# Transforming Functions

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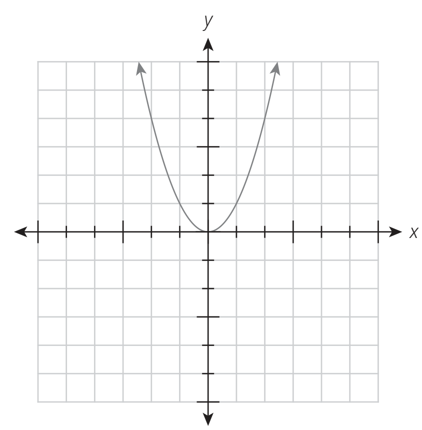
## Overview

The simplest equation of one shape (e.g., line, parabola, absolute value) is called a parent equation. Changing a parent equation by addition or multiplication can move the parent graph and change its size and orientation, but does not change the basic shape. These changes are called **transformations**. The first set of examples shows how the parent graph of a parabola can be moved on an *xy*-coordinate system. In later courses, you will also learn how to make the parabola wider or narrower. Transformations of other functions are done in a similar manner.

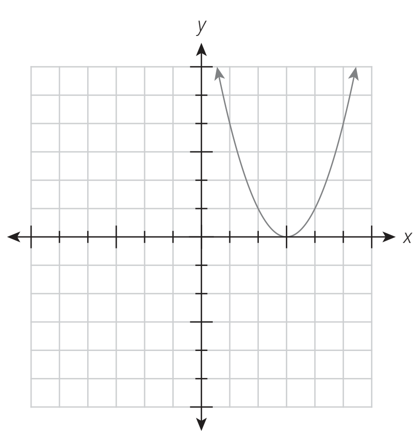
## Examples

**1.** For each problem, the parent graph is $f(x) = x^2$. Describe each transformation and graph the function.



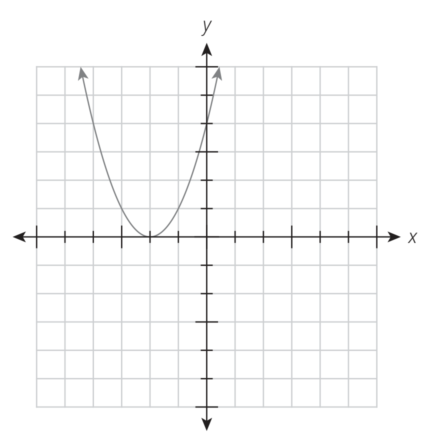

a. $f(x - 3) = (x - 3)^2$

right 3 units




b. $f(x + 2) = (x + 2)^2$

left 2 units

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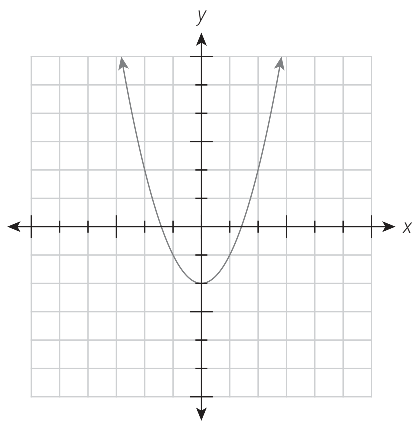
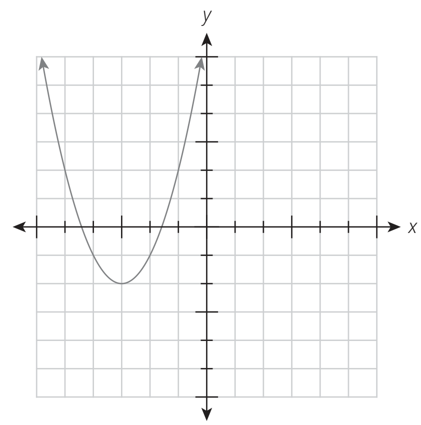
c. $f(x) - 2 = x^2 - 2$

down 2 units



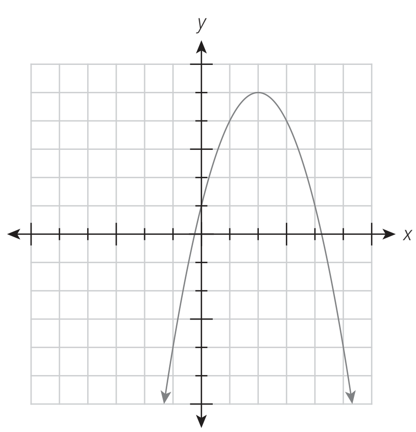
d. $f(x + 3) - 2 = (x + 3)^2 - 2$

left 3 and down 2 units



e. $-f(x - 2) + 5 = -(x - 2)^2 + 5$

right 2 and up 5 units, and reflected across the $x$-axis



# Practice

Predict how each parabola is different from the parent graph. For each of these problems, the parent graph is $f(x) = x^2$.

**1.** $y = f(x + 5)$

**2.** $y = f(x) + 5$

**3.** $y = -f(x)$

**4.** $y = f(x) - 4$

**5.** $y = f(x - 5)$

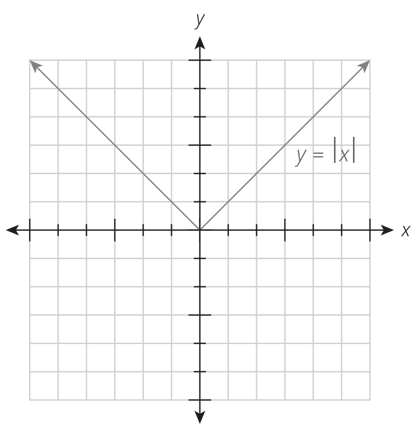
**6.** $y = -f(x + 2)$

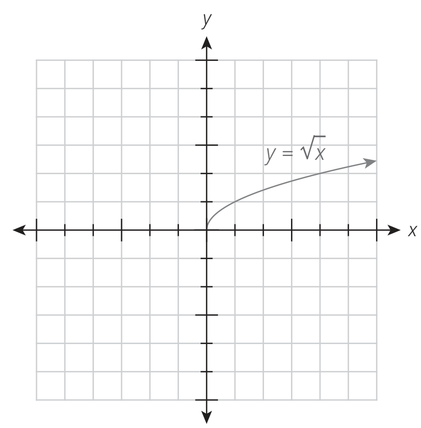
**7.** $y = f(x - 5) - 3$

**8.** $y = -f(x + 2) + 1$

**9.** $y = f(x - 3) - 5$

The parent graphs for absolute value and square root are shown. They can be transformed in exactly the same ways as parabolas. Predict how the graph of each equation below is different from the parent graph.




**10.** $f(x - 4)$ where $f(x) = |x|$

**11.** $f(x) + 5$ where $f(x) = \sqrt{x}$

**12.** $f(x) + 3$ where $f(x) = |x|$

**13.** $f(x + 2)$ where $f(x) = \sqrt{x}$

**14.** $-f(x) + 5$ where $f(x) = |x|$

**15.** $-f(x)$ where $f(x) = \sqrt{x}$

**16.** $-f(x + 3)$ where $f(x) = \sqrt{x}$

**17.** $-f(x - 5)$ where $f(x) = |x|$

**18.** $-f(x - 2) + 5$ where $f(x) = \sqrt{x}$

**19.** $f(x) + 3$ where $f(x) = \sqrt{x}$

**20.** $f(x - 7)$ where $f(x) = \sqrt{x}$

**21.** $-f(x + 3) + 7$ where $f(x) = |x|$

# Answers

**1.** left 5

**2.** up 5

**3.** reflected across the $x$-axis

**4.** down 4

**5.** left 5

**6.** left 2, reflected across the $x$-axis

**7.** right 5, down 3

**8.** left 2, up 1, reflected across the $x$-axis

**9.** right 3, down 5

**10.** right 4

**11.** up 5

**12.** up 3

**13.** left 2

**14.** up 5, reflected across the $x$-axis

**15.** reflected across the $x$-axis

**16.** left 3, reflected across the $x$-axis

**17.** right 5, reflected across the $x$-axis

**18.** right 2, up 5, reflected across the $x$-axis

**19.** up 3

**20.** right 7

**21.** left 3, up 7, reflected across the $x$-axis

# Comparing and Representing Data

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## Overview

A **distribution** of a data set is an organized way of showing all the possible values of the data set and how often they occur. Histograms and box plots are examples of data distributions. Two distributions of data can be compared by examining their centers, shapes, spreads, and outliers.

The center of a data distribution can be described by the **mean** or **median**. If the distribution is symmetric and has no outliers, the mean can be used to describe the center. Otherwise, the median can be used. An **outlier** is a data value that is far away from the bulk of the data. Outliers shift the mean towards the outlier, making the data set skewed and the mean less representative of a typical value.

Common shapes of data distributions can be found in the Methods & Meanings box in Lesson 11.2.2.

The spread of a distribution can be described using the **interquartile range** (IQR), described in the Methods & Meanings box in Lesson 11.2.1, or the **standard deviation**, described in the Methods & Meanings box in Lesson 11.2.3. Since the standard deviation is based upon the mean, it should be used only to describe the spread of distributions that are symmetric and without outliers.

## Examples

**1.** University professors wonder how the number of novels they assign in their English Literature classes compares to the number of novels assigned in English Literature classes at community colleges. A community college statistics student collected the following data from 42 university and community college literature courses in the state. Compare the number of novels read in the two types of colleges.

COMMUNITY COLLEGE
13, 10, 15, 12, 14, 9, 11, 15, 12, 14, 9, 10, 13, 15, 12, 9, 11, 15, 12, 10, 15, 14
Checksum: 270

UNIVERSITY
11, 8, 14, 13, 25, 11, 7, 13, 8, 16, 11, 10, 20, 7, 8, 13, 14, 16, 18, 10
Checksum: 253

Any comparison of data should begin with graphical representations using the same scale on the $x$-axes.

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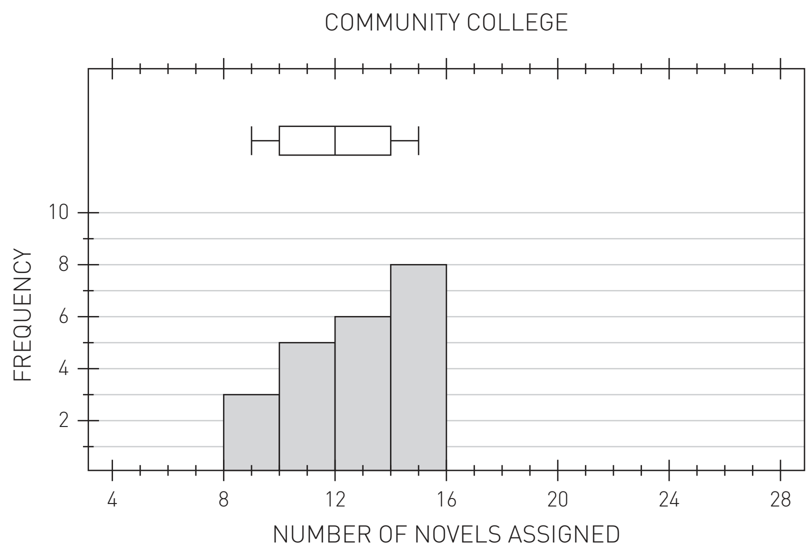
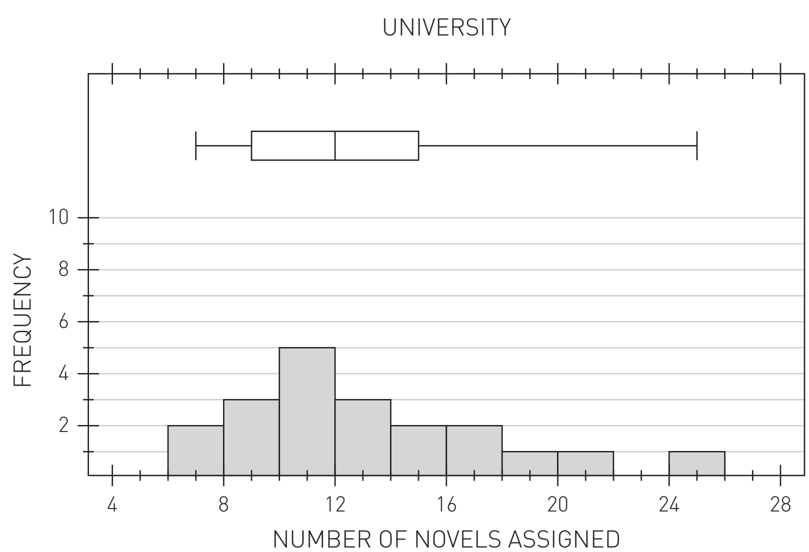
Since neither of the distributions are symmetrical and one of the distributions appears to have an outlier, it would not be appropriate to use means and standard deviations to compare their centers and spreads. The five-number summaries (see the Methods & Meanings box in Lesson 10.1.3) can be located on each graph.

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CENTER

Both types of colleges have the same median of 12 novels.

SHAPE

The distribution for community colleges is skewed to the left, while the distribution for universities is skewed to the right.

SPREAD

The variability in the number of novels assigned in the community college courses is smaller than the variability between courses at the university level. The IQR for community colleges is 4 novels (14 – 10 = 4), while the IQR for universities of 6 (15 – 9 = 6) is one-and-a-half times as wide.

OUTLIERS

One literature course at a university assigns 25 novels, which is an outlier. 25 novels is far away from the bulk of novels assigned in university courses.

CONCLUSIONS

The median number of novels assigned at both types of schools is the same, 12. 25% of university courses assign more novels than any of the community college courses. But, 25% of the university courses also assign fewer novels than any of the community colleges. Community colleges are more consistent from course to course in the number of novels they assign, with an IQR of 4, than are the universities, with an IQR of 6.

**2.** A group of five friends recorded their scores after playing an arcade game. Their scores were: 243, 215, 184, 280, and 148. Calculate the mean and standard deviation of their scores.

The results shown can be calculated using a statistics calculator. The following steps show the required calculations to obtain these measures.

The mean is the sum of all values divided by the number of values: $\frac{243+215+184+280+148}{5} = 214$.

To calculate the standard deviation, first determine the distance each score is from the mean: $243 - 214 = 29$, $215 - 214 = 1$, $184 - 214 = -30$, $280 - 214 = 66$, $148 - 214 = -66$.

Square each distance: $29^2 = 841$, $1^2 = 1$, $(-30)^2 = 900$, $66^2 = 4{,}356$, $(-66)^2 = 4{,}356$.

Then calculate the mean of the squared distances: $\frac{841+1+900+4{,}356+4{,}356}{5} = 2{,}090.8$.

The square root of 2,090.8 is 45.725. The mean score is 214 with a standard deviation of 45.7.

# Practice

**1.** The following data was collected from two different toad species. Compare the number of eggs laid by American toads to the number laid by Fowler toads. Is it appropriate to summarize the distributions by using mean and standard deviation? Use a bin width of 250 eggs.

AMERICAN TOADS
9,100, 8,700, 10,300, 9,500, 7,800, 8,900, 9,200, 9,300, 8,900, 8,300, 9,400, 8,000, 9,000, 8,400, 9,700, 10,000, 8,600, 8,900, 9,900, 9,300
Checksum: 181,200

FOWLER TOADS
9,500, 9,100, 9,400, 8,800, 9,000, 8,400, 9,200, 9,200, 8,900, 9,100, 8,600, 9,200, 8,700, 9,800, 9,300, 8,800, 9,200, 9,300, 9,000, 9,100
Checksum: 181,600

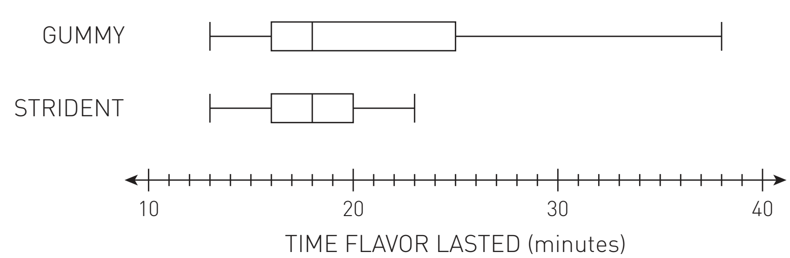
Calculate the standard deviation of the number of eggs laid by each of the first five American toads.

**2.** Do babies with a low birth weight start crawling at a later age than babies born with an average weight? The following data was collected for the age (in months) at which children started crawling:

LOW BIRTH WEIGHT
10, 12, 11, 11, 7, 13, 10, 12, 11, 13, 10, 11, 15, 11, 14, 10
Checksum: 181

AVERAGE BIRTH WEIGHT
7, 6, 13, 9, 8, 7, 5, 7, 9, 8, 10, 8, 11, 7, 7, 10, 6, 8, 7, 6, 12, 8, 7
Checksum: 186

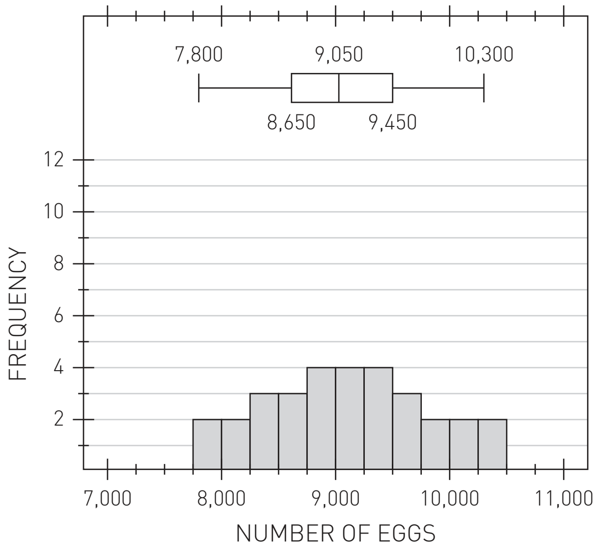
**3.** The following graph shows the amount of time people can spend chewing gum from two different brands, Gummy and Strident, before the flavor runs out. Compare the brands. What is the mean for each type of gum?



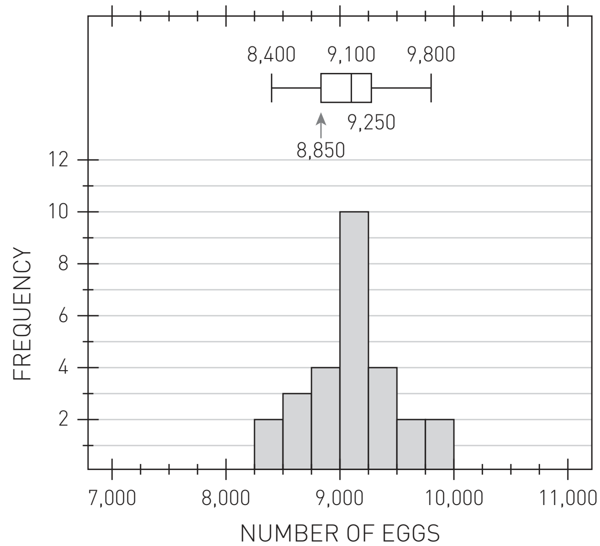
# Answers

**1.**

AMERICAN TOADS




FOWLER TOADS



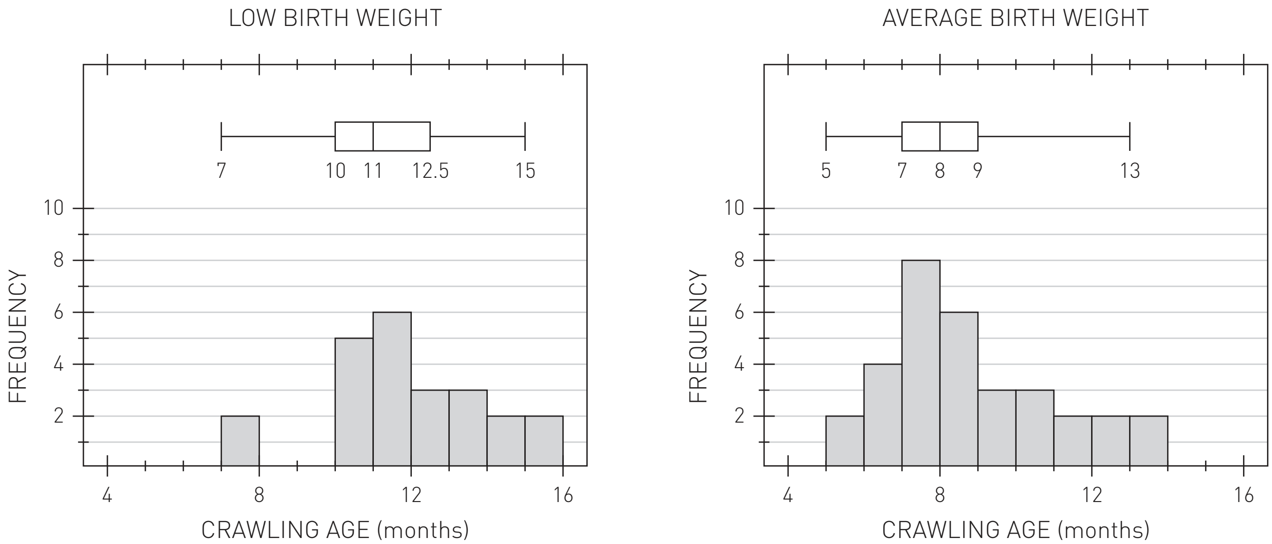

Mean and standard deviation are appropriate statistics because both distributions are fairly symmetric with no outliers.

Both types of toads lay a mean between 9,000 and 9,100 eggs. Both distributions are single-peaked and symmetric with no apparent outliers. However, there is much greater variability in the number of eggs that American toads lay. The standard deviation for American toads is about 637, while the standard deviation for Fowler toads is only about 316 eggs.

$$\frac{\sqrt{20^2 + (-380)^2 + 1{,}220^2 + 420^2 + (-1{,}280)^2}}{5} \approx 830 \text{ eggs}$$

**2.**




Mean and standard deviation are not appropriate to compare these distributions because both are skewed and one has an outlier. Therefore, the median and IQR should be used.

The median age at which low-weight babies start crawling is 11 months, while the median age for average-weight babies is 8 months.

Both distributions are single-peaked and skewed. The low-weight babies have no outliers, and the average-weight babies have an outlier at 13 months.

The variability in the crawling age is roughly the same for low-weight babies (IQR is 2.5 months) as for average-weight babies (IQR is 2 months).

About 75% of low-weight babies have not yet started crawling at an age when 75% of average-weight babies are already crawling.

**3.** The median for both types of gum was about 18 minutes of flavor. The times for Gummy were skewed, while the times for Strident were symmetric. The lower half of the distributions for both gums was the same. But there was much more variability in the upper half of people chewing Gummy than in the upper half of Strident. Indeed, more than 25% of Gummy chewers reported flavor lasting longer than any of the Strident chewers. Neither gum had outliers in how long the flavor lasted.

There was more variability for Gummy, as the IQR was about 9 minutes (25 – 16 = 9). The IQR of 4 minutes (20 – 16 = 4) for Strident was less than half that of Gummy. That variability is an advantage. If you chew Gummy, you will probably be no worse off than chewing Strident, and you could have a much longer-lasting flavor.

The distribution is symmetric, so the mean for Strident is about the same as the median, about 18 minutes. However, the mean for Gummy is greater than 18 minutes due to the skew in the distribution, possibly around 22 minutes.