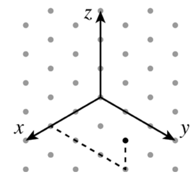
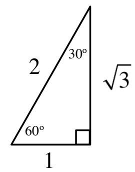
Glossary

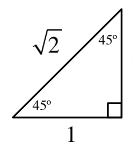
3-dimensional coordinate system In three dimensions the z-axis is perpendicular to the x-y plane at $(0, 0)$. Points in 3-dimensions are represented with coordinates (x, y, z). The first octant where x, y, and z are positive is shown at right with the point $(2, 3, 1)$. (p. 257)



30º-60º-90º triangle A special right triangle with acute angle measures of 30º and 60º. The side lengths are always in the ratio of $1:\sqrt{3}:2$. (p. 320)



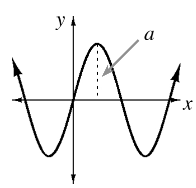
45º-45º-90º triangle A special right triangle with acute angle measures of 45º. The side lengths are always in the ratio of $1:1:\sqrt{2}$. (p. 320)



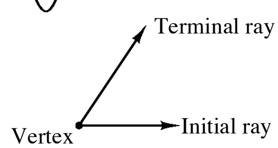
absolute value The absolute value of a number is the distance of the number from zero. Since the absolute value represents a distance without regard to direction, it is always a positive real number. For example, $|-7| = 7$, and $|7| = 7$. The absolute value of a complex number is its distance from zero in the complex plane. For a complex number $a + bi$, $|a+bi| = \sqrt{a^2+b^2}$. For example, $|3-2i| = \sqrt{3^2+(-2)^2} = \sqrt{13}$. (p. 404)

algebraic strategies Using algebraic strategies means to write an algebraic representation of the problem and then to rewrite those expressions to get equivalent, but more useful results that lead to a solution for the problem or that reveal more information to help solve it.

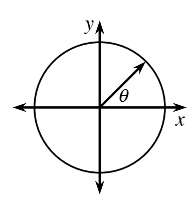
amplitude The amplitude of a cyclic graph is one-half the distance between the highest and lowest points. In the graph at right, a is the amplitude. (p. 360)



angle An angle is formed by two rays joined at a common endpoint (the vertex). In geometric figures, angles are usually formed by two segments with a common endpoint.



angles of rotation. Angles with one vertex at the origin and formed by counter-clockwise rotation from the positive x-axis, are referred to as angles of rotation in standard position. For an angle θ, in standard position, the positive x-axis is the initial ray and the terminal ray may point in any direction. The measure of such an angle may have any real value. (p. 319)



Angle Sum and Difference Identities Trigonometric identities that involve angle sums or differences. They are: $\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$ and $\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$. (p. 649)

annual Occurring once every year.

appreciation An increase in value. (p. 742)

arccosine of *x* See *cosine inverse (* $\cos^{-1} x$ *).*

arcsine of *x* See *sine inverse (* $\sin^{-1} x$ *).*

arctangent of *x* See *tangent inverse (* $\tan^{-1} x$ *).*

area For a 2-dimensional region, the number of non-overlapping square units needed to cover the region.

area model See *generic rectangle*. (p. 126)

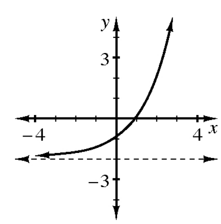
argument Used with sigma notation for sequences to describe the n^{th} term. In the expression, $\sum_{k=1}^{10}(2k+1)$ the expression $2k+1$ is the argument. (p. 519)

arithmetic sequence In an arithmetic sequence the difference between sequential terms is constant. Each term of an arithmetic sequence can be generated by adding the common difference to the previous term. For example in the sequence, 4, 7, 10, 13, …, the common difference is 3. (p. 692)

arithmetic series An arithmetic series is the sum of the terms of an arithmetic sequence. Given the arithmetic sequence $3, 7, 11, \ldots, 43$, the corresponding arithmetic series is $3+7+11+\ldots+43$. (p. 510)

association A relationship between two (or more) variables. An association between numerical variables can be displayed on a scatterplot, and described by its form, direction, strength, and outliers. Possible association between two categorical variables can be studied in a relative frequency table. Also see *scatterplot*.

asymptote A line that a graph of a curve approaches as the *x*-values approach positive or negative infinity. An asymptote is often represented by a dashed line on a graph. For example, the graph at right has an asymptote at $y = -2$. (p. 37)



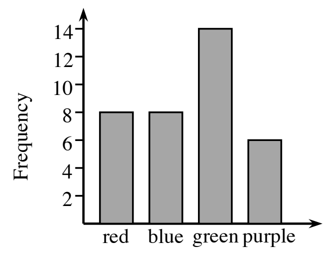
average See *mean*.

average rate of change The average rate of change of a function over an interval is the change in the value of the dependent quantity divided by the change in the independent quantity. It is the change in the *y*-value divided by the change in the *x*-value for two distinct points on the graph, which is the slope of the line through these two distinct points. (p. 137)

axes In a coordinate plane, the two perpendicular number lines that intersect at the origin (0, 0). The x-axis is horizontal and the y-axis is vertical. In 3-dimensions a third number line, the z-axis is perpendicular to the x-y plane at the origin (0, 0, 0). See *coordinate axes* and *3-dimensional coordinate system* for an illustration.

b When the equation of a line is expressed in $y = mx + b$ form, the parameter b gives the y-intercept of the line. For example, the y-intercept of the line $y=-\frac{1}{3}x+7$ is (0, 7).

bar graph A bar graph is a set of rectangular bars that have height proportional to the number of data elements in each category. Usually the bars are separated from each other. It is a way of displaying one-variable data that can be put into categories (like what color you prefer, your gender, or the state you were born in). Also see *histogram*.



base When working with an exponential expression in the form a^b, a is called the base. For example, 2 is the base in 2^5. (5 is the exponent, and 32 is the value.) Also see *exponent*.

best-fit line See *line of best fit*.

bias A systematic inaccuracy in data due to a process that favors certain outcomes. (p. 442)

biased wording Words used in a survey question that intentionally or unintentionally influence results. (p. 443)

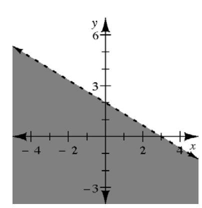
bin width An interval, or the width of a bar, on a histogram. (p. 763)

binomial An expression that is the sum or difference of exactly two terms, each of which is a monomial. For example, $-2x + 3y^2$ is a binomial. (p. 135)

The Binomial Theorem The formula for the expansion of $(x + y)^n$ is called the Binomial Theorem. $(x+y)^n={}_nC_nx^n+{}_nC_{n-1}x^{n-1}y+{}_nC_{n-2}x^{n-2}y^2+\ldots+{}_nC_1xy^{n-1}+{}_nC_0y^n$
For example, $(x+y)^3={}_3C_3x^3+{}_3C_2x^2y+{}_3C_1xy^2+{}_3C_0y^3=1x^3+3x^2y+3xy^2+1y^3$. (p. 544)

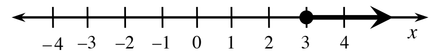
bound The highest or lowest value that a statistical prediction is within. Also see *margin of error* and *boundary line*. (p. 589)

boundary line or curve (1) A line or curve on a two-dimensional graph that divides the graph into two regions. A boundary line or curve is used when graphing inequalities with two variables. For example, the inequality $y<-\frac{2}{3}x+2$ is graphed at right. The dashed boundary line has equation $y=-\frac{2}{3}x+2$. A boundary line is also sometimes called a "dividing line." (2) A line drawn parallel to and above or below the least squares regression line at a distance equivalent to the largest residual. The line determines the upper or lower limit on the values that a prediction is likely to be. (p. 197)

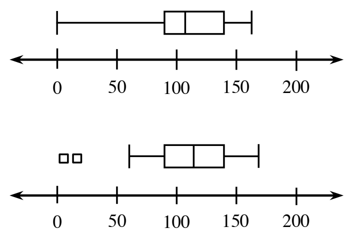


$y<-\frac{2}{3}x+2$

boundary point The endpoint of a ray or segment on a number line where an inequality is true. For strict inequalities (that is, inequalities involving < or >), the point is not part of the solution. We find boundary points by solving the equality associated with our inequality. For example, the solution to the equation $2x + 5 = 11$ is $x = 3$, so the inequality $2x + 5 \geq 11$ has a boundary point at 3. The solution to that inequality is illustrated on the number line at right. A boundary point is also sometimes called a "dividing point." (p. 186)



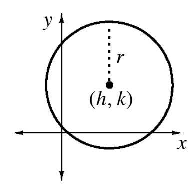
boxplot A graphic way of displaying the five number summary of a distribution of data: minimum, first quartile, median, third quartile, and maximum. A **modified boxplot** displays outliers separately from the boxplot. Also see *five number summary*. (p. 757)



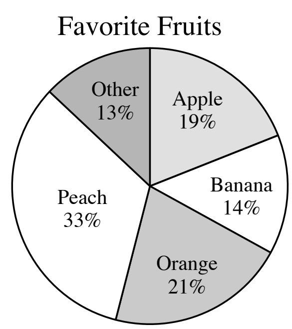
census The process of measuring every member of the population. (p. 445)

center (of a data distribution) A number that represents the middle of a data set, or that represents a "typical" value of the set. Two ways to measure the center of a data set are the mean and the median. When dealing with measures of center, it is often useful to consider the distribution of the data. For symmetric distributions with no outliers, the mean or median can represent the middle, or "typical" value, of the data well. However, in the presence of outliers or non-symmetrical data distributions, the median may be a better measure. Also see *mean* and *median*. (p. 764)

circle In a plane, the set of all points equidistant from a single point. The general equation of a circle is $(x-h)^2 + (y-k)^2 = r^2$ where the point (h, k) is the center of the circle of radius r.



circle graph A way of displaying data that can be put into categories (like what color you prefer, your gender, or the state you were born in). A circle graph shows the proportion each category is of the whole.



circular functions The periodic functions based on the unit circle, including $y = \sin x$, $y = \cos x$, and $y = \tan x$. See *sine function*, *cosine function*, and *tangent function*.

closed question A question that limits respondents to some set number of responses from which to choose. (p. 444)

closed set A set of numbers is said to be closed under an operation if the result of applying the operation to any two numbers in the set produces a number in the set. For example, the whole numbers are a closed set under addition, because if you add any two whole numbers the result is always a whole number. However, the whole numbers are not closed under division: if you divide any two whole numbers you do not always get a whole number. (p. 139)

closure properties A closure properties states that a particular set of numbers is a closed set under a specific operation. For example, the closure property of rational numbers states that the product or sum of two rational numbers is a rational number. For example, $\frac{1}{2}$ and $\frac{3}{4}$ are both rational numbers; $\frac{1}{2}+\frac{3}{4}$ is $\frac{5}{4}$; and $\frac{5}{4}$ is a rational number. Also, 2.2 and 0.75 are both rational numbers; $2.2 \cdot 0.75$ is 1.65; and 1.65 is a rational number.

coefficient When variable(s) are multiplied by a number, the number is called a coefficient of that term. The numbers that are multiplied by the variables in the terms of a polynomial are called the coefficients of the polynomial. For example, 3 is the coefficient of $3x^2$. (p. 135)

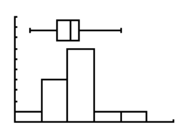
cofunction A trigonometric function whose value for the complement of an angle is equal to the value of a given trigonometric function of the angle itself. For example, the cofunction of sine is cosine and the cofunction of tangent is cotangent. (p. 651)

cofunction identities Trigonometric identities that relate sine and cosine, tangent and cotangent, and secant and cosecant. The value of a trigonometric function of an angle equals the value of the co-function of the complement of the angle. (p. 651)

coincide Two graphs coincide if they have all their points in common. For example, the graphs of $y = 2x + 4$ and $3y = 6x + 12$ coincide; both graphs are lines with a slope of 2 and a *y*-intercept of 4. When the graphs of two equations coincide, those equations share all the same solutions and have an infinite number of intersection points.

combination A combination is the number of ways we can select items from a larger set without regard to order. For instance, choosing a committee of 3 students from a group of 5 volunteers is a combination since the order in which committee members are selected does not matter. We write ${}_nC_r$ to represent the number of combinations of *n* things taken *r* at a time (or *n* choose *r*). For instance, the number of ways to select a committee of 3 students from a group of 5 is ${}_5C_3$. You can use Pascal's Triangle to find ${}_5C_3$ or use permutations and divided by the number of arrangements $\frac{{}_5P_3}{{}_3P_3} = \frac{5!}{3!2!}$. Formulas for combinations include: ${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}$. (p. 503)

combination histogram and box plot A way to visually represent a distribution of data. The box plot is drawn with the same *x*-axis as the histogram.



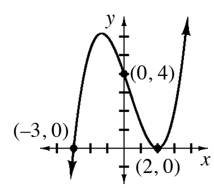
common difference The difference between consecutive terms of an arithmetic sequence or the *generator* of the sequence. When the common difference is positive the sequence increases; when it is negative the sequence decreases. In the sequence $3, 7, 11, \ldots, 43$, the common difference is 4. (p. 692)

common factor A common factor is a factor that is the same for two or more terms. For example, x^2 is a common factor of $3x^2$ and $-5x^2y$.

common logarithm The logarithm with base 10. If no base is given, the logarithm is understood to be base 10. (p. 239)

common ratio Common ratio is another name for the multiplier or *generator* of a geometric sequence. It is the number to multiply one term by to get the next one. In the sequence: 96, 48, 24, … the common ratio is $\frac{1}{2}$. (p. 537)

complete graph A complete graph is one that includes everything that is important about the graph (such as intercepts and other key points, asymptotes, or limitations on the domain or range), and that makes the rest of the graph predictable based on what is shown. For example, a complete graph of the equation $y = \frac{1}{3}(x-2)^2(x+3)$ is shown at right. (p. 9)

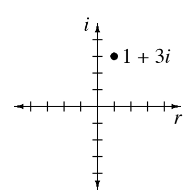


completing the square A standard procedure for rewriting a quadratic equation from standard form to graphing (or vertex) form is called completing the square. Completing the square is also used to solve quadratic equations in one variable. For example, the expression $x^2 - 6x + 4$ starts with the first two terms of $(x-3)^2$. To "complete the square" we need to add 9 to $x^2 - 6x$. Since the original expression only adds 4, completing the square would increase the expression by 5, so $(x-3)^2 - 5$ is an equivalent form that is useful for solving or graphing. (p. 75)

complex conjugates The complex number $a + bi$ has a complex conjugate $a - bi$. Similarly, the conjugate of $c - di$ is $c + di$. What is noteworthy about complex numbers conjugates is that both their product $(a+bi)(a-bi) = a^2 - b^2i^2 = a^2 + b^2$ and their sum $(a+bi)+(a-bi) = 2a$ are real numbers. If a complex number is a zero (or root) of a real polynomial function, then its complex conjugate is also a zero (or root). (p. 396)

complex numbers Numbers written in the form $a + bi$ where a and b are real numbers, are called complex numbers. Each complex number has a real part, a, and an imaginary part, bi. Note that real numbers are also complex numbers with $b = 0$, and imaginary numbers are complex numbers where $a = 0$. (p. 392)

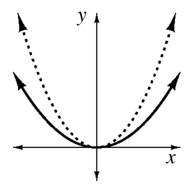
complex plane A set of coordinate axes with all the real numbers on the horizontal axis (the real axis) and all the imaginary numbers on the vertical axis (the imaginary axis) defines the complex plane. Complex numbers are graphed in the complex plane using the same method we use to graph coordinate points. Thus, the complex number $1 + 3i$ is located at the point (1, 3) in the complex plane. (p. 404)



composite function A function that is created as the result of using the outputs of one function as the inputs of another can be seen as a composite function. For example the function $h(x)=|\log x|$ can be seen as the composite function $f(g(x))$ where $f(x)=|x|$ and $g(x)=\log x$. See *composition*. (p. 225)

composition When the output of one function is used as the input for a second function, a new function is created which is a composition of the two original functions. If the first function is $g(x)$ and the second is $f(x)$, the composition can be written as $f(g(x))$ or $f\circ g(x)$. Note that the order in which we perform the composition matters, and $g(f(x))$ will usually be a different function.

compound interest Interest that is paid on both the principal and the previously accrued interest. (p. 550)



compress A term used informally to describe the relationship of a graph to its parent graph when the graph increases or decreases more slowly than the parent. For example, the solid parabola shown at right is a compressed version of its parent, shown as a dashed curve. (p. 66)

conjugate Every complex number $a+bi$ has a conjugate, $a-bi$, and both the sum and product of the conjugates are real. Similarly, an irrational number that can be written $a+\sqrt{b}$ where a and b are rational, has a conjugate, $a-\sqrt{b}$, and the product and sum of these conjugates are rational. (p. 396)

constant term A number that is not multiplied by a variable. In the expression $2x+3(5-2x)+8$, the number 8 is a constant term. The number 3 is not a constant term, because it is multiplied by a variable inside the parentheses. (p. 135)

constraint A limitation or restriction placed on a function or situation, either by the context of the situation or by the nature of the function.

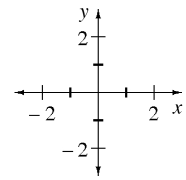
continuous For this course, when the points on a graph are connected and it makes sense to connect them, we say the graph is continuous. Such a graph will have no holes or breaks in it. This term will be more completely defined in a later course. (p. 668)

continuously compounded interest For this course, continuously compounded interest can be thought of as interest that is computed and added to the balance of an account every instant.

control limit The upper and lower limits of a critical measurement that determines acceptable quality in a manufacturing process. See *statistical process control*. (p. 599)

convenience sample A subgroup of the population for which it was easy to collect data. A convenience sample is not a random sample. (p. 456)

coordinate axes For two dimensions, two perpendicular number lines, the x- and y-axes, that intersect where both are zero and that provide the scale(s) for labeling each point in a plane with its horizontal and vertical distance and direction from the origin $(0,0)$. In three dimensions, a third number line, the z-axis, is perpendicular to a plane and intersects it at origin $(0,0,0)$. The z-axis provides the scale for the height of a point above or below the plane. See *3-dimensional coordinate system*.

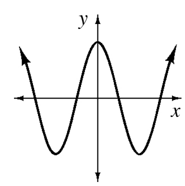


coordinates The numbers in an ordered pair (a, b) or triple (a, b, c) used to locate a point in the plane or in space in relation to a set of coordinate axes.

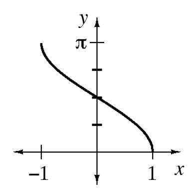
correlation coefficient, r A measure of how much or how little data is scattered around the least squares regression line. It is a measure of the strength of an association that has already been determined to be linear. The correlation coefficient takes on values between –1 and 1. The closer to 1 or –1 the correlation coefficient is, the less scattered the data is around the LSRL.

cosecant The cosecant is the reciprocal of the sine. (p. 651)

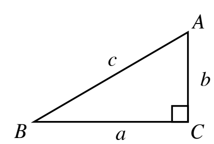
cosine function For any real number θ, the cosine of θ, denoted $\cos\theta$, is the x-coordinate of the point on the unit circle reached by a rotation angle of θ radians in standard position. The general equation for the cosine function is $y = a\cos b(x-h)+k$. The graph of this function has amplitude a, period $\frac{2\pi}{b}$, horizontal shift h, and vertical shift k. (p. 327)



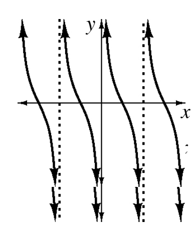
cosine inverse ($\cos^{-1} x$) Read as the inverse of cosine x, $\cos^{-1} x$ is the measure of the angle with cosine x. We can also write $y = \arccos x$. Note that the notation refers to the inverse of the cosine function, *not* $\frac{1}{\cos x}$. Because $y = \cos^{-1} x$ is equivalent to $x = \cos y$ and there are infinitely many angles y such that $\cos y = x$, the inverse *function* is restricted to select the *principal* value of y such that $0 \le y \le \pi$. The graph of the inverse cosine function is at right. (p. 636)



cosine ratio In a right triangle the ratio $\frac{\text{adjacent side}}{\text{hypotenuse}}$ is known as the cosine of the acute angle. At right, $\cos B = \frac{a}{c}$ since the side length a is adjacent to angle B. (p. 327)



cotangent The cotangent is the reciprocal of the tangent. The graph of the cotangent function is at right. (p. 651)

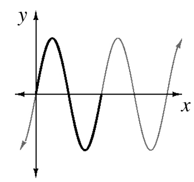


counterexample An example showing that a statement has at least one exception; that is, a situation in which the statement is false. For example, the number 4 is a counterexample to the hypothesis that all even numbers are greater than 7. (p. 283)

cubic A cubic polynomial is a polynomial of the form $y = ax^3 + bx^2 + cx + d$. "Cube" refers to the third (3) power.

cube root In the equation $a = b^3$, the value b that is multiplied by itself three times to give the value a. For example, the cube root of 8 is 2 because $8 = 2 \cdot 2 \cdot 2 = 2^3$. This is written $\sqrt[3]{8} = 2$.

cycle One cycle of a graph of a trigonometric function is the shortest piece that represents all possible outputs. The length of one cycle is the distance along the x-axis needed to generate all possible output values or the distance around the unit circle needed to generate all possible outcomes.



cyclic function The term cyclic function is sometimes used to describe trigonometric functions, but it also includes any function that sequentially repeats its outputs at regular intervals. (p. 349)

data distribution See *distribution*.

degree One degree is an angle measure that is $\frac{1}{360}$ of a full circle or $\frac{\pi}{180}$ radians. (p. 333)

degree of a polynomial The degree of a monomial is the sum of the exponents of its variables. For example, the degree of $3x^2y^5z$ is 8. For a polynomial the degree is the degree of the monomial term with the highest degree. Example: for the polynomial $2x^5y^2 - 4x^4z^6 + x^7z$ the degree is 10. (p. 374)

degree of a polynomial in one variable The highest power of the variable. The degree of a polynomial function also indicates the maximum number of factors of the polynomial and provides information for predicting the number of "turns" the graph can take. (p. 374)

dependent variable When one quantity depends for its value on one or more others, it is called the dependent variable. For example, we might relate the speed of a car to the amount of force you apply to the gas pedal. Here, the speed of the car is the dependent variable; it depends on how hard you push the pedal. The dependent variable appears as the output value in an $x \rightarrow y$ table, and is usually placed relative to the vertical axis of a graph. We often use the letter y and the vertical y-axis for the dependent variable. When working with functions or relations, the dependent variable represents the output value. In Statistics, the dependent variable is often called the response variable. Also see *independent variable*. (p. 6)

depreciation A decrease in value possibly because of normal wear and tear, age, decay, decrease in price. (p. 742)

description of a function A complete description of a function includes: a description of the shape of the graph, where it increasing and/or decreases, all the intercepts, domain and range, description of special points, and description of lines of symmetry.

difference of squares A polynomial that can be factored as the product of the sum and difference of two terms. The general pattern is $x^2 - y^2 = (x+y)(x-y)$. For example, the difference of squares $4x^2 - 9$ can be factored as $(2x+3)(2x-3)$. (p. 128)

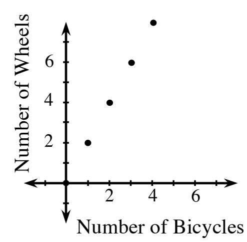
difference of cubes A polynomial of the form $x^3 - y^3$. It can be factored as follows: $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$. For example, the difference of cubes $8x^3 - 27$ can be factored as $(2x-3)(4x^2 + 6x + 9)$. (p. 427)

dilation A transformation which vertically compresses or stretches the graph of a function. For example, the graph of $g(x) = 2(x^2 + 3x - 5)$ is a dilation of the function $f(x) = x^2 + 3x - 5$ by a factor of 2.

dimensions The dimensions of a flat region or space tell how far it extends in each direction. For example, the dimensions of a rectangle might be 16 cm wide by 7 cm high.

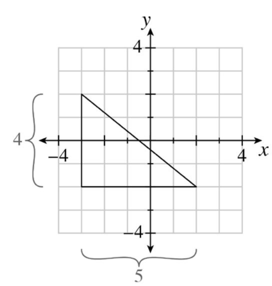
direction (of an association) If one variable in a relationship increases as the other variable increases, the direction is said to be a positive association. If one variable decreases as the other variable increases, there is said to be a negative association. If there is no apparent pattern in the scatterplot, then the variables have no association. When describing a linear association, you can use the slope, and its numerical interpretation in context, to describe the direction of the association.

discrete graph A graph that consists entirely of separated points is called a discrete graph. For example, the graph shown at right is discrete. Also see *continuous*. (p. 668)



discriminant For quadratic equations in standard form $ax^2 + bx + c = 0$, the discriminant is $b^2 - 4ac$. If the discriminant is positive, the equation has two roots; if the discriminant is zero, the equation has one root; if the discriminant is negative, the equation has no real-number roots. For example, the discriminant of the quadratic equation $2x^2 - 4x - 5$ is $(-4)^2 - 4(2)(-5) = 56$, which indicates that that equation has two roots (solutions). (p. 396)

distance formula An application of the Pythagorean Theorem to find the distance between two points in a plane. The distance between any two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. In the example at right the distance is $\sqrt{4^2 + 5^2}$. (p. 76)

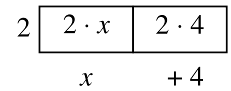


$a^2 + b^2 = c^2$

$4^2 + 5^2 = \text{distance}^2$

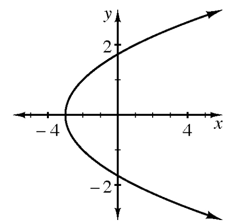
distribution The statistical distribution of a variable is a description (usually a list, table, or graph) of the number of times each possible outcome occurs. A distribution of data is often summarized using its center, shape, spread, and outliers, and can be displayed with a combination histogram and boxplot. (p. 765)

Distributive Property We use the Distributive Property to write a product of expressions as a sum of terms. The Distributive Property states that for any numbers or expressions a, b, and c, $a(b + c) = ab + ac$. For example, $2(x + 4) = 2 \cdot x + 2 \cdot 4 = 2x + 8$. We can demonstrate this with a generic rectangle.



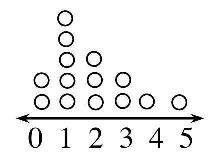
dividing line See *boundary line*.

dividing point See *boundary point*.



domain The set of all input values for a relation or function. For example, the domain of the relation graphed at right is $x \geq -3$. For variables, the domain is the set of numbers the variable may represent. Also see *input*. (p. 6)

Dot Plot

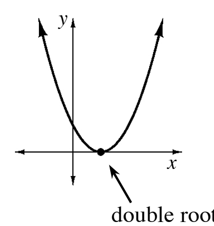


dot plot A way of displaying one-variable data that has an order and can be placed on a number line. Dot plots are generally used when the data is discrete (separate and distinct) and numerous pieces of data are of equal value.

Double Angle Identities Trigonometric identities that involve doubling angle measures. They are: $\sin(2x) = 2\sin(x)\cos(x)$ and $\cos(2x) = \cos^2(x) - \sin^2(x)$. (p. 651)

double-peaked See *shape (of a data distribution)*. (p. C13)

double root A root of a function that occurs exactly twice. If an expression of the form $(x - a)^2$ is a factor of a polynomial, then the polynomial has a double root at $x = a$. The graph of the polynomial does not pass through the x-axis at $x = a$ but is tangent to the axis at $x = a$. (p. 380)



e In mathematics, the base of a natural logarithm. $e \approx 2.71828\ldots$ (p. 548)

Elimination Method A method for solving a system of equations. The key step in using the Elimination Method is to add or subtract both sides of two equations to eliminate one of the variables. For example, the two equations in the system at right can be added together to get the simplified result $7x = 14$. We can solve this equation to find x, then substitute the x-value back into either of the original equations to find the value of y.

$$5x + 2y = 10$$
$$2x - 2y = 4$$

Equal Values Method A method for solving a system of equations. To use the equal values method, take two expressions that are each equal to the same variable and set those expressions equal to each other. For example, in the system of equations at right, $-2x + 5$ and $x - 1$ each equal y. So we write $-2x + 5 = x - 1$, then solve that equation to find x. Once we have x, we substitute that value back into either of the original equations to find the value of y.

$$y = -2x + 5$$
$$y = x - 1$$

equation A mathematical sentence in which two expressions appear on either side of an "equals" sign (=), stating that the two expressions are equivalent. For example, the equation $7x + 4.2 = –8$ states that the expression $7x + 4.2$ has the value –8. In this course, an equation is often used to represent a rule relating two quantities. For example, a rule for finding the area y of a tile pattern with figure number x might be written $y = 4x – 3$.

equivalent Two expressions are equivalent if they have the same value. For example, $2 + 3$ is equivalent to $1 + 4$. Two equations are equivalent if they have all the same solutions. For example, $y = 3x$ is equivalent to $2y = 6x$. Equivalent equations have the same graph. (pp. 123, 133)

evaluate To evaluate an expression, substitute the value(s) given for the variable(s) and perform the operations according to the order of operations. For example, evaluating $2x + y – 10$ when $x = 4$ and $y = 3$ gives the value 1.

even function A function where $f(–x) = f(x)$ for all values of x defined for the function. The graph of the function will be symmetric about the y-axis. (p. 108)

event One or more results of an experiment.

excluded value A value that is undefined for a given function and therefore excluded from the domain of the function. (p. 152)

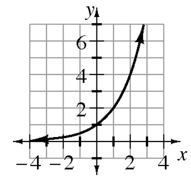
expected value The expected value for an outcome is the product of the probability of the outcome and the value placed on that outcome. The expected value of an event is the sum of the expected values for its possible outcomes. For example, in a lottery where 7 numbers are drawn from 77 and you have to have chosen all seven to win, the probability that your ticket is the \$1,000,000 winner is $\frac{1}{2404808340}$ and the expected value is \$0.000416.

experiment A process that applies a treatment (change) to a group of subjects to determine if there is a measurable difference from another group. If an experiment properly implements randomization, controls (balances) the effects of lurking variables between the groups, and is replicated with many subjects, cause and effect can often be determined. (p. 461)

explicit equation (for a sequence) An equation for a term in a sequence that determines the value of any term $t(n)$ directly from n, without necessarily knowing any other terms in the sequence. Also see *recursive equation*. (p. 683)

exponent In an expression of the form a^b, b is called the exponent. For example, in the expression 2^5, 5 is called the exponent. (2 is the base, and 32 is the value.) The exponent indicates how many times to use the base as a multiplier. For example, in 2^5, 2 is used 5 times: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$. For exponents of zero, the rule is: for any number $x \neq 0$, $x^0 = 1$. For negative exponents, the rule is: for any number $x \neq 0$, $x^{-n} = \frac{1}{x^n}$, and $\frac{1}{x^{-n}} = x^n$. Also see *laws of exponents*. (pp. 724, 740)

exponential function An exponential function in this course has an equation of the form $y = ab^x + c$, where a is the initial value, b is positive and is the multiplier, and $y = c$ is the equation of the horizontal asymptote. An example of an exponential function is graphed at right. (p. 57)



expression An expression is a combination of individual terms separated by plus or minus signs. Numerical expressions combine numbers and operation symbols; algebraic (variable) expressions include variables. For example, $4 + (5 - 3)$ is a numerical expression. In an algebraic expression, if each of the following terms, $6xy^2$, 24, and $\frac{y-3}{4+x}$, are combined, the result may be $6xy^2 + 24 - \frac{y-3}{4+x}$. An expression does not have an "equals" sign. (p. 135)

extraneous solution Sometimes in the process of solving equations, multiplying or squaring expressions involving a variable will lead to a numerical result that does not make the original equation true. This false result is called an extraneous solution. For example, in the process of solving the equation $\sqrt{x+3} = 9 - x$ both sides of the equation are squared to get $x + 3 = x^2 - 18x + 81$ which has solutions 6 and 13. 6 is a solution of the original equation, but 13 is extraneous, because $\sqrt{13+3} \neq 9 - 13$. (p. 173)

extrapolate A prediction made outside the range of the observed data. Often extrapolations are not very reliable predictions.

$\boldsymbol{f^{-1}(x)}$ Read this as "f inverse of x," the inverse function for $f(x)$. (p. 219)

factor A factor is part of a product. A polynomial expression $p(x)$ is a factor of another polynomial expression $P(x)$ when there is a polynomial $q(x)$ such that $p(x)q(x) = P(x)$. In the equation $3x^2 - 9x + 6 = 3(x - 2)(x - 1)$, the expressions $(x - 2)$, $(x - 1)$, and 3 are factors.

Factor Theorem States that if a is a root of a polynomial then $x - a$ is a factor, and if $x - a$ is a factor then a is a root. For example, the polynomial $x^2 - 5x - 6 = (x - 6)(x + 1)$ and the roots are 6 and -1. (p. 418)

factored completely A polynomial is factored completely if none of the resulting factors can be factored further using integer coefficients. For example, $-2(x + 3)(x - 1)$ is the completely factored form of $-2x^2 - 4x + 6$. (p. 371)

factored form A quadratic equation in the form $a(x + b)(x + c) = 0$, where a is nonzero, is said to be in factored form. For example, $-7(x + 2)(x - 1.5) = 0$ is a quadratic equation in factored form. (p. 371)

factorial A shorthand notation for the product of a list of consecutive positive integers from the given number down to 1: $n! = n(n - 1)(n - 2)(n - 3) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$. For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. (p. 648)

family of functions A group of functions that have at least one common characteristic, usually the shape and the form of the equation. For example the quadratic family of functions have graphs that are parabolas, and equations of the form $y = ax^2 + bx + c$. Examples of other families are linear functions, exponential functions, and absolute value functions. (p. 98)

Fibonacci Sequence The sequence of numbers 1, 1, 2, 3, 5, 8, 13, ... Each term of the Fibonacci sequence (after the first two terms) is the sum of the two preceding terms. (p. 684)

first quartile (Q1) The median of the lower half of a set of data which has been written in numerical order. (p. 757)

first term form (of a sequence) An equation of a sequence written in first term form uses the first term of the sequence and its common difference or ratio. The first term form of an arithmetic sequence is $a_n = a_1 + d(n-1)$, where a_1 is the first term and d is the common difference. The first term form of a geometric sequence is $a_n = a_1 r^{n-1}$, where a_1 is the first term and r is the common ratio. (p. 744)

five number summary A way of summarizing the center and spread of a one-variable distribution of data. The five number summary includes the minimum, first quartile, median, third quartile, and maximum of the set of data. Also see *boxplot*. (p. 757)

form (of an association) The form of an association can be linear of non-linear. The form can contain cluster of data. A residual plot can help determine if a particular form is appropriate for modeling the relationship.

fraction The quotient of two quantities in the form $\frac{a}{b}$ where b is not equal to 0. (p. 142)

fractional exponents Raising a number to a fractional exponent indicates a power as well as a root. $x^{a/b} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$. (p. 740)

function A relationship in which for each input value there is one and only one output value. For example, the relationship $f(x) = x + 4$ is a function; for each input value (x) there is exactly one output value. In terms of ordered pairs (x, y), no two ordered pairs of a function have the same first member (x). See also *description of a function* and *function notation*. (p. 6)

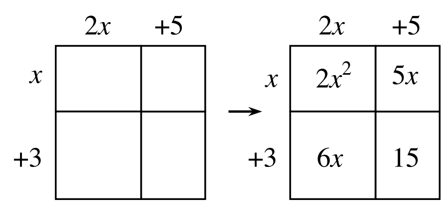
function notation When a rule expressing a function is written using function notation, the function is given a name, most commonly "f," "g," or "h." The notation $f(x)$ represents the output of a function, named f, when x is the input. It is pronounced "f of x." For example, $g(2)$, pronounced "g of 2", represents the output of the function g when $x = 2$. If $g(x) = x^2 + 3$, then $g(2) = 7$. (p. 6)

Fundamental Theorem of Algebra The Fundamental Theorem of Algebra states that a polynomial of degree n with (real or) complex coefficients has exactly n roots, which may be real or complex. This also means that the polynomial has n linear factors since for every root a, $(x - a)$ is a linear factor. (p. 418)

general equation If $y = f(x)$ is a parent equation, then the general equation for that function is given by $y = a\,f(x - h) + k$ where (h, k) is the point corresponding to $(0, 0)$ in the parent graph and, relative to the parent graph, the function has been: 1) vertically stretched if the absolute value of a is greater than 1; 2) vertically compressed if the absolute value of a is less than 1; and/or 3) reflected across the x-axis if a is less than 0. (p. 98)

generator The generator of a sequence tells what you do to each term to get the next term. Note that this is different from the function for the n^{th} term of the sequence. The generator only tells you how to find the following term, when you already know one term. In an arithmetic sequence the generator is the common difference; in a geometric sequence it is the multiplier or common ratio. (p. 692)

generic rectangle A type of diagram used to visualize multiplying expressions without algebra tiles. Each expression to be multiplied forms a side length of the rectangle, and the product is the sum of the areas of the sections of the rectangle. For example, the generic rectangle at right can be used to multiply $(2x + 5)$ by $(x + 3)$.



$(2x + 5)(x - 3) = 2x^2 + 11x + 15$

area as a product area as a sum

geometric sequence A geometric sequence is a sequence that is generated by a multiplier. This means that each term of a geometric sequence can be found by multiplying the previous term by a constant. For example: 5, 15, 45… is the beginning of a geometric sequence with generator (common ratio) 3. In general a geometric sequence can be represented $a, ar, ar^2, \ldots, ar^{n-1}$. (p. 537)

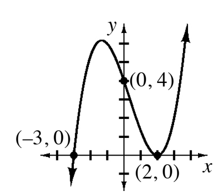
geometric series The sum of a geometric sequence is a geometric series, for example: $5 + 15 + 45 + \ldots$ The sum of the first n terms of a geometric sequence, $a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}$ is given by the formula at right. (p. 510)

$$S = \frac{a(r^n - 1)}{r - 1}$$

Giant One A fraction that is equal to 1. Multiplying any fraction by a Giant One will create a new fraction equivalent to the original fraction. (p. 152)

$$\frac{x-2}{x-3} \cdot \boxed{\frac{x-2}{x-2}} = \frac{(x-2)^2}{(x-3)(x-2)}$$

graph The graph of an equation is the set of points representing the coordinates that make the equation true. The direction to "graph an equation" or "draw a graph" means use graph paper, scale your axes appropriately, label key points, and plot points accurately. This is different from *sketching* a graph. The equation $y = \frac{1}{3}(x - 2)^2(x + 3)$ is graphed at right. (p. 9)



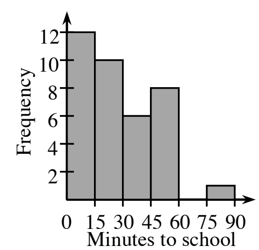
graphing form A form of the equation of a function or relation that clearly shows key information about the graph. For example, the graphing form for the general equation of a quadratic function (also called vertex form) is $y = a(x-h)^2 + k$. The vertex (h, k), orientation (whether a is positive or negative) and amount of stretch or compression based on $|a| > 1$ or $|a| < 1$ can be appear in the equation. (p. 68)

growth factor One way to analyze how the output value in a mathematical relation changes as the input value increases. Growth can be represented by constant addition, as the slope of a linear function or the constant difference in an arithmetic sequence, or by multiplication as the base of an exponential function or the multiplier in a geometric sequence. (p. 10)

$\boldsymbol{(h, k)}$ In this course h and k are used as parameters in general equations for families of functions $f(x) = af(x-h) + k$ and families of relations to represent the horizontal and vertical shifts of the parent graph. The point (h, k) represents location of a point that corresponds to $(0, 0)$ for parent graphs where $(0, 0)$ is on the graph.

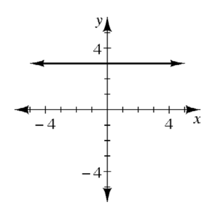
half-life When material decays, the half-life is the time it takes until only half the material remains. (p. 723)

histogram A way of displaying one-variable data that is like a bar graph in that the height of the bars is proportional to the number of elements. However, a histogram is for numerical data. Each bin (bar) of a histogram represents the number of data elements in a range of values, such as the number of people who take from 15 minutes up to, but not including, 30 minutes to get to school. Each bin should the same width and the bins should touch each other. Also see *bar graph*. (p. 763)



horizontal Parallel to the horizon. The x-axis of a coordinate graph is the horizontal axis.

horizontal lines Horizontal lines are "flat" and run left to right in the same direction as the x-axis. Horizontal lines have equations of the form $y = b$, where b can be any number. For example, the graph at right shows the horizontal line $y = 3$. The slope of any horizontal line is 0. The x-axis has the equation $y = 0$ because $y = 0$ everywhere on the x-axis.



horizontal shift Used with parent graphs and general equations for functions and relations such as $y = a(x-h)^2 + k$. This type of transformation is a horizontal translation of a graph that moved left or right in relation to its parent graph. The horizontal shift will be h units to the right if h is positive, to the left if h is negative. (p. 66)

hypotenuse The longest side of a right triangle, the side opposite the right angle.

identity (trigonometric) Equations that are true for all values for which the functions are defined. For example $\sin^2 x + \cos^2 x = 1$ which is true for all values of x, or $\cot x = \frac{1}{\tan x}$ which is true whenever the tangent and cotangent are defined.

imaginary numbers The set of numbers created by multiplying the imaginary number i by every possible real number. The imaginary number i is defined as the square root of -1, or $i = \sqrt{-1}$. Therefore $i^2 = -1$. They are not positive, negative, or zero. These numbers are solutions of equations of the form x^2 = (a negative number). In general, imaginary numbers follow the rules of real number arithmetic (e.g. $i + i = 2i$). (p. 392)

independent variable When one quantity changes in a way that does not depend on the value of another quantity, the value that changes independently is represented with the independent variable. For example, we might relate the speed of a car to the amount of force you apply to the gas pedal. Here, the amount of force applied may be whatever the driver chooses, so it represents the independent variable. The independent variable appears as the input value in an $x \rightarrow y$ table, and is usually placed relative to the horizontal axis of a graph. We often use the letter x and the horizontal x-axis for the independent variable. When working with functions or relations, the independent variable represents the input value. In Statistics, the independent variable is often called the explanatory variable. Also see *dependent variable*. (p. 6)

index (plural indices) In summation notation, the indices are the numbers below and above the sigma that indicate which term to start with and which to end with. For a series they show the first and last replacement values for n. When the symbol above sigma is ∞ the series continues without ending. (p. 519)

Example: $\sum_{n=1}^{8} 5n - 7 = -2 + 3 + 8 + ... + 33$.

inequality An inequality consists of two expressions on either side of an inequality symbol. For example, the inequality $7x + 4.2 < -8$ states that the expression $7x + 4.2$ has a value less than -8. (p. 197)

inequality symbols The symbol $\leq$ read from left to right means "less than or equal to." The symbol $\geq$ read from left to right means "greater than or equal to." The symbols $<$ and $>$ mean "less than" and "greater than," respectively. For example, "7 < 13" means that 7 is less than 13.

inequalities with absolute value If k is any positive number, an inequality of the form: $|f(x)| > k$ is equivalent to the statement $f(x) > k$ or $f(x) < -k$; and $|f(x)| < k$ is equivalent to the statement $-k < f(x) < k$. For example, you can solve the inequality, $|5x - 6| > 4$ by solving the two inequalities $5x - 6 > 4$ or $5x - 6 < -4$. (p. 193)

inference A statistical prediction.

infinite geometric series An infinite geometric series is a geometric series which never ends. The sum of such a series with an initial value a and common ratio r, with $-1 < r < 1$, is given by the formula at right. (p. 537)

$$S = \frac{a}{1-r}$$

infinity The concept that something is without limit or unending. The symbol for infinity is ∞. Note that infinity is an idea, not a number, even though it is often treated as such. (p. 18)

initial value The initial value of a sequence is the first term of the sequence. (p. 57)

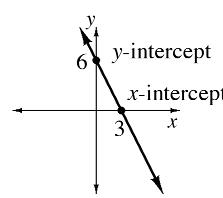
input A replacement value for a variable in a function or relation. The first number in an ordered pair. The set of all possible input values is the domain of a function. (p. 26)

integer Any whole number or the opposite of a whole number: … –3, –2, –1, 0, 1, 2, 3, …

integral roots Roots (or zeros) of functions that are integers. (p. 414)

Integral Zero Theorem For any polynomial with integral coefficients, if an integer is a zero of the polynomial, it must be a factor of the constant term. (p. 418)

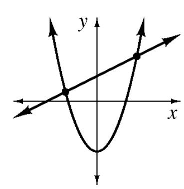
intercepts Points where a graph crosses the axes. x-intercepts are points at which the graph crosses the x-axis and y-intercepts are points at which the graph crosses the y-axis. On the graph at right the x-intercept is (3, 0) and the y-intercept is (0, 6). (p. 10)



interest An amount paid which is a percentage of the principal.
For example, a savings account may offer 4% annual interest rate, which means they will pay $4.00 in interest for a principal of $100 kept in the account for one year. (p. 719)

interquartile range (IQR) A way to measure the spread of data. It is calculated by subtracting the first quartile from the third quartile.

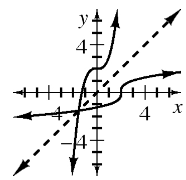
intersection A point of intersection is a point that the graphs of two or more equations have in common. Graphs may intersect in one, two, several, many or no points. The set of coordinates of a point of intersection are a solution to the equation for each graph. (p. 21)



interval A set of numbers between two given numbers.

inverse circular functions See *inverse trigonometric functions*.

inverse function A function that "undoes" what the original function does. It can also be seen as the *x*-*y* interchange of the function. The inverse of a function performs in reverse order the inverse operation for each operation of the function. The graph of an inverse function is a reflection of the original function across the line $y = x$. For example, $y = x^3 + 2$ is equivalent to $x = \sqrt[3]{y-2}$, its inverse function is written $y = \sqrt[3]{x-2}$. (p. 216)



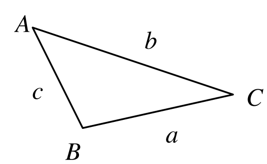
inverse operations Subtraction is the inverse operation for addition and vice versa, division for multiplication, square root for squaring, and more generally taking the n^{th} root for raising to the n^{th} power.

inverse trigonometric functions For each trigonometric function sin(*x*), cos(*x*), and tan(*x*), there is an inverse function written $\sin^{-1}(x), \cos^{-1}(x)$, and $\tan^{-1}(x)$. Note: This symbol does not mean $\frac{1}{\sin(x)}$. It is a new function that "undoes" the original trig function, thus giving a specific angle measure when the input is sin *x*, cos *x*, or tan *x*. For example: $\sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$. Note that the range of the inverse function is restricted to outputs for *y* such that $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ for $\sin x$, $-\frac{\pi}{2} < y < \frac{\pi}{2}$ for $\tan x$ and $0 \le y \le \pi$ for $\cos x$. Also see *cosine inverse*, *sine inverse*, and *tangent inverse*. (p. 629)

investigating a function To investigate a function means to make a complete graph of the function and to write down everything you know about the function. Some things to consider are: domain, range, intercepts, asymptotes, inverse, and symmetry. (p. 33)

irrational numbers The set of numbers that cannot be expressed in the form $\frac{a}{b}$, where *a* and *b* are integers and $b \ne 0$. For example, π and $\sqrt{2}$ are irrational numbers. (p. 389)

Law of Cosines For any ΔABC, $a^2 = b^2 + c^2 - 2bc\cos A$. (p. 29)



Law of Large Numbers A theorem stating that as the number of trials of a random process increases (for example, as the number of trials in a simulation increases), the results will approach the theoretical value closer. (p. 567)

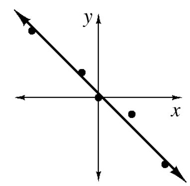
Law of Sines For any ΔABC, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. (p. 29)

laws of exponents The laws of exponents we study in this course are: (pp. 724, 740)

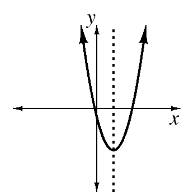
Law	Examples	
$x^m x^n = x^{m+n}$ for all x	$x^3x^4 = x^{3+4} = x^7$	$2^5 \cdot 2^{-1} = 2^4$
$\frac{x^m}{x^n} = x^{m-n}$ for $x \ne 0$	$x^{10} \div x^4 = x^{10-4} = x^6$	$\frac{5^4}{5^7} = 5^{-3}$
$(x^m)^n = x^{mn}$ for all x	$(x^4)^3 = x^{4\cdot 3} = x^{12}$	$(10^5)^6 = 10^{30}$
$x^0 = 1$ for $x \ne 0$	$\frac{y^2}{y^2} = y^0 = 1$	$9^0 = 1$
$x^{-1} = \frac{1}{x}$ for $x \ne 0$	$\frac{1}{x^2} = (\frac{1}{x})^2 = (x^{-1})^2 = x^{-2}$	$3^{-1} = \frac{1}{3}$
$x^{m/n} = \sqrt[n]{x^m}$ for $x \ge 0$	$\sqrt{k} = k^{1/2}$	$y^{2/3} = \sqrt[3]{y^2}$

least squares regression line (LSRL) A unique best-fit line that is found by making the squares of the residuals as small as possible.

line Graphed, a line is made up of an infinite number of points, is one-dimensional and extends without end in two directions. In two dimensions a line is the graph of an equation of the form $ax + by = c$.



line of best fit A line that represents, in general, data on a scatterplot. The line of best fit is a model of numerical two-variable data that helps describe the data. It is also used to make predictions for other data. Also see *least square regression line*. (p. 667)



line of symmetry A line that divides a figure into two congruent shapes which are reflections of each other across the line. (p. 217)

linear equation An equation with at least one variable of degree one and no variables of degree greater than one. The graph of a linear equation of two variables is a line in the plane. $ax + by = c$ is the standard form of a linear equation. (p. 10)

linear factor A factor of the form ($ax + b$).

linear function A polynomial function of degree one or zero, with general equation $f(x) = a(x - h) + k$. The graph of a linear function is a line. (p. 41)

linear inequality An inequality with a boundary line represented by a linear equation. (p. 197)

linear programming A method for solving a problem with several conditions or constraints that can be represented as linear equations or inequalities. (p. 195)

linear regression A method for finding a best- fit line through a set of points on a scatterplot. Also see *regression* and *least squares regression line*.

locator point A locator point is a point which gives the position of a graph with respect to the axes. For a parabola, the vertex is a locator point. (p. 82)

Log base 10 $\log_{10} m = n$ if $10^n = m$. See *common logarithm*. (p. 282)

Log-Power Property See *Power Property of Logs*.

Log-Product Property See *Product Property of Logs*.

Log-Quotient Property See *Quotient Property of Logs*.

logarithm The logarithm of a given number, x, with respect to a base, b, is the exponent to which b must be raised to give x. For example, since we know that $32 = 2^5$, it follows that 5 is the logarithm, base 2, of 32, or $\log_2 32 = 5$. (p. 233)

logarithmic and exponential notation $m = \log_b(n)$ is the logarithmic form of the exponential equation $b^m = n$ ($b > 0$).

logarithmic functions Inverse exponential functions. The base of the logarithm is the same base as that of the exponential function. For instance $y = \log_2 x$ can be read as "y is the exponent needed for base 2 to get x," and is equivalent to $x = 2^y$. The short version is stated "log, base 2, of x," and written $\log_2 x$. (p. 240)

Looking Inside "Looking Inside" is a method of solving one-variable equations containing parentheses or an absolute value symbol. To use "looking inside," we first determine what the value of the entire expression inside the parentheses (or absolute value symbol) must be. We then use that fact to solve for the value of the variable. For example, to use "looking inside" to solve the equation $4(x + 2) = 36$, we first determine that $x + 2$ must equal 9. We then solve the equation $x + 2 = 9$ to find that $x = 7$. (p. 170)

lower control limit (LCL) See *control limit*. (p. 599)

lurking variable A hidden variable that was not part of the statistical study under investigation. Sometimes a lurking variable explains the true cause of an association between two other variables that are linked. (p. 461)

m When the equation of a line is expressed in $y = mx + b$ form, the parameter m gives the slope of the line. For example, the slope of the line $y = -\frac{1}{3}x + 7$ is $-\frac{1}{3}$.

margin of error Half of the spread between the upper and lower bounds of a statistical prediction. For example, if the predicted weight is likely to be between a lower bound of 12 kg and an upper bound of 18 kg, the margin of error is 3 kg, and the prediction of the weight can be written as 15 ± 3 kg. (p. 584)

maximize Make as large as possible. (p. 192)

maximum point The highest point on a graph. For example, the vertex of a downwardly oriented parabola.

maximum value The largest value in the range of a function. For example, the y-coordinate of the vertex of a downwardly oriented parabola. (p. 757)

mean The arithmetic mean, or average, of several numbers is one way of defining the "center" or "middle" or "typical value" of a set of numbers. The mean represents the center of a set of data well if the distribution of the data is symmetric and has no outliers. To find the mean of a numerical data set, add the values together then divide by the number of values in the set. For example, the mean of the numbers 1, 5, and 6 is $(1 + 5 + 6) \div 3 = 4$.

measure of central tendency Mean and median are measures of central tendency, representing the "center" or "middle" or "typical value" of a set of data. See *center (of a data distribution)*.

mean absolute deviation A way to measure the spread, or the amount of variability, in a set of data. The mean absolute deviation is the average of the distances to the mean, after the distances have been made positive with the absolute value. The standard deviation is a much more commonly used measure of spread than the mean absolute deviation. (p. 769)

median The middle number of a set of data which has been written in numerical order. If there is no distinct middle, then the mean of the two middle numbers is the median. The median is generally more representative of the "middle" or of a "typical value" than the mean if there are outliers or if the distribution of the data in not symmetric. (p. 757)

midline The horizontal axis of the graph of a trigonometric function. The midline is halfway between the maximum and minimum values of the function. (p. 360)

minimize Make as small as possible.

minimum point The lowest point on a graph. For example, the vertex of an upwardly oriented parabola.

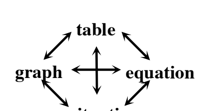
minimum value The smallest value in the range of a function. For example, the y-coordinate of the vertex of an upwardly oriented parabola. (p. 757)

model A mathematical summary (often an equation) of a trend in data, after making assumptions and approximations to simplify a complicated situation. Models allow us to describe data to others, compare data more easily to other data, and allow us to make predictions. For example, mathematical models of weather patterns allow us to predict the weather. No model is perfect, but some models are better at describing trends than other models. Regressions are a type of model. Also see *regression*. (p. 113)

modified boxplot See *boxplot*. (p. 765)

monomial An expression with only one term. It can be a number, a variable, or the product of a number and one or more variables. For example, 7, $3x$, $-4ab$, and $3x^2y$ are each monomials.

multiple representations web An organizational tool we use to keep track of connections between the four representations of relationships between quantities emphasized in this course. In this course, we emphasize four different ways of representing a numerical relationship: with a graph, table, situation (pattern), or rule (equation or inequality). (p. 9)



multiplicative identity The number 1 is called the multiplicative identity because multiplying any number by 1 does not change the number. For example, 7(1) = 7.

Multiplicative Identity Property The Multiplicative Identity Property states that multiplying any expression by 1 leaves the expression unchanged. That is, $a(1) = a$. For example, $437x \cdot 1 = 437x$.

multiplier In a geometric sequence the number multiplied times each term to get the next term is called the multiplier or the common ratio or generator. The multiplier is also the number you can multiply by in order to increase or decrease an amount by a given percentage in one step. For example, to increase a number by 4%, the multiplier is 1.04. We would multiply the number by 1.04. The multiplier for decreasing by 4% is 0.96. (p. 692)

natural logarithm A logarithm base e. Abbreviated "ln". (p. 548)

negative exponent The value of a number raised to a negative exponent is the reciprocal of the number raised to the absolute value of the exponent. $x^{-a} = \frac{1}{x^a}$ for $x \neq 0$. (p. 740)

non-function A relation that has more than one output for one or more of its inputs.

normal distribution A distribution of data that can be modeled with the normal probability density function. See *normal probability density function*. (p. 477)

normal probability density function A very specific function that is used to model single-peaked, symmetric, bell-shaped data. (p. 485)

observational study A study where data is collected by observing an existing situation without the researcher (or observer) imposing any type of change or treatment. (p. 461)

observed value An actual measurement, as opposed to a prediction made from a model.

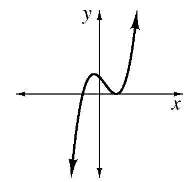
odd function A function where $f(-x) = -f(x)$ for all values of x defined for the function. The graph of the function will be symmetric about the origin. (p. 108)

open question A question that allows respondents to offer any answer they like. (p. 444)

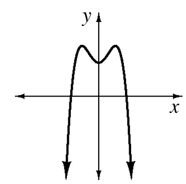
one-dimensional Not having any width or depth. Lines and curves are one-dimensional.

ordered pair A pair of numbers written (x, y) used to represent the coordinates of a point in an xy-plane where x represents the horizontal distance from 0 and y is the vertical. The input and output values of a function or relation can be represented as ordered pairs where x is the input, and y is the output.

ordered triple Three real numbers written in order (x, y, z) represent a point in space or replacement values for a situation involving three variables. See *3-dimensional coordinate system*. (p. 263)



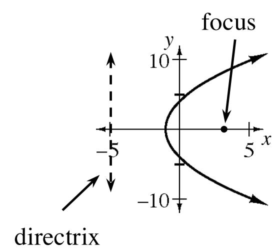
orientation Used informally in this course to describe some graphs. For example the direction a parabola opens might be referred to as its orientation. When describing the graph of a polynomial function, a positive orientation would mean the graph eventually continues upward as the value of x increases, as in the example above right. A negative orientation would mean it eventually heads downward as the value of x continues to increase, as in the example below right. (p. 358)



origin The point on a coordinate plane where the x- and y-axes intersect is called the origin. This point has coordinates $(0, 0)$. The point assigned to zero on a number line is also called the origin. (p. 258)

outlier A number in a set of data that is far away from the bulk of the data. (p. 765)

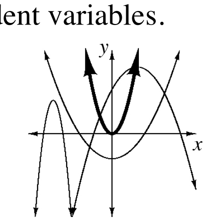
output Used to describe the result of applying a function or relationship rule to an input value. For the function $f(x) = x^2 - 73$ when the input is 10, the output is 27. Function notation shows how the function operates on the input to produce the output: $f(10) = 10^2 - 73 = 27$. (p. 26)



parabola The set of all points that are equidistant from a single point (the focus) and a line (the directrix). The general equation for a parabola that is a function (or a quadratic function) in graphing (or vertex) form, is $y = a(x-h)^2 + k$. The general equation of a quadratic function in standard form is $y = ax^2 + bx + c$. A general equation for parabolas that are not functions, "sleeping" parabolas, is $x - h = a(y-k)^2$. (p. 66)

parallel Two lines in a plane are parallel if they never intersect. Parallel lines have the same slope. Two line segments in a plane are parallel if they lie on lines that never intersect. Two lines in space are parallel if they lie in the same plane and they do not intersect. There is a constant distance between parallel lines.

parameter In a general equations where x and y represent the inputs and outputs of the function, variables such as a, b, c, m, h, and k are often referred to as parameters, and they are often replaced with specific values. For example, in the equation $y = a(x-h)^2 + k$ representing all parabolas that are functions, the a, h, and k are (variable) parameters that give the shape and location, while x and y are the independent and dependent variables. (p. 41)



parent graph The simplest version of a family of graphs. For instance, the graph of $y = x^2$, is considered the parent graph for parabolas that are functions. (p. 98)

Pascal's Triangle The array of numbers at right. The triangular pattern continues downward. This array shows all the values of ${}_nC_r$ where n is the row number when the vertex is ${}_0C_0$. r is the number of places to the right in row n (when the counting begins with 0). For instance, ${}_5C_2$ is equal to 10. (p. 540)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

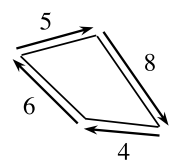
percent A ratio that compares a number to 100. Percents are often written using the "%" symbol. For example, 0.75 is equal to $\frac{75}{100}$ or 75%.

percentile A percentile ranking indicates the percentage of scores which are below the score in question. For example, if you scored at the 90th percentile on a test, your score was higher than the scores of 90% of the other test takers. (p. 482)

perfect square Usually, a quadratic polynomial $ax^2 + bx + c$ that can be rewritten as the second power of a binomial, $(cx + d)^2$. For example, $x^2 - 6x + 9$ is a perfect square that can be rewritten as $(x - 3)^2$. Also, any polynomial of even degree that can be rewritten as the square of one polynomial factor. For numbers, a whole number that can be written as the second power of another whole number. For example, 1, 4, 9, 16, and 25 are perfect squares.

perfect square form A quadratic equation in the form $(ax + b)^2 = c$, where a is nonzero, is said to be in perfect square form. For example, $(2x + 3)^2 = 20$ is a quadratic equation in perfect square form. (p. 68)

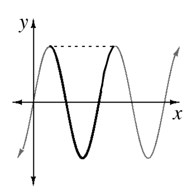
perfect square trinomials Trinomials of the form $a^2x^2 + 2abx + b^2$, where a and b are nonzero real numbers, are known as perfect square trinomials and factor as $(ax + b)^2$. For example, the perfect square trinomial $9x^2 - 24x + 16$ can be factored as $(3x - 4)^2$.



Perimeter =
5 + 8 + 4 + 6 = 23 units

perimeter The distance around a figure on a flat surface.

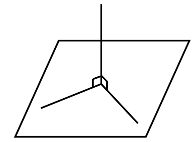
period The length of one cycle of a graph, as shown by the dashed line in the graph at right. (p. 360)



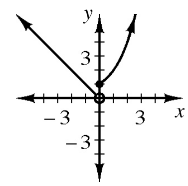
periodic function A function which has a repetitive section or cycle such as the sine, cosine and tangent functions. In a periodic function, the cyclic pattern continues forever both to the left and to the right.

permutation A permutation is an arrangement in which the order of selection matters. For example a batting line-up is a permutation because it is an ordered list of players. If each of five letters, A, B, C, D, E is printed on a card, the number of 3-letter sequences can you make by selecting three of the five cards is a permutation. Permutations can be represented with tree diagrams, decision charts, and their value calculated by using the formula for ${}_nP_r = \frac{n!}{(n-r)!}$. In the example given above, ${}_5P_3 = \frac{5!}{2!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{2\cdot1} = 5 \cdot 4 \cdot 3 = 60$. (p. 503)

perpendicular Two lines, rays, or line segments that intersect to form a right angle. A line and a plane can also be perpendicular if the line does not lie in the plane, but intersects it and forms a right angle with every line in the plane that passes through the point of intersection.

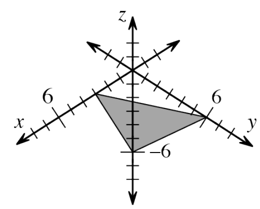


piecewise function A function composed of parts of two or more functions. Each part is a function with a restricted (limited) domain. (p. 105)



placebo A treatment with no effect. A placebo is sometimes used in experiments involving people so that the psychological effects of participating in an experiment are the same in all the treatment groups. (p. 462)

plane A plane is an undefined term in geometry. It is a two-dimensional flat surface that extends without end. It is made up of points and has no thickness. The part of a plane outlined by its *xy*-, *xz*- and *yz*-traces is often used to represent a plane on a 3-dimensional coordinate system. (p. 263)



point An undefined term in geometry. A point has no dimensions but can be located by its coordinates on a number line, in a plane, or in space. (p. 263)

Point-Slope form $y-k=m(x-h)$ is called the point slope form of a linear equation or function because it shows the slope m and a point (h, k) that is on the graph of the line. For example, given a line that has slope $\frac{5}{3}$ and contains the point with coordinates (3, –4) its equation can be written $y+4=\frac{5}{3}(x-3)$. (p. 92)

polynomial An algebraic expression that involves at most the operations of addition, subtraction, and multiplication. A polynomial in one variable is an expression that can be written as the sum of terms of the form: $(\text{any number}) \cdot x^{(\text{whole number})}$. These polynomials are usually arranged with the powers of x in order, starting with the highest, left to right. The numbers that multiply the powers of x are called the coefficients of the polynomial. See *degree of a polynomial* for an example. (p. 135)

population A collection of objects or group of people about whom information is gathered. (p. 445)

power A number or variable raised to an exponent in the form x^n. See *exponent*.

power model or power curve A best-fit curve of the form $y=ax^b$ that represents data on a scatterplot.

Power Property of Logs $\log_m(a^n)=n\log_m(a)$. For example, $\log_3 625=4\log_3 5$. (p. 289)

predicted value (of an association) The dependent (y-value) that is predicted for an independent (x-value) by the best-fit model for an association.

principal Initial investment or capital. An initial value.

probability The probability that an event A. with a finite number of equally likely outcomes, will occur is the number of outcomes for event A divided by the total number of equally likely outcomes. This can be written as $\frac{\text{number of outcomes for event A}}{\text{total number of possible outcomes}}$. A probability p is a ratio, $0 \le p \le 1$. (p. 567)

process control See *statistical process control*. (p. 598)

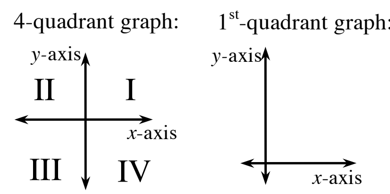
Product Property of Logs $\log_m(a \cdot b) = \log_m(a) + \log_m(b)$. For example, $\log_3 30 = \log_3 5 + \log_3 6$. (p. 289)

profit The amount of money after expenses have been accounted for.

proportion Equal ratios are described as a proportion. For example, the equation $\frac{x-8}{2x+1} = \frac{5}{6}$. (p. 472)

Pythagorean Identity For trigonometric functions, $\cos^2 x + \sin^2 x = 1$ for any value of x. (p. 343)

quadrants The coordinate plane is divided by its axes into four quadrants. The quadrants are numbered as shown in the first diagram at right. When graphing data that has no negative values, we sometimes use a graph showing only the first quadrant.



quadratic equation Any equation where at least one term has degree 2 and no term has degree higher than 2. The standard equation $Ax^2 + By^2 + Cxy + Dx + Ey + F = 0$ represents all quadratic relations in one or two variables. (p. 24)

quadratic expression An expression that can be written in the form $ax^2 + bx + c$, where a, b, and c are real numbers and a is nonzero. For example, $3x^2 - 4x + 7.5$ is a quadratic expression.

Quadratic Formula This formula gives you the solutions $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, for a quadratic equation in one variable that can be written in the standard form $ax^2 + bx + c = 0$. (p. 24)

quadratic function A quadratic equation that can be written $y = ax^2 + bx + c$ is also a quadratic function where x is the independent variable and y is the dependent variable. Its graph is a parabola with a vertical orientation. The graphing form of a quadratic function is $f(x) = a(x-h)^2 + k$.

quarterly Occurring four times a year. (p. 718)

quartile Along with the median, the quartiles divide a set of data into four groups of the same size. Also see *boxplot*. (p. 757)

quotient The result of a division problem is a quotient with a remainder (which could be 0). When a polynomial $p(x)$ is divided by a polynomial $d(x)$ the polynomial $q(x)$ will be the quotient with a remainder $r(x)$. The product of $q(x)$ and $d(x)$ plus the remainder $r(x)$ will equal the original polynomial. $p(x) = d(x)q(x) + r(x)$.

Quotient Property of Logs $\log_m(\frac{a}{b}) = \log_m(a) - \log_m(b)$. For example, $\log_3 \frac{37}{5} = \log_3 37 - \log_3 5$. (p. 289)

***R*²** See *R*-squared.

radian measure An arc of a unit circle equal to the length of the radius of the circle is one radian. The central angle for this arc has measure one radian. 1 radian = $\frac{180}{\pi}$ degrees. (p. 334)

radical An expression in the form $\sqrt{a}$, where $\sqrt{a}$ is the positive square root of a. For example, $\sqrt{49} = 7$. Also see *square root*. (p. 68)

radicand The expression under a radical sign. For example, in the expression $3 + 2\sqrt{x-7}$, the radicand is $x - 7$.

random sample A sample which was chosen as a result of a random process. A random sample can represent the whole population well.

range (of a data set) The range of a set of data is the difference between the highest and lowest values. (p. 771)

range (of a function) The range of a function is the set of possible outputs for a function. It consists of all the values of the dependent variable, that is every number that y can represent for the function $f(x) = y$. (p. 18)

ratio The comparison of two quantities or expressions by division.

rational equation An equation that includes at least one rational expression. For example, $5 - \frac{x+2}{x} = 7$. (p. 608)

rational expression An expression in the form of a fraction in which the numerator and denominator are polynomials. For example, $\frac{x-7}{x^2+8x-9}$ is a rational expression. (p. 135)

rational function A function that contains at least one rational expression. For example, $f(x) = \frac{5}{x-3}$. (p. 138)

rational number A number that can be written as a fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

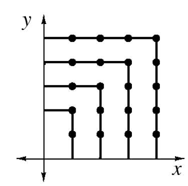
real numbers The set of all rational numbers and irrational numbers is referred to as the set of real numbers. Any real number can be represented by a point on a number line. (p. 389)

rebound height The height a ball reaches after a bounce.

rebound ratio The ratio of the height a ball bounces after one bounce to the height from which it dropped. (p. 666)

reciprocal The multiplicative inverse of a number or an expression.

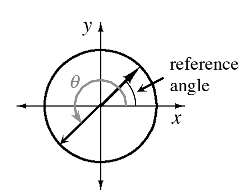
reciprocal trigonometric functions The functions $y = \frac{1}{\sin x} = \csc x$, $y = \frac{1}{\cos x} = \sec x$, and $y = \frac{1}{\tan x} = \cot x$. (p. 636)



rectangular numbers The terms of the sequence $0, 2, 6, 12, 20, \ldots$. These numbers are called rectangular because they count the number of dots in rectangular arrays with the dimensions $n(n+1)$ where $n = 0, 1, 2, 3, 4, \ldots$

recursive equation (for a sequence) An equation for a term in a sequence that requires knowing other terms in the sequence first. For example, the equation for the Fibonacci sequence is recursive because you need to know the previous two terms to find any other term. The equation for the Fibonacci sequence is $t(n) = t(n-2) + t(n-1)$, $t(1) = 1$, $t(2) = 1$. Also see *explicit equation*. (p. 692)

recursive sequence A sequence which can be described by a recursive equation. See *recursive equation*. (p. 692)



reference angle For every angle of rotation in standard position, the reference angle is the angle in the first quadrant $0 \leq \theta \leq \frac{\pi}{2}$ whose cosine and sine have the same absolute values as the cosine and sine of the original angle. (p. 338)

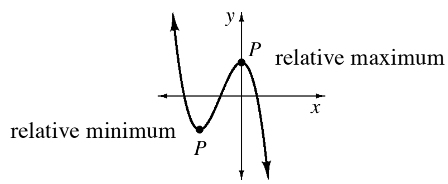
reflective symmetry A type of symmetry where one half of the image is a reflection across a line of the other half of the image. You can fold the image on the line of symmetry and have both halves match exactly. Also see *line of symmetry*.

regression A method for finding a best-fit line or curve through a set of points on a scatterplot. The most common type of regression is finding the least squares regression line. In this course we also do curved regressions: exponential regressions (fitting $y = a \cdot b^x$), quadratic regressions (fitting $y = ax^2 + bx + c$) and power regressions (fitting $y = ax^b$).

relation Functions are also relations, but relations are not necessarily functions. The equations for parabolas, ellipses, hyperbolas, and circles are all relations but only the equations that describe vertically oriented parabolas are functions.

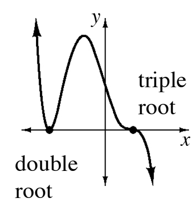
relative frequency A ratio or percent. If 60 people are asked, and 15 people prefer "red," the relative frequency of people preferring red is $\frac{15}{60}=25\%$. (p. 472)

relative maximum (minimum) A function that has a "peak" (or "valley") at a point *P* is said to have a relative maximum (minimum) at the point *P*. This is the point where the function changes direction.



remainder When dividing polynomials in one variable, the remainder is what is left after the constant term of the quotient has been determined. The degree of the remainder must be less than the degree of the divisor. In the example below the remainder is $3x-3$. $(x^4+3x^3-x^2+x-7)\div(x^2+x+1)=x^2+2x-4+\frac{3x-3}{x^2+x+1}$. Also see *quotient*. (p. 410)

Remainder Theorem For any number c, when a polynomial $p(x)$ is divided by $(x-c)$, the remainder is $p(c)$. For example, if the polynomial $p(x)=x^4-x^3+20x-48$ is divided by $(x-5)$, the remainder is $p(5)=552$. (p. 418)



repeated root A root of a polynomial that occurs more than once. The root r will occur as many times as $(x-r)$ is a factor of the polynomial. See *double root* and *triple root*. (p. 380)

residual The distance a prediction is from the actual observed measurement in an association. The residual is the y-value predicted by the best-fit model subtracted from the actual observed y-value. A residual can be graphed with a vertical segment that extends from the observed point to the line or curve made by the best-fit model. It has the same units as the y-axis.

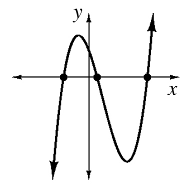
residual plot A display of the residuals of an association. A residual plot is created in order to analyze the appropriateness of a best-fit model. If a model fits the data well, no apparent pattern will be made by the residuals—the residuals will be randomly scattered.

rewrite To rewrite an equation or expression is to write an equivalent equation or expression. Rewriting could involve using the Distributive Property, following the Order of Operations, using properties of 0 or 1, substitution, inverse operations, Properties of Logarithms, or use of trigonometric identities. We usually rewrite in order to change expressions or equations into more useful forms or sometimes, just simpler forms. (p. 170)

right angle An angle with measure 90°.

right triangle A triangle with a right angle.

roots of a function The number r is a root (or zero) of the function $f(x)$ if $f(r) = 0$. A root may be a real or a complex number. Real roots occur where the graph of the function $f(x)$ crosses the x-axis. Complex roots must be found algebraically. (p. 382)



***R*-squared** The correlation coefficient squared, and usually expressed as a percent. R^2 is a measure of the strength of a linear relationship. Its interpretation is that R^2 % of the variability in the dependent variable can be explained by a linear relationship with the independent variable. The rest of the variability is explained by other variables not part of the study.

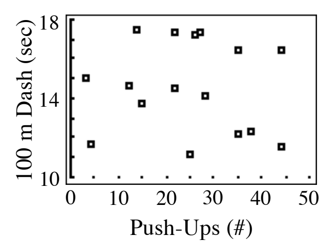
rule An algebraic representation or a written description of a mathematical relationship.

sample A subset (group) of a given population with the same characteristics as the whole population. (p. 445)

sample space With probability, all the possible outcomes.

scatter The variability in data. When the scatter of data forms a pattern, we can often describe and model the data. Random scatter has no discernable pattern. (Note that random scatter does not mean evenly scattered—randomly scattered data often forms clusters and gaps.) See *variability* and *spread*.

scatterplot A way of displaying two-variable numerical data where two measurements are taken for each subject (like height and forearm length, or surface area of cardboard and volume of cereal held in a cereal box). To create a scatterplot, the two values for each subject are written as coordinate pairs and graphed on a pair of coordinate axes (each axis representing a variable). Also see *association*.



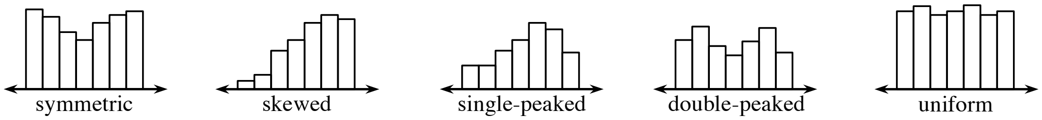
secant The reciprocal of the cosine. (p. 651)

sequence A sequence is an ordered set of objects, usually written as a list of numbers. A sequence can be thought of as a function in which the independent variable is the term number and the dependent variable is the term value. (p. 674)

series The sum of the terms of a sequence. (p. 500)

set A collection of data or items. (p. 139)

shape (of a data distribution) Statisticians use the following words to describe the overall shape of a data distribution: symmetric, skewed, single-peaked, double-peaked, and uniform. Examples are shown below. (p. 765)



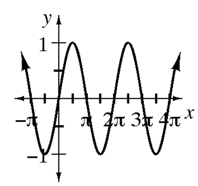
sigma The Greek letter Σ which is used to mean sum. Using Σ provides a way to write a short, compact mathematical representation for a series. See *summation notation*. (p. 519)

simple interest Interest paid on the principal alone. (p. 719)

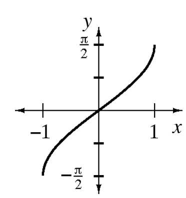
simplify To rewrite an expression or equation as an equivalent expression or equation in a form that is considered to be simpler or less cumbersome than the original.

simulation (probability) When conducting an experiment with an event that is unrealistic to perform, a simulation can be used. A simulation is a similar experiment that has the same probabilities as the original experiment. (p. 565)

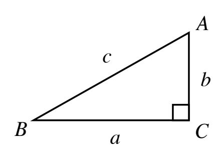
sine function For any real number θ, the sine of θ, denoted $\sin\theta$, is the y-coordinate of the point on the unit circle reached by a rotation angle of θ radians in standard position. The general equation for the sine function is $y = a \sin b(x - h) + k$. This function has amplitude a, period $\frac{2\pi}{b}$, horizontal shift h, and vertical shift k. (p. 360)



sine inverse ($\sin^{-1} x$) Read as the inverse of sine x, $\sin^{-1} x$ is the measure of the angle with sine x. We can also write $y = \arcsin x$. Note that the notation refers to the inverse of the sine function, *not* $\frac{1}{\sin x}$. Because $y = \sin^{-1} x$ is equivalent to $x = \sin y$ and there are infinitely many angles y such that $\sin y = x$, the inverse *function* is restricted to select the *principal* value of y such that $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. The graph of the inverse function, $y = \sin^{-1} x$ with $\frac{\pi}{2} \le y \le \frac{\pi}{2}$ is at right. (p. 636)



sine ratio In a right triangle the ratio $\frac{\text{opposite side}}{\text{hypotenuse}}$ is known as the sine of the acute angle. At right, $\sin B = \frac{b}{c}$ since the side length b is opposite angle B. (p. 317)

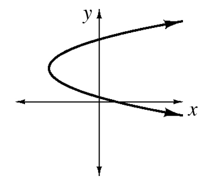


single-peaked See *shape (of a data distribution)*. (p. 765)

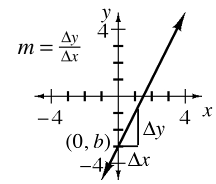
sketch a graph To sketch the graph of an equation means to show the approximate shape of the graph in the correct location with respect to the axes with key points clearly labeled. (p. 8)

skewed See *shape (of a data distribution)*. (p. 765)

"sleeping" parabola A "sleeping" parabola is a parabola which opens to the left or right, rather than upward or downward. These parabolas are not the graphs of functions. (p. 102)



slope The ratio of the vertical change to the horizontal change between any two points on a line. For any two points (x_1, y_1) and (x_2, y_2) on a given line, the slope is $\frac{y_2 - y_1}{x_2 - x_1}$. For example, the slope of a line between points with coordinates (3, –5) and (–7, 2) is $\frac{2-(-5)}{-7-3} = -\frac{7}{10}$. (p. 10)



Slope-Intercept Form A linear equation written in the form $y = mx + b$ is written in slope-intercept form. In this form, m is the slope of the line and the point $(0, b)$ is the y-intercept. (p. 10)

solution Of an equation or inequality is a number or expression that makes the equation or inequality true when substituted for the variable. To find the numerical solution means to identify all the numbers that make a mathematical equation or inequality true. There may be any number of solutions for an equation or inequality, from 0 to infinitely many. Solutions to equations and inequalities in one variable are single numbers. For two variables they are ordered pairs, for three variables ordered triples. Equations with no solutions, such as $|x-5| = -3$, are never true. Equations with one, several, or many solutions, such as $x(x + 3)(x - 7) = 0$ are sometimes true, and equations or identities such as $5(2x-4) = 10x - 20$ are always true. (p. 200)

solve (1) To find all the solutions to an equation or an inequality (or a system of equations or inequalities). For example, solving the equation $x^2 = 9$ gives the solutions $x = 3$ and $x = -3$. (2) Solving an equation for a variable gives an equivalent equation that expresses that variable in terms of other variables and constants. For example, solving $2y - 8x = 16$ for y gives $y = 4x + 8$. The equation $y = 4x + 8$ has the same solutions as $2y - 8x = 16$, but $y = 4x + 8$ expresses y in terms of x and some constants. (p. 169)

special right triangles A right triangle with particular notable features that can be used to solve problems. Sometimes, these triangles can be recognized by the angles, such as the 45º-45º-90º triangle (also known as an "isosceles right triangle") and a 30º-60º-90º triangle. (p. 320)

spread (of a data distribution) A measure of the amount of variability, or how "spread out" a set of data is. Some ways to measure spread are the range, the mean absolute deviation, the standard deviation, and the interquartile range. For symmetric distributions with no outliers, the standard deviation or the interquartile range can represent the spread of the data well. However, in the presence of outliers or non-symmetrical data distributions, the interquartile range may be a better measure. Also see *variability*. (p. 757)

square root A number a is a square root of b if $a^2 = b$. For example, the number 9 has two square roots, 3 and –3. A negative number has no real square roots; a positive number has two; and zero has just one square root, namely, itself. Other roots, such as cube root, will be studied in other courses. Also see *radical*. (p. 83)

standard deviation A way to measure the spread, or the amount of variability, in a set of data. The standard deviation is the square root of the average of the distances to the mean, after the distances have been made positive by squaring. The standard deviation represents the spread of a set of data well if the distribution of the data is symmetric and has no outliers. (p. 771)

Standard Form (linear function) A linear equation written in the form $Ax + By = C$ is written in standard form. For linear functions, $B \neq 0$. (p. 10)

Standard Form (quadratic function) The standard form for the equation of a quadratic function is $y = ax^2 + bx + c$ where $a \neq 0$. (p. 68)

Standard Form (quadratic relations in general) The standard for parabolas, ellipses, and hyperbolas with axes parallel to the x- or y-axes is $Ax^2 + By^2 + Cx + Dy + E = 0$.

standard position An angle is in standard position if its vertex is located at the origin and one ray is on the positive x-axis. (p. 343)

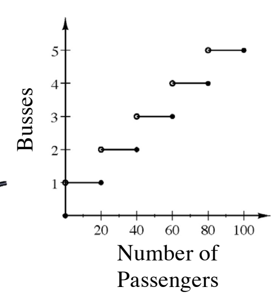
standard window The graphing window on a calculator set to show the x- and y-axes for values $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

statistic A numerical fact or calculation that is formed from data. For example, a mean or a correlation coefficient is a statistic. With a capital "S," Statistics is the field of study which is concerned with summarizing data, and using probability to make predictions from data with natural variability. (p. 445)

statistical process control The process of checking whether manufacturing quality remains stable over time. A process is "in control" if critical measurements of quality remain within the upper control limit and lower control limit as expected. (p. 598)

statistical test A method of making decisions based on a statistical analysis of data. (p. 581)

step function A special kind of piecewise function (a function composed of parts of two or more functions). A step function has a graph that is a series of line segments that often looks like a set of steps. (p. 728)



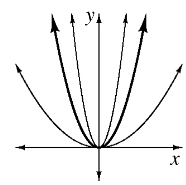
stoplight icon The icon (shown at right) will appear periodically throughout the text. Problems that display this icon contain errors of some type.

strength (of an association) A description of how much scatter there is in the data away from the line or curve of best fit. If an association is linear, the correlation coefficient, r, or R-squared can be used to numerically describe and interpret the strength.

streak Consecutive identical outcomes in a probabilistic situation. For example, if a baseball team wins four consecutive games, they have a winning streak of 4. (p. 572)

study See *observational study*. (p. 461)

stretch factor Used to describe the effect of a in the graphing form of a quadratic, cubic, absolute value, or exponential function. For $a > 1$ or $a < -1$ the outputs increase or decrease faster than the outputs for the parent functions, and the graphs are described as being stretched upwards or downwards in relation to the parent graph. (p. 66)



subjects The people or items being measured in a statistical study.

substitution Replacing a variable or expression with a number, another variable, or another expression. For example, when evaluating the function $f(x) = 5x - 1$, for $x = 3$, substitute 3 for x to get $f(3) = 5(3) - 1 = 14$.

Substitution Method A method for solving a system of equations by replacing one variable with an expression involving the remaining variable(s). For example, in the system of equations at right the first equation tells you that y is equal to $-3x + 5$. We can substitute $-3x + 5$ in for y in the second equation to get $2(-3x + 5) + 10x = 18$, then solve this equation to find x. Once we have x, we substitute that value back into either of the original equations to find the value of y.

$$y = -3x + 5$$
$$2y + 10x = 18$$

sum The result of adding two or more numbers. For example, the sum of 4 and 5 is 9.

sum of cubes A polynomial of the form $x^3 + y^3$. It can be factored as follows: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$. For example, the sum of cubes $27y^3 + 64$ can be factored as $(3y + 4)(9y^2 - 12y + 16)$. (p. 427)

summation notation A convenient way to represent a series is to use summation notation. The Greek letter sigma, Σ, indicates a sum.

For example, $\sum_{n=1}^{4} 3n = 3(1) + 3(2) + 3(3) + 3(4) = 30$. (p. 519)

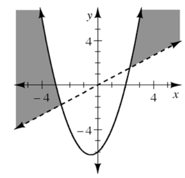
The numbers below and above the sigma are called the indices. The index below, $n = 1$, tells us what value to start with for n. The top index tells us how high the value can go. In the example shown above, n starts at 1 and increases to 4.

symmetry A figure that appears not to change when reflected across a line is said to have reflection symmetry. A figure that appears not to change when rotated through an angle of less than 360° is described as having rotation symmetry.

system of equations A system of equations is a set of equations with more than one unknown or variable. The systems we solve most often in this course have two or three equations and two or three variables. Systems of equations are often solved using substitution or elimination to reduce the number of variables. A system of quadratic equations is shown at right. (p. 171)

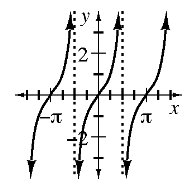
$$y = x^2 + 8x - 4$$
$$y = 2x^2 + 5x - 8$$

system of inequalities A system of inequalities is a set of inequalities with the same variables. Solving a system of inequalities means finding one or more regions on the coordinate plane whose points represent solutions to each of the inequalities in the system. There may be zero, one, or several such regions for a system of inequalities. For example, the shaded region at right is a graph of the system of inequalities that appears below it. (p. 187)

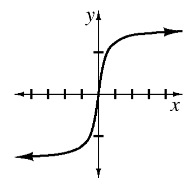


$y \le x^2 + x - 6$
$y > \frac{2}{3}x$

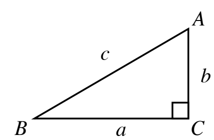
tangent function For any real number θ, the tangent of θ, denoted $\tan\theta$, is the slope of the line containing the ray which represents a rotation of θ radians in standard position. The general equation for the tangent function is $y = a \tan b(x - h) + k$. This function has period of $\frac{\pi}{b}$, vertical asymptotes at $\frac{\pi}{2b} + h \pm \frac{n\pi}{b}$ for $n = 1, 2, \ldots$, horizontal shift h, and vertical shift k. (p. 341)



tangent inverse ($\tan^{-1} x$) Read as the inverse of tangent x, $\tan^{-1} x$ is the measure of the angle that has tangent x. We can also write $y = \arctan x$. Note that the notation refers to the inverse of the tangent function, *not* $\frac{1}{\tan x}$. Because $y = \tan^{-1} x$ is equivalent to $x = \tan y$ and there are infinitely many angles y such that $\tan y = x$, the inverse *function* is restricted to select the *principal* value of y such that $-\frac{\pi}{2} < x < \frac{\pi}{2}$. The graph of the inverse tangent function is at right. (p. 636)



tangent ratio In a right triangle the ratio $\frac{\text{opposite side}}{\text{adjacent side}}$ is known as the tangent of an acute angle. At right, $\tan B = \frac{b}{a}$ since the side of length b is opposite angle B and the side length a is adjacent to angle B.



term A single number, variable, or product of numbers and variables. A monomial is a term. Also a component of a sequence. (p. 135)

term number In a sequence, a number that gives the position of a term in the sequence. A replacement value for the independent variable in a function that determines the sequence. See *sequence*. (p. 676)

theoretical probability A probability calculation based on counting possible outcomes.

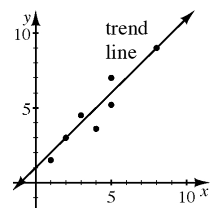
third quartile (Q3) The median of the upper half of a set of data which has been written in numerical order. (p. 757)

three-dimensional An object that has height, width, and depth. (p. 257)

transcendental number A number that is not the root of *any* polynomial equation with rational coefficients. Transcendental numbers are very common. The most prominent examples of transcendental numbers are π and e. (p. 548)

transformation (of a function) The conversion of a function to a corresponding function. Transformations often slide, reflect, stretch, and/or compress graphs of functions. For example, the transformation of $f(x) + k$ moves the graph of the function up or down by an amount k. The transformation $f(-x)$ reflects the graph of the function across the y-axis. (p. 83)

translation The result of moving a graph horizontally, vertically, or both but without changing its orientation.



treatment Some type of change imposed on one or more groups in an experiment so that a direct comparison can be made. (p. 461)

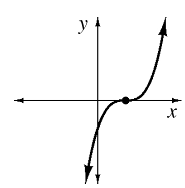
trend line Another name for a line of best fit. See *line of best fit*.

trigonometric identities The different trigonometric functions are related by many different equations, termed identities. A few examples of these identities are: $\cos(x) = \cos(-x)$ and $\sin(-x) = -\sin(x)$. (p. 651)

trigonometric ratios See *sine ratio*, *cosine ratio*, *tangent ratio*, *secant*, *cosecant*, and *cotangent*. (p. 29)

trinomial A polynomial that is the sum or difference of exactly three terms, each of which is a monomial. For example, $x^2 + 6x + 9$ is a trinomial.

triple root A root of a function that occurs exactly three times. If an expression of the form $(x - a)^3$ is a factor of a polynomial, then the polynomial has a triple root at $x = a$. The graph of the polynomial has an inflection point at $x = a$. (p. 380)

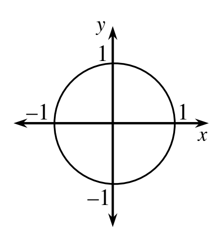


two-dimensional An object having length and width. (p. 257)

Undoing Using inverse operations and reversing the order of operations to solve an equation in one variable or to "undo" a function rule in order to write its inverse. For example, in the equation $y = x^3 - 7$, add y and write the cube root. (p. 170)

uniform See *shape (of a data distribution)*. (p. 765)

unit circle A circle with a radius of one unit is called a unit circle. (p. 337)

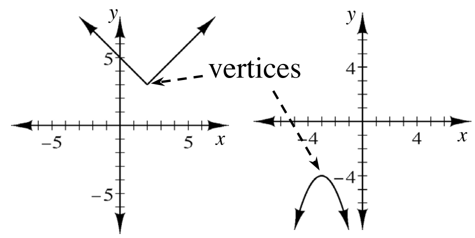


upper control limit (UCL) See *control limit*. (p. 599)

variable A symbol used to represent one or more numbers. In this course, letters of the English alphabet are used as variables. For example, in the expression $3x-(8.6xy+z)$, the variables are x, y, and z. (p. 41)

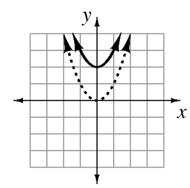
variability The inconsistency in measured data. Variability comes from two sources. Measurement error is the inconsistency in measurement of the same element repeated times and/or by different people. Element variability is the natural variation of the elements themselves. For example, even though in general people with larger shoe sizes are taller, not all people with the same shoe size have the same height. That is element variability. See *spread*. (p. 581)

vertex (of a parabola) The vertex of a parabola is the highest or lowest point on the parabola (depending on the parabola's orientation). See *parabola*. (p. 60)



vertex form The vertex form for the equation of a quadratic function (also called graphing form) is written $y=a(x-h)^2+k$. (p. 68)

vertical shift Used to describe the location of a graph in relation to its parent graph, the shift is the vertical distance (up or down) of each point on the graph from the corresponding point on the parent graph. This type of transformation is a vertical translation. For example, each point on the graph of $y=x^2+2$ is two units higher that its corresponding point on $y=x^2$. In a general equation such as $y=a(x-h)^2+k$ the vertical shift is represented by k. (p. 66)



volume For a three-dimensional figure the number of non-overlapping cubic units that will fit inside.

***x*-bar process control chart** A plot of the mean measurement of samples over time. x-bar process control charts are used to monitor whether a manufacturing process is proceeding within limits as expected. (p. 599)

***x*-intercept** A point where a graph intersects the x-axis. In two dimensions, the coordinates of the x-intercept are $(x, 0)$. In three dimensions they are $(x, 0, 0)$. See *intercept*. (p. 10)

***x-y* interchange** The result of exchanging the x and y variables and then solving for y. The resulting equation is the inverse of the original function.

$x \rightarrow y$ table A table that represents pairs of related values. The input value x appears in the first row or column, and the output value, y, appears in the second. The $x \rightarrow y$ table at right contains input and output pairs for the equation $y=x^2-3$.

x	y
–3	6
–2	1
–1	–2
0	–3
1	–2
2	1

***y*-intercept** A point where a graph intersects the y-axis. In two dimensions the coordinates of the y-intercept are $(0, y)$. In three dimensions, the coordinates are $(0, y, 0)$. See *intercept*. (p. 10)

***z*-score** A z-score indicates how many standard deviations a data value is from the mean. More formally, $z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}}$.

zero factorial Zero factorial is 1, $0! = 1$. (p. 648)

zero power The result of raising any number (except zero) to the zero power is 1. $x^0 = 1$ for any number $x \neq 0$. (p. 740)

Zero Product Property When the product of two or more factors is zero, at least one of the factors must equal zero. Used to solve equations in factored form. For example, given the equation $(x-4)(x+5)=0$ you can see that 4 and –5 are solutions and that they are the only possible solutions because there are no other numbers that make either factor zero. (p. 24)

zeros of a function The roots of a function or the values of x for which the function value $y = 0$. The x-intercepts of the graph of a function in the real plane are zeros. These are also called the roots of the function. A function can have complex zeros. These complex zeros cause $y = 0$, but they are not x-intercepts since they do not exist in the real plane. See *roots of a function*. (p. 382)