Slope – A Measure of Steepness

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Overview

The equation $y = mx + b$ can be used to graph lines and describe linear patterns. When the equation of a line is written in slope–intercept form, $y = mx + b$, the coefficient m represents the slope of the line. Slope indicates the direction of the line and its steepness. Steeper lines have a value of m with a larger absolute value, or in other words, $m > 1$ or $m < -1$. When $m = 1$, as in $y = x$, the line goes upward by one unit each time it goes over one unit to the right. Flatter lines have an m-value between -1 and 1, often in the form of a fraction. The constant b represents the y-intercept, written $(0, b)$, and indicates where the line crosses the y-axis.

A *slope triangle* can be drawn between any two points on a line to determine the slope of the line. The slope of a line is constant, so the slope ratio is the same for any two points on the line.

Using points where grid lines cross is convenient. To determine the slope, count the vertical distance (Δy) and the horizontal distance (Δx) on the dashed sides of the slope triangle. Write the distances in a ratio: slope $= m = \frac{\Delta y}{\Delta x}$. The symbol Δ means "change in." The order in the fraction is important: the numerator (the top of the fraction) must be the vertical distance and the denominator (the bottom of the fraction) must be the horizontal distance. If $\Delta y = 0$, then the line is horizontal and has a slope of zero, or in other words, $m = 0$. If $\Delta x = 0$, then the line is vertical and its slope is undefined.

Additional information about the slope of a line can be found in the Lesson 2.1.4 Methods & Meanings box.

Examples

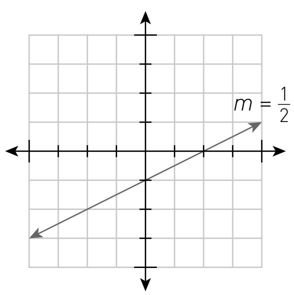
1. Identify the slope in each equation. Then, state whether the graph of the line is also any of the following:

- steeper or flatter than $y = x$ or $y = -x$;
- increasing or decreasing from left to right; or
- horizontal or vertical.

$y = \frac{1}{2}x - 1$ $\qquad$ $y = -\frac{1}{2}x - 1$ $\qquad$ $y = 0x + 3$ or $y = 3$

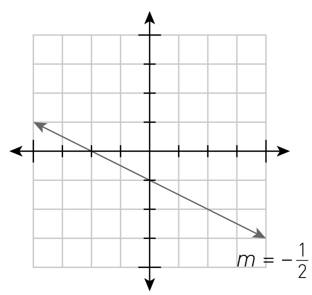
If m, the slope, is positive, the line goes upward from left to right. If m is negative, the line goes downward from left to right. If $m = 0$, the line is horizontal. In the examples:

- the slope of $y = \frac{1}{2}x - 1$ is flatter than $y = x$ and the line goes upward;
- the slope of $y = -\frac{1}{2}x - 1$ is flatter than $y = -x$ and the line goes downward; and
- the slope of $y = 0x + 3$ or $y = 3$ is zero and the line is horizontal.



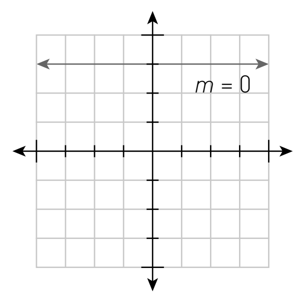
POSITIVE SLOPE

$y = \frac{1}{2}x - 1$



NEGATIVE SLOPE

$y = -\frac{1}{2}x - 1$



ZERO SLOPE

$y = 0x + 3$ or $y = 3$

2. What are the slope and y-intercept of each line? Describe whether the slope is steeper or flatter than $y = x$ or $y = -x$, and whether the line goes upward or downward.

$y = x + 2$ $\qquad$ $y = -3x + 2$ $\qquad$ $3y - x = 6$

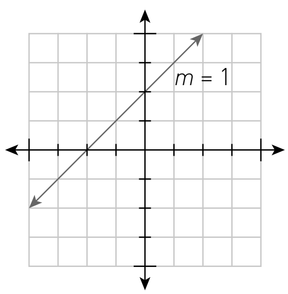
In all three examples, the y-intercept is (0, 2), $b = 2$. However, each of the slopes, m, are different.

The equations $y = x + 2$ and $y = -3x + 2$ are in slope–intercept form and can be analyzed in that form. To rewrite the equation $3y - x = 6$ in slope–intercept form, add x to both sides of the equation and then divide both sides of the equation by 3.

$$3y - x = 6$$
$$3y = 6 + x$$
$$y = 2 + \tfrac{1}{3}x$$

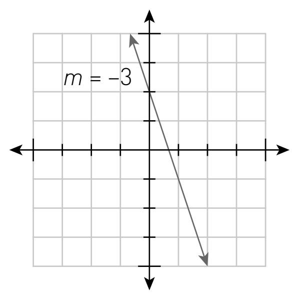
Descriptions for the slope of each line might include the following.

- The slope of the line $y = x + 2$ is 1. It has the same steepness and is going upward.
- The slope of the line $y = -3x + 2$ is -3. It is steeper and going downward.
- The slope of the line $3y - x = 6$ is $\frac{1}{3}$. It is flatter and going upward.



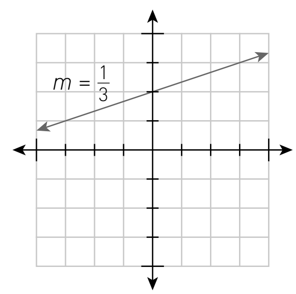
POSITIVE SLOPE

$y = x + 2$



NEGATIVE SLOPE

$y = -3x + 2$

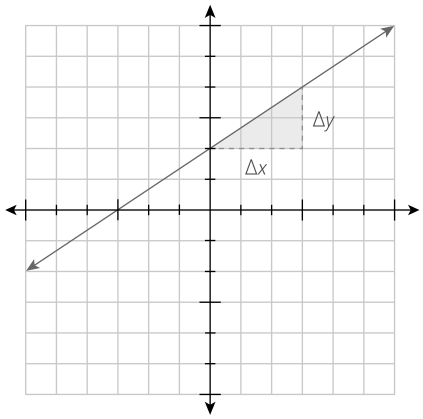


POSITIVE SLOPE

$3y - x = 6$ or $y = \frac{1}{3}x + 2$

3. What is the slope of each line? Use the slope triangles to justify your answers.

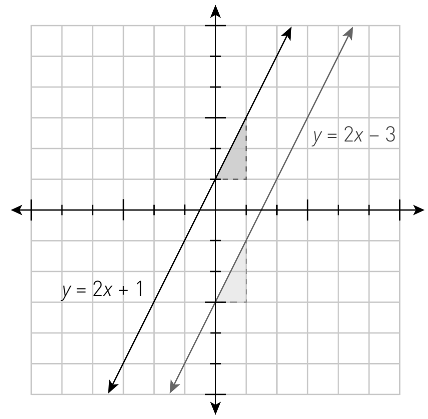
a.



Count the vertical distance (Δy) and the horizontal distance (Δx) on the dashed sides of the slope triangle.

The slope ratio is $\frac{\Delta y}{\Delta x} = \frac{2}{3}$. From left to right, the line goes up and to the right, so the slope is positive, $m = \frac{2}{3}$.

b.



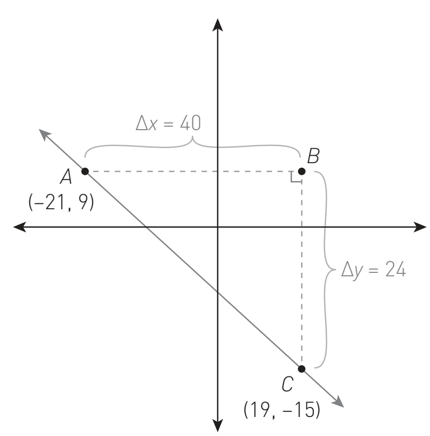
Count the vertical distance (Δy) and the horizontal distance (Δx) on the dashed sides of the slope triangle.

Both lines have the same slope, $\frac{\Delta y}{\Delta x} = \frac{2}{1}$, and direction, which means the lines are parallel. From left-to-right, the lines go up and to the right, so the slope is positive, $m = 2$.

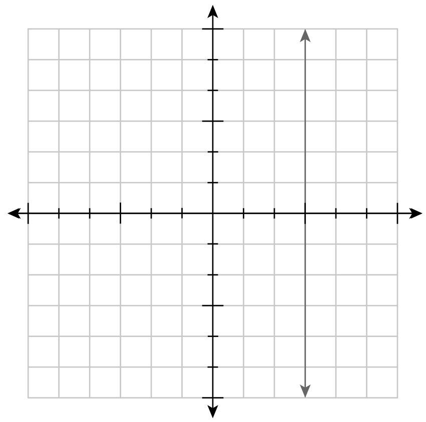
4. What is the slope of a line that passes through the points (–21, 9) and (19, –15)? Use a slope triangle to justify your answer.

Graph the points on unscaled axes by approximating where they are located, and then draw a slope triangle. Next, calculate and label the distance along the vertical side: Δy is 24. Then determine and label the horizontal distance: Δx is 40.

This slope is negative because the line goes downward from left to right, so $\frac{\Delta y}{\Delta x} = -\frac{24}{40}$. The slope is $-\frac{3}{5}$.



5. What is the slope of the line?



The slope of vertical lines is undefined. Consider the equation of the vertical line, which in this case is $x = 3$. Every point on the line has an x-coordinate of 3. Three points on the line include (3, −8), (3, 5), and (3, 14.8). The slope of the line is $\frac{\Delta y}{\Delta x} = \frac{-8-5}{3-3}$. For any two points on the line, the slope is $\frac{\Delta y}{0}$. Since division by zero is undefined, the slope is undefined.

6. What is the slope of a line that passes through points (−12, 53) and (6, −28)? Determine the slope without graphing.

Any two points on a line can be used to determine the slope of the line. It is not necessary to draw a graph to determine the slope of the line. Imagine the slope triangle and use the algebraic process of writing a slope ratio as shown. The vertical change is the difference between the y-values, and the horizontal change is the difference between the x-values.

Δy

(−12, 53) (6, −28)

Δx

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{53-(-28)}{-12-6}$$

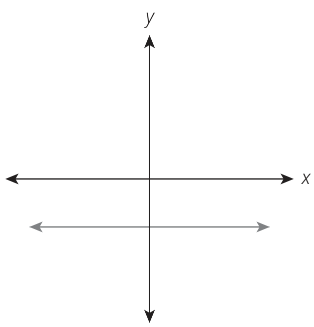
$$m = \frac{81}{-18}$$

$$m = -\frac{9}{2}$$

Practice

Is the slope of each line negative, positive, or zero?

1.



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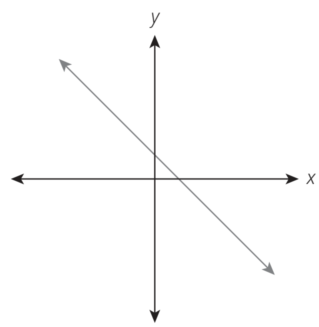

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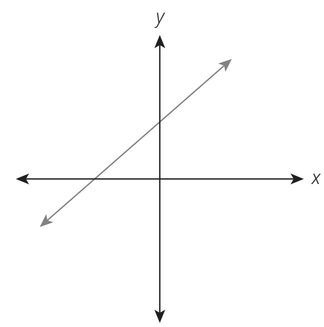


Identify the slope in each equation. Then, describe the slope using terms from the list or similar terms.

- steeper or flatter than $y = x$ or $y = -x$
- going up or down from left to right
- horizontal or vertical

4. $y = 3x + 2$

5. $y = -\frac{1}{2}x + 4$

6. $y = \frac{1}{3}x - 4$

7. $4x - 3 = y$

8. $y = 2$

9. $x = 5$

What is the slope of each line? Use a slope triangle to support your answer.

10. $y = -2 + \frac{1}{2}x$

11. $10 + 4y = -16x$

12. $6x + 3y = 8$

Without graphing, determine the slope of each line based on the given information.

13. $\Delta y = 27$, $\Delta x = -8$

14. $\Delta x = 15$, $\Delta y = 3$

15. $\Delta y = 7$, $\Delta x = 0$

16. horizontal $\Delta = 6$, vertical $\Delta = 0$

17. between (5, 28) and (64, 12)

18. between (−3, 2) and (5, −7)

Answers

1. zero

2. negative

3. positive

4. $m = 3$; It is steeper and goes upward.

5. $m = -\frac{1}{2}$; It is flatter and goes downward.

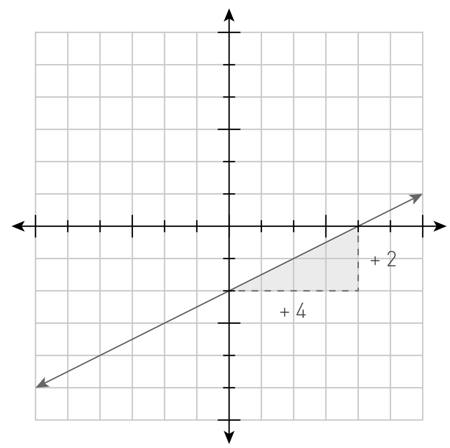
6. $m = \frac{1}{3}$; It is flatter and goes upward.

7. $m = 4$; It is steeper and goes upward.

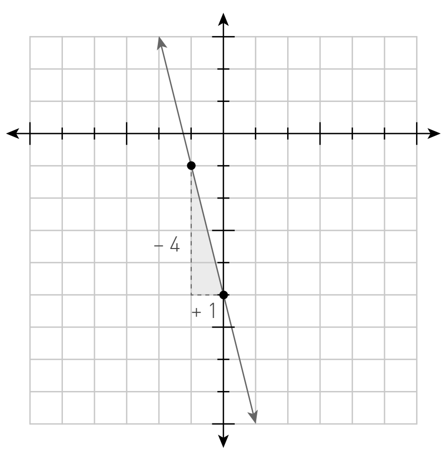
8. $m = 0$; It is flatter and horizontal.

9. $m =$ undefined; It is steeper and vertical.

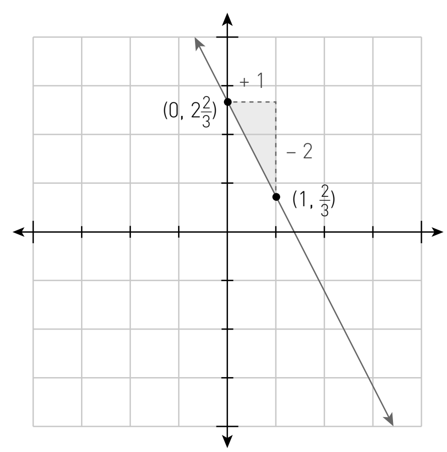
10. $m = \frac{1}{2}$



11. $m = -4$; It is steeper and goes downward.



12. $m = -2$; It is steeper and goes downward.



13. $m = -\frac{27}{8}$

14. $m = \frac{3}{15}$ or $\frac{1}{5}$

15. $m =$ undefined

16. $m = 0$

17. $m = -\frac{16}{59}$

18. $m = -\frac{9}{8}$

Writing the Equation of a Line Given the Slope and a Point on the Line

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Overview

Equations written in slope–intercept form, $y = mx + b$, are useful to identify b as the y-intercept of the line, the m as the slope. The variables x and y represent the coordinates of any point on the line (x, y). Each line has a unique value for m and a unique value for b, but there are infinite (x, y) points for each linear equation. The slope of the line is the same between any two points on that line. This information can be used to write equations without creating tables or graphs.

Additional information about how to write an equation can be found in the Lesson 3.3.3 Methods & Meanings box.

Examples

For problems 1 and 2, write the equation for the line that fits each description.

1. The line with a slope of 2 that passes through the point (10, 17).

Identify how the given information can be used to write an equation in slope–intercept form. In this example, 2 is the slope, m, and the point (10, 17) can be substituted for x and y. The y-intercept, b, is unknown. Substitute the known values, then solve for the y-intercept.

$y = 17 \quad m = 2 \quad x = 10$

$y = mx + b$	Write the general equation of a line.
$17 = 2(10) + b$	Substitute the known values: m, x, and y.
$17 = 20 + b$	Solve for b.
$-3 = b$	

$m = 2$

$y = mx + b$	Write the equation using the values $m = 2$ and $b = -3$.
$y = 2x + (-3)$	
$y = 2x - 3$	

2. What is the equation of the line parallel to $y = 3x - 4$ that goes through the point (2, 8)?

This algebraic method helps with writing equations of parallel lines. Parallel lines never intersect or meet. They have the same slope, m, but *different* y-intercepts, b. Use the slope of the given equation and the point (x, y) to determine the value of b.

$y = 8$ $m = 3$ $x = 2$

$y = mx + b$	Write the general equation of a line.
$8 = 3(2) + b$	Substitute the known values: m, x, and y.
$8 = 6 + b$	Solve for b.
$2 = b$	

$m = 3$

$y = mx + b$	Write the equation using the values $m = 3$ and $b = 2$.
$y = 3x + 2$	

Practice

Write the equation of the line with the given slope that passes through the given point.

1. slope = 5, (3, 13)

2. slope = $-\frac{5}{3}$, (3, −1)

3. slope = −4, (−2, 9)

4. slope = $\frac{3}{2}$, (6, 8)

5. slope = 3, (−7, −23)

6. slope = 2, $(\frac{5}{2}, -2)$

Write the equation of the line *parallel* to the given line that goes through the given point.

7. $y = \frac{3}{5}x + 2$; (0, 0)

8. $y = 4x - 1$; (−2, −6)

9. $y = -2x + 5$; (−4, −2)

10. $y = 4x + 5$; (−6, −28)

11. $y = \frac{1}{3}x - 1$; (6, 9)

12. $y = 3x + 8$; $(0, \frac{1}{2})$

Answers

1. $y = 5x - 2$	**2.** $y = -\frac{5}{3}x + 4$	**3.** $y = -4x + 1$
4. $y = \frac{3}{2}x - 1$	**5.** $y = 3x - 2$	**6.** $y = 2x - 7$
7. $y = \frac{3}{5}x$	**8.** $y = 4x + 2$	**9.** $y = -2x - 10$
10. $y = 4x - 4$	**11.** $y = \frac{1}{3}x + 7$	**12.** $y = 3x + \frac{1}{2}$

Writing the Equation of a Line Given Two Points on the Line

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Overview

The equation of a line in $y = mx + b$ form requires a slope, m, and a y-intercept, b. The equation of a line from two points can be written by creating a slope triangle and calculating $\frac{\Delta y}{\Delta x}$. The slope ratio compares the vertical change, Δy, to the horizontal change, Δx. Sometimes it is easy to identify the y-intercept when the points are plotted on a coordinate plane. After the slope, m, and y-intercept, b, are known, the values can be substituted into the equation $y = mx + b$.

Sometimes, using a graph is not convenient. Writing an equation without a graph uses the same steps as previously described but without using a coordinate grid. The equation from a line given two points can be written by first determining the slope. Calculate $\frac{\Delta y}{\Delta x}$ by subtracting the y-values and then the x-values.

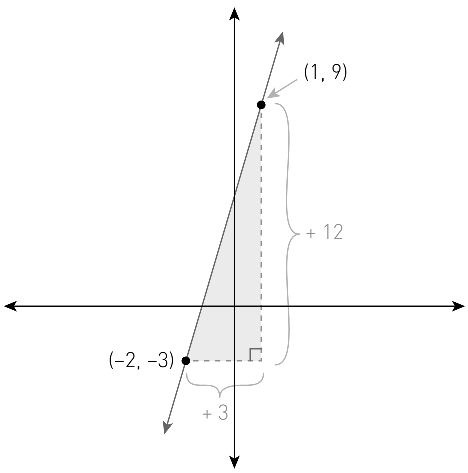
Next, determine the y-intercept. Substitute the value of the slope and use one point on the line to also substitute values for x and y, and solve for b. After the slope, m, and y-intercept, b, are known, these values can be substituted into the equation $y = mx + b$.

Additional information about how to write an equation can be found in the Lesson 3.3.3 Methods & Meanings box. Additional examples and more practice can be found in the Checkpoint 5B materials.

Examples

1. Write the equation of the line that passes through the points (1, 9) and (–2, –3) using slope–intercept form.

First, determine the slope. Position the two points approximately where they belong on a coordinate plane. Sketch a line through the two points and draw a generic slope triangle using the given points. Then, calculate the slope by first determining the change in *y*, and then the change in *x* using the values of the two points.



Δy

(–2, –3)   (1, 9)

Δx

$$m = \frac{\Delta y}{\Delta x}$$
$$m = \frac{9-(-3)}{1-(-2)}$$
$$m = \frac{12}{3}$$
$$m = 4$$

Using slope–intercept form, $y = mx + b$, substitute m and either one of the given points into the equation. In this example, the point (1, 9) is used. Solve for b, the y-intercept.

$m = 4$ $(1, 9)$

$$y = mx + b$$
$$9 = 4(1) + b$$
$$9 = 4 + b$$
$$5 = b$$

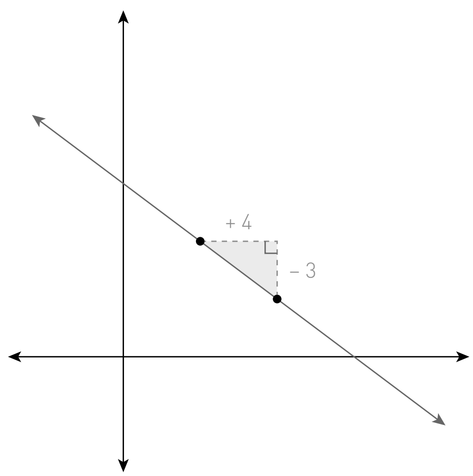
Write the equation in slope–intercept form by only substituting in the values for m and b.

$m = 4$ $b = 5$

$$y = mx + b$$
$$y = 4x + 5$$

2. Write the equation of the line that passes through the points (8, 3) and (4, 6) using slope–intercept form.

First, determine the slope. Position the two points approximately where they belong on a coordinate plane. Sketch a line through the two points and draw a generic slope triangle using the given points. Then, calculate the slope, m, $\frac{\Delta y}{\Delta x}$. The slope is negative since the line goes down from left to right.



$$(4,\ 6) \quad (8,\ 3)$$

Δy

Δx

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{3-6}{8-4}$$

$$m = \frac{-3}{4}$$

Using slope–intercept form, $y = mx + b$, substitute m and either one of the given points into the equation. In this example, the point (8, 3) is used. Solve for b, the y-intercept.

$$m = -\frac{3}{4} \qquad (8, 3)$$

$$y = mx + b$$

$$3 = \left(-\frac{3}{4}\right)(8) + b$$

$$3 = -\frac{24}{4} + b$$

$$3 = -6 + b$$

$$9 = b$$

Write the equation in slope–intercept form by only substituting in the values for m and b.

$$m = -\frac{3}{4} \qquad b = 9$$

$$y = mx + b$$

$$y = \left(-\frac{3}{4}\right)x + 9$$

Practice

For each pair of points, write the equation of the line that contains them. Use slope–intercept form.

1. (1, 1) and (0, 4)

2. (5, 4) and (1, 1)

3. (1, 3) and (–5, –15)

4. (–2, 3) and (3, 5)

5. (2, –1) and (3, –3)

6. (4, 5) and (–2, –4)

7. (1, –4) and (–2, 5)

8. (–3, –2) and (5, –2)

9. (–4, 1) and (5, –2)

Answers

1. $y = -3x + 4$

2. $y = \frac{3}{4}x + \frac{1}{4}$

3. $y = 3x$

4. $y = \frac{2}{5}x + 3\frac{4}{5}$

5. $y = -2x + 3$

6. $y = \frac{3}{2}x - 1$

7. $y = -3x - 1$

8. $y = -2$

9. $y = -\frac{1}{3}x - \frac{1}{3}$

