Describing Graphs of Functions

GUIDED SKILL SUPPORT

Overview

A function can be described using various features of its graph, and graph investigation questions can be useful when examining the graph of a function. The following sample summary statements are for the square root function, $y = \sqrt{x}$.

Graph Investigation Question	Sample Summary Statement
What does the graph look like?	The graph looks like half of a parabola on its side.
Is the graph increasing or decreasing?	As x increases, y increases.
What are the x- and y-intercepts?	The graph intersects both the x- and y-axes at (0, 0).
What x-values (inputs) are possible?	The possible inputs are $x \geq 0$.
What y-values (outputs) are possible?	The possible outputs are $y \geq 0$.
Are there any special points?	The graph has a "starting" point at (0, 0).
Does the graph have any symmetry? If so, where?	The graph has no symmetry.

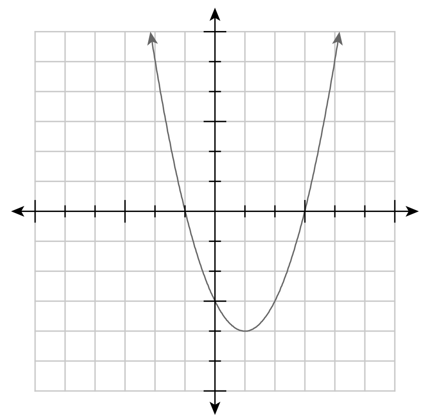
For more information, see the Lesson 1.2.3 Reflection Journal.

Examples

1. For the function $y = x^2 - 2x - 3$, make a table, draw a graph, and fully describe the features of the graph.

Including points around $x = 0$ is a common approach when creating a table. Add more x-values as necessary. Be careful with substitution and Order of Operations when calculating values. For example, when $x = -2$, $y = (-2)^2 - 2(-2) - 3 = 4 + 4 - 3 = 5$.

x	−3	−2	−1	0	1	2	3
y	12	5	0	−3	−4	−3	0

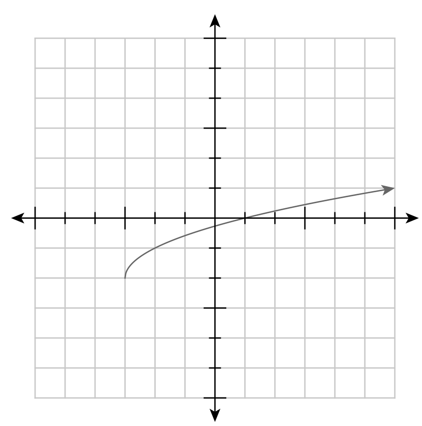


The graph is a parabola; it opens upward. The x-intercepts are (−1, 0) and (3, 0). The y-intercept is (0, −3). Reading from left to right, the graph decreases until $x = 1$ and then increases. The minimum (lowest) point on the graph, called the vertex, is (1, −4). The vertical line $x = 1$ is a line of symmetry. All inputs are possible, and the outputs are greater than or equal to −4.

2. For the function $y = \sqrt{x+3} - 2$, make a table, draw a graph, and describe the features of the graph.

Note that the smallest possible input for the function is $x = -3$. Any value less than −3 will require the square root of a negative number, which is not a real number.

x	−4	−3	0	1	3	6
y		−2	−0.3	0	0.4	1



The graph is half a parabola on its side and is always increasing. The x-intercept is (1, 0) and there is a y-intercept at $\sqrt{0+3} - 2 \approx -0.3$, or approximately (0, −0.3). The inputs are greater than or equal to −3, and the outputs are greater than or equal to −2. There is a special point at (−3, −2) which "starts" the curve. There is a minimum value at $y = -2$. There are no lines of symmetry.

Practice

For each function, make a table, draw a graph, and describe the features of the graph.

1. $y = x^2 - 2x$

2. $y = x^2 + 2x - 3$

3. $y = 4 - x$

4. $y = -\sqrt{x} + 3$

5. $y = -x^2 + 2x - 1$

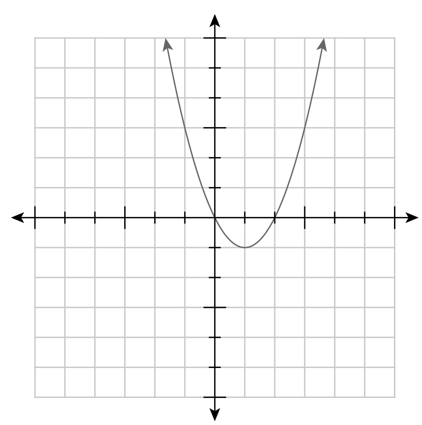
6. $y = |x + 2|$

Answers

1.

x	−1	0	1	2	3
y	3	0	−1	0	3

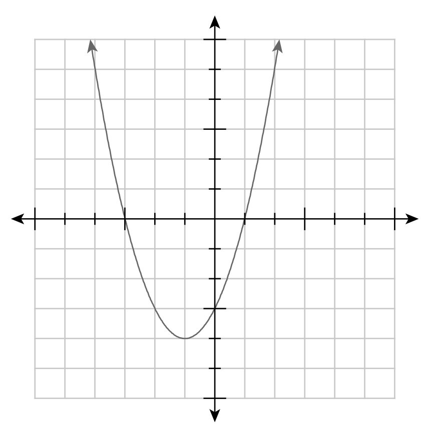
parabola; intercepts (0, 0), (2, 0); decreasing until $x = 1$, then increasing; minimum value at (1, −1); $x = 1$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to −1.



2.

x	−3	−2	−1	0	1	2
y	0	−3	−4	−3	0	5

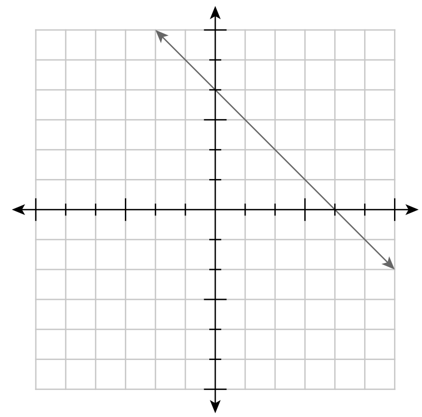
parabola; intercepts (−3, 0), (1, 0), and (0, −3); decreasing until $x = -1$, then increasing; minimum value at (−1, −4); $x = -1$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to −4.



3.

x	–2	–1	0	1	2	3	4
y	6	5	4	3	2	1	0

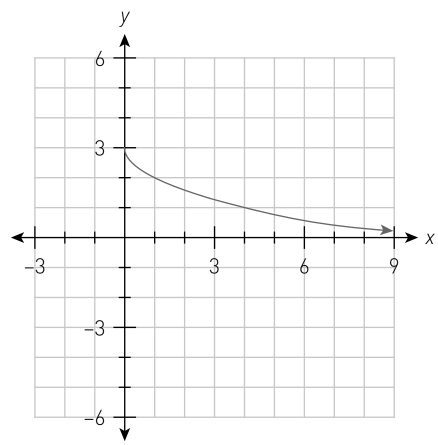
line; intercepts (0, 4) and (4, 0); always decreasing; The inputs and outputs are all real numbers.



4.

x	0	1	2	3	4
y	3	2	≈ 1.59	≈ 1.27	1

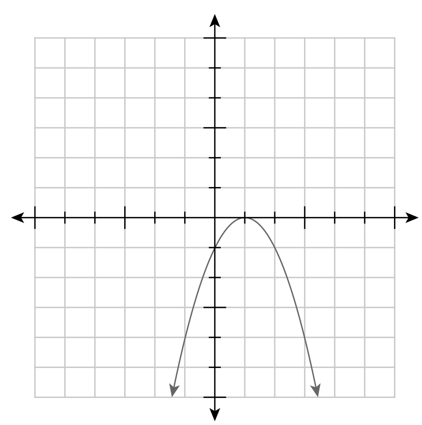
half-parabola on its side; starting point, intercept, and maximum point (0, 3); decreasing for $x > 0$; Inputs can be any number greater than or equal to 0. Outputs are less than or equal to 3.



5.

x	–1	0	1	2	3
y	–4	–1	0	–1	–4

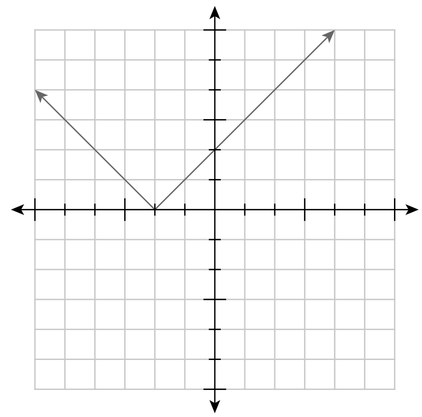
parabola; intercepts (1, 0) and (0, –1); increasing for $x < 1$, decreasing for $x > 1$; maximum value at (1, 0); $x = 1$ is a line of symmetry. Inputs can be any real number. Outputs are less than or equal to 0.



6.

x	–3	–2	–1	0	1
y	1	0	1	2	3

V-shape; intercepts (–2, 0) and (0, 2); decreasing for $x < -2$, increasing for $x > -2$; minimum value at (–2, 0); $x = -2$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to 0.



Functions

GUIDED SKILL SUPPORT

Overview

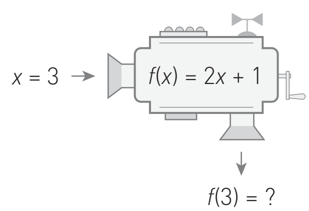
A relationship between input and output values is called a **function** if, for each input value, there is only one corresponding output value.

The set of all possible inputs of a relation is called the **domain**, while the set of all possible outputs of a relation is called the **range**.

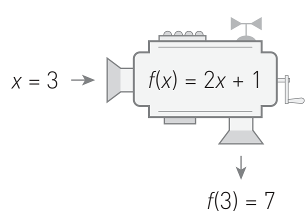
For additional information about functions, function notation, and domain and range, see the Methods & Meanings box in Lesson 1.3.3.

Examples

1. Numbers, represented by a letter or symbol such as x, are input into the function machine labeled f one at a time, and then the function performs the operation on each input to determine each output, $f(x)$. What is $f(3)$?



When $x = 3$ is input into the function f, the function machine multiplies 3 by 2 and adds 1 to get the output, 7. The notation $f(3) = 7$ shows that the function named f connects the input $x = 3$ with the output 7. This also means the point (3, 7) lies on the graph of the function.



2. a. If $f(x) = \sqrt{x-2}$, then $f(11) = ?$

$f(11) = \sqrt{11-2} = \sqrt{9} = 3$

b. If $g(x) = 3 - x^2$, then $g(5) = ?$

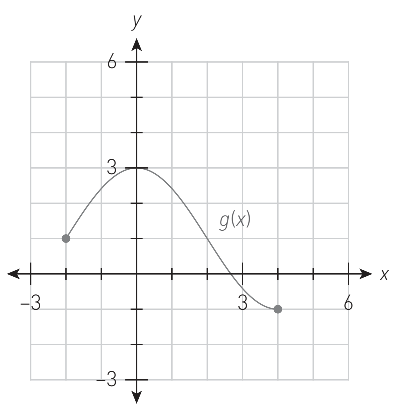
$g(5) = 3 - (5)^2 = 3 - 25 = -22$

c. If $f(x) = \frac{x+3}{2x-5}$, then $f(2) = ?$

$f(2) = \frac{2+3}{2(2)-5} = \frac{5}{-1} = -5$

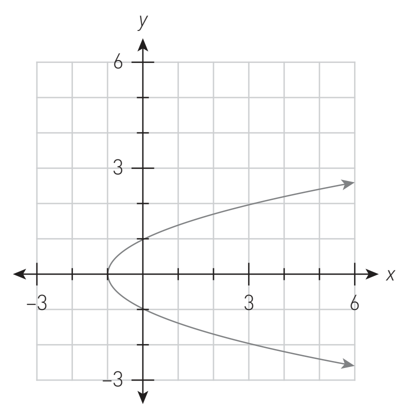
3. A relation in which each input has only one output is called a **function**.

a. Is $g(x)$ a function?



$g(x)$ is a function because each input (x) has only one output (y).

b. Is the graph shown a function?



It is not a function because each input greater than −1 has two outputs. For example, at $x = 0$, $y = 1$ and −1, and at $x = 3$, $y = 2$ and −2.

4. The set of all possible inputs of a relation is called the **domain**, while the set of all possible outputs of a relation is called the **range**.

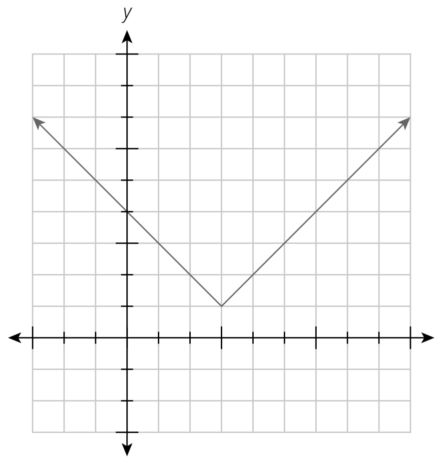
a. What are the domain and range of $g(x)$ from part (a) of Example 3?

The domain of $g(x)$ is $-2 \le x \le 4$. The range of $g(x)$ is $-1 \le y \le 3$.

b. What are the domain and range of the graph from part (b) of Example 3?

The domain is $x \ge -1$. The range is all real numbers.

5. What are the domain and range of the graph shown?

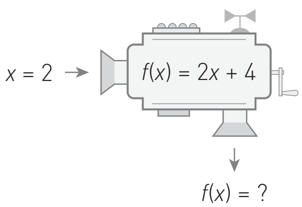


Since the x-values extend forever in both directions, the domain is all real numbers. The y-values start at 1 and go higher, so the range is $y \geq 1$ or "all numbers greater than or equal to 1."

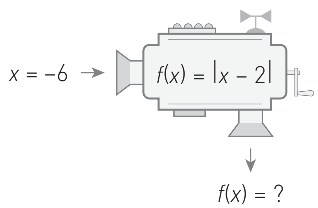
Practice

For problems 1 through 9, determine the outputs for the given inputs for each function, if possible.

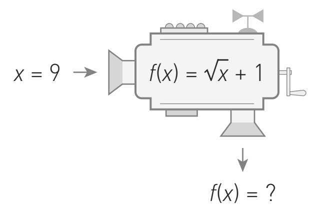
1.



2.



3.



4. $f(x) = (5 - x)^2$
$f(8) = ?$

5. $g(x) = x^2 - 5$
$g(-3) = ?$

6. $f(x) = \frac{2x + 7}{x^2 - 9}$
$f(3) = ?$

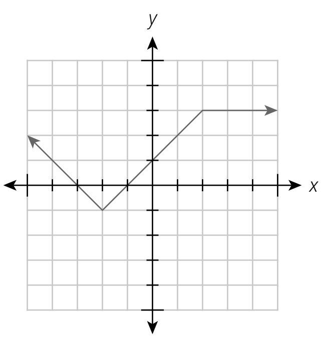
7. $h(x) = 5 - \sqrt{x}$
$h(9) = ?$

8. $h(x) = \sqrt{5 - x}$
$h(9) = ?$

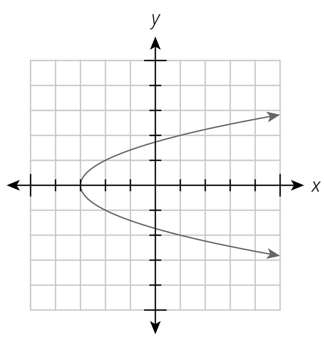
9. $f(x) = -x^2$
$f(4) = ?$

For problems 10 through 15, determine whether each relation is a function. Then state its domain and range.

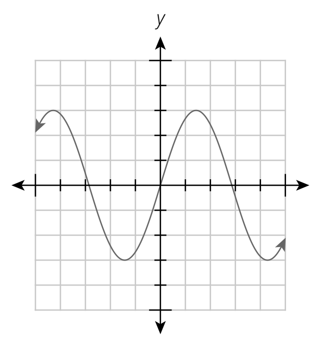
10.



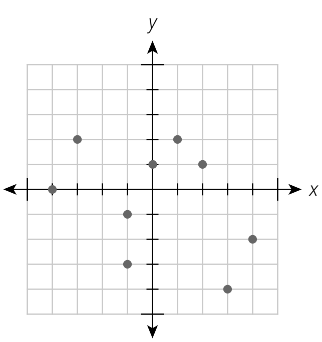
11.



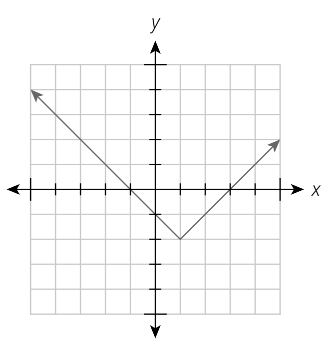
12.



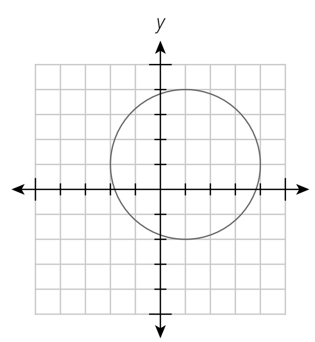
13.



14.



15.



Answers

1. $f(2) = 8$

2. $f(-6) = 8$

3. $f(9) = 4$

4. $f(8) = 9$

5. $g(-3) = 4$

6. not possible

7. $h(9) = 2$

8. not possible

9. $f(4) = -16$

10. function; domain is all real numbers, range is $-1 \leq y \leq 3$

11. not a function; domain is $x \geq -3$, range is all real numbers

12. function; domain is all real numbers, range is $-3 \leq y \leq 3$

13. not a function; domain is –4, –3, –1, 0, 1, 2, 3, 4; range is –4, –3, –2, –1, 0, 1, 2

14. function; domain is all real numbers, range is $y \geq -2$

15. not a function; domain is $-2 \leq x \leq 4$, range is $-2 \leq y \leq 4$