by 2.5 feet?

Answers

1.	13 units	**2.**	11.31 units	**3.**	20 units	**4.**	8.66 units
5.	10 units	**6.**	17.32 units	**7.**	46.10 in	**8.**	8.94 m
9.	9.43 mi	**10.**	4.47 cm	**11.**	114.13 yd	**12.**	2.92 ft