Introduction to Sequences

GUIDED SKILL SUPPORT

Overview

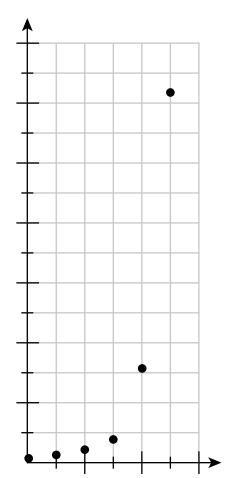
In Lessons 5.1.1 through 5.1.4, students are introduced to patterns that have a common multiplier between terms. They investigate these situations using tables and graphs. These situations can be modeled using exponential functions. If the function is increasing, it represents exponential growth. If the function is decreasing, it represents exponential decay. The graphs of these functions are curved and have a horizontal asymptote (a line that the graph of a curve approaches).

Examples

1. Three friends decide to pay it forward by sharing appreciation for others in their school. They will each put a positive message on a sticky note and place it on the locker of a different person. Each person who receives a sticky note will do the same for four more people. Create a table and graph to model the number of sticky notes given each round. Assume that everyone maintains the same schedule, so that all sticky notes in a round are placed on lockers simultaneously. Does this situation involve exponential growth or decay? During which round will over 10,000 sticky notes be distributed?

If three friends (round 0) each give out four sticky notes, twelve sticky notes will be distributed (round 1). If those twelve friends each give out four sticky notes, 48 sticky notes will be distributed (round 2). The pattern is multiplication by 4. This can be modeled using the following table and graph.

Round	Number of Sticky Notes Distributed
0	3
1	12
2	48
3	192
4	768
5	3,072



This situation involves exponential growth because the number of sticky notes is increasing as the round number increases. Round 6 would have 12,288 sticky notes distributed, which exceeds 10,000 sticky notes in one round.

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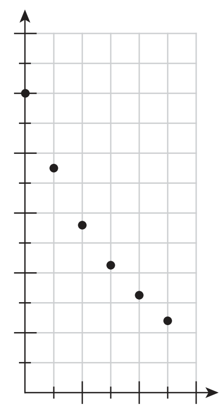
2. Bryson inherited $1,000,000. He wants to make it last as long as possible, so he decides to spend only 25% of the remaining amount each year. Create a table and graph to model the amount of money Bryson has remaining after each year. Does this situation involve exponential growth or decay? When will Bryson have less than $1,000?

Bryson will keep 75% of the remaining amount each year, so 0.75 is the multiplier to determine the amount that remains after each year.

This situation represents exponential decay because the amount of money decreases as time increases.

Bryson will have less than $1,000 after 25 years.

Year	Money ($)
0	1,000,000
1	750,000
2	562,500
3	421,875
4	316,406
5	237,304



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Practice

For problems 1 through 3, complete each table assuming that the pattern shown continues. What is the multiplier? Then decide whether the table represents exponential growth or decay.

1.

x	y
5	
6	
7	
8	1.9
9	6.65
10	23.275
11	

2.

x	y
−2	
−1	1.481
0	8
1	43.2
2	
3	
4	

3.

x	y
0	5,200
1	
2	52
3	5.2
4	
5	
6	

4. A non-native species of fish is taking over a lake in Florida. Each pair of fish creates 30 new fish each year. Scientists predict that there are currently 50 non-native fish in the lake. How long will it take for the non-native fish population to exceed 1,000,000? Create a table and graph to model this situation. Then decide whether the situation represents exponential growth or decay.

5. Researchers were testing out a new antibiotic on a certain bacterium. They started with 45,000 bacteria. After one treatment with antibiotics, 40,500 bacteria were present. After a second treatment, 36,450 bacteria were present. How many treatments of antibiotics will it take to kill at least half of the bacteria? Create a table and graph to model this situation. Then decide whether the situation represents exponential growth or decay.

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Answers

1.

x	y
5	0.0443
6	0.1551
7	0.5429
8	1.9
9	6.65
10	23.275
11	81.463

multiplier: 3.5, growth

2.

x	y
–2	0.2743
–1	1.481
0	8
1	43.2
2	233.28
3	1,259.71
4	6,802.445

multiplier: 5.4, growth

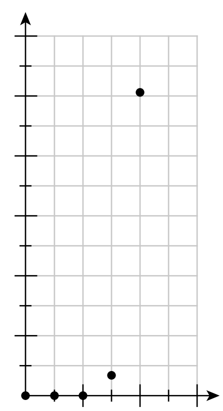
3.

x	y
0	5,200
1	520
2	52
3	5.2
4	0.52
5	0.052
6	0.0052

multiplier: 0.1, decay

4.

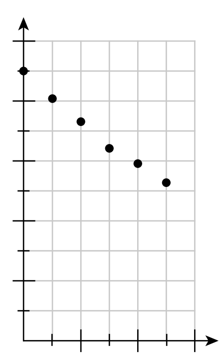
Year	Number of Fish
0	50
1	750
2	11,250
3	168,750
4	2,531,250
5	37,968,750



The non-native fish population is expected to exceed 1,000,000 between 3 and 4 years.

5.

Round	Number of Bacteria
0	45,000
1	40,500
2	36,450
3	32,805
4	29,525
5	26,572



It will take 7 rounds of antibiotics to reduce the number of bacteria to less than half the original 45,000 bacteria.

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Functions for Sequences

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Overview

In these lessons, students learn multiple representations for sequences: as a list of numbers, as a table, as a graph, and as a function. The terms of an arithmetic sequence are generated by adding a constant value to the previous term. The terms of a geometric sequence are generated by multiplying a constant value by the previous term.

Functions for sequences can also be written recursively. An explicit equation tells exactly how to calculate any specific term in a sequence. A recursive equation names the first term (or any other term) and how to get from one term to the next. For an explanation of sequences, see the Methods & Meanings box in Lesson 5.3.2.

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Examples

1. Consider the two sequences:

SEQUENCE A	SEQUENCE B
−8, −5, −2, 1, ...	256, 128, 64, ...

a. For each sequence, is it arithmetic, geometric, or neither? How can you tell? Explain completely.

To determine the type of sequence for A and B, look at the pattern of growth for each sequence. Sequence A is generated by adding 3 to each term to get the next term. When each term has a **common difference** (in this case, "+3"), the sequence is **arithmetic**.

$$-8 \xrightarrow{+3} -5 \xrightarrow{+3} -2 \xrightarrow{+3} 1$$

Sequence B, however, is different. The terms do not have a common difference. Instead, these terms have a **multiplier**. A sequence with a common ratio is a **geometric sequence**.

$$256 \xrightarrow{-128} 128 \xrightarrow{-64} 64 \qquad 256 \xrightarrow{\times \frac{1}{2}} 128 \xrightarrow{\times \frac{1}{2}} 64$$

b. What are the "zeroth term" and the generator for each sequence?

The first term for sequence A is −8, and it has a generator or common difference of +3. Therefore, the zeroth term is −11 (because −11 + 3 = −8).

For sequence B, the first term is 256 with a generator or common ratio of $\frac{1}{2}$.

Therefore, the zeroth term is 512, because $512 \cdot \frac{1}{2} = 256$.

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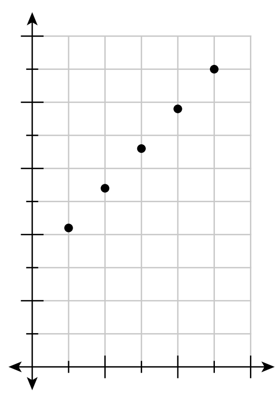
2. Peachy Orchard Developers are preparing land to create a large subdivision of single-family homes. They have already built 15 houses on the site. Peachy Orchard plans to build six new homes every month. Write a sequence for the number of houses built, then write a function for the sequence. Fully describe a graph of this sequence.

The sequence is 21, 27, 33, 39, ... Note that all sequences begin with the first term, in this case, it is when the number of months $n = 1$.

The common difference is 6, and the zeroth term is 15. The function can be written $f(n) = mn + b = 6n + 15$. Note that for a sequence, $f(n) =$ is used instead of $y =$. $f(n) =$ indicates the function is for a discrete sequence (separate points), as opposed to a continuous function (connected points). Students compare sequences to functions in Lesson 5.3.3.

The graph of the sequence is shown. There are no intercepts. There is no point at (0, 15) because sequences are written starting with the first term, where $n = 1$. The domain consists of whole numbers greater than or equal to one. The range consists of the output values of the points that result from the function $f(n) = 6n + 15$ when $n \geq 1$. There are no asymptotes. The graph is linear. This graph is discrete.



3. List the first five terms of the arithmetic sequence.

$$f(n) = 5n + 2$$

$f(1) = 5(1) + 2 = 7$
$f(2) = 5(2) + 2 = 12$
$f(3) = 5(3) + 2 = 17$
$f(4) = 5(4) + 2 = 22$
$f(5) = 5(5) + 2 = 27$

The sequence is: 7, 12, 17, 22, 27, ...

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4. List the first five terms of the arithmetic sequence.

$$f(1) = 3$$
$$f(n + 1) = f(n) - 5$$

$f(1 + 1)$ or $f(2) = f(1) - 5 = 3 - 5 = -2$
$f(2 + 1)$ or $f(3) = f(2) - 5 = 2 - 5 = -7$
$f(3 + 1)$ or $f(4) = f(3) - 5 = -7 - 5 = -12$
$f(4 + 1)$ or $f(5) = f(4) - 5 = -12 - 5 = -17$

$f(1) = 3$ means that the first term is 3. If the term you are on is $f(n)$, then the next term is $f(n + 1)$. For example, if $n = 1$, $f(n)$ is $f(1)$, or the first term. Then $f(n + 1)$ is $f(1 + 1)$ or $f(2)$, the second term.

The sequence is: 3, −2, −7, −12, −17, ...

5. Write an explicit and a recursive equation for the sequence: −2, 1, 4, 7, ...

The sequence increases by 3 for each term. Therefore, this is an arithmetic sequence.

Explicit: $f(n) = mn + b$; $m = 3$, $b = -5$, so the function is: $f(n) = 3n - 5$

Recursive: $f(1) = -2$, $f(n + 1) = f(n) + 3$

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Practice

List the first five terms of each sequence, then state whether the sequence is arithmetic, geometric, or neither.

1. $f(n) = 5n + 2$

2. $s(n) = 3 - 8n$

3. $u(n) = 9n - n^2$

4. $f(n) = (-4)^n$

5. $s(n) = \left(\frac{1}{4}\right)^n$

6. $u(n) = n(n + 1)$

7. $f(n) = 8$

8. $s(n) = \frac{3}{4}n + 1$

List the first five terms of each arithmetic sequence. Note: Problems 9 through 12 are given as explicit functions, while problems 13 through 16 are given as recursive functions.

9. $f(n) = 5n - 2$

10. $f(n) = -3n + 5$

11. $f(n) = -15 + \frac{1}{2}n$

12. $f(n) = 5 + 3(n - 1)$

13. $f(1) = 5$, $f(n + 1) = f(n) + 3$

14. $f(1) = 5$, $f(n + 1) = f(n) - 3$

15. $f(1) = -3$, $f(n + 1) = f(n) + 6$

16. $f(1) = \frac{1}{3}$, $f(n + 1) = f(n) + \frac{1}{2}$

Determine the 30th term of each arithmetic sequence.

17. $f(n) = 5n - 2$

18. $f(n) = -15 + \frac{1}{2}n$

19. $f(31) = 53$, $d = 5$

20. $f(29) = 25$, $f(n + 1) = f(n) - 3$

For each arithmetic sequence in problems 21 through 24, write an explicit and a recursive function.

21. 4, 8, 12, 16, 20, ...

22. −2, 5, 12, 19, 26, ...

23. 27, 15, 3, −9, −21, ...

24. $3, 3\frac{1}{3}, 3\frac{2}{3}, 4, 4\frac{1}{3}, \ldots$

25. Graph the following two sequences on the same set of axes.

a. $f(n) = -6n + 20$

b. 1, 4, 16, 64, ...

26. Fully describe the graph of the sequence $f(n) = -4n + 18$.

27. Calculate the missing terms for this arithmetic sequence and a function for it.

__, 15, 11, __, 3

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28. For this sequence, each term is $\frac{1}{5}$ of the previous one. Determine the missing terms.

$$\_\_, \_\_, \frac{2}{3}, \_\_, \_\_$$

29. The 30th term of a sequence is 42. If each term in the sequence is four greater than the previous number, what is the first term?

30. The microscopic length of a crystalline structure grows so that each day it is 1.005 times as long as the previous day. If, on the third day, the structure was 12.5 nm long, write a sequence for how long it was on the first five days. (The unit nm stands for "nanometer," or 1×10^{-9} meters.)

31. Davis loves to ride the mini-cars at the amusement park, but riders must be no more than 125 cm tall. If, on his fourth birthday, he is 94 cm tall, and he grows approximately 5.5 cm per year, at what age will he no longer be able to go on the mini-car ride?

32. Davis has \$5.40 in his bank account on his fourth birthday. If his parents add \$0.40 to his bank account every week, when will he have enough to buy the new Smokin' Derby racecar set, which retails for \$24.99?

Answers

1. 7, 12, 17, 22, 27; arithmetic, common difference is 5

2. –5, –13, –21, –29, –37; arithmetic, common difference is –8

3. 8, 14, 18, 20, 20; neither

4. –4, 16, –64, 256, –1,024; geometric, common ratio is –4

5. $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}, \frac{1}{1{,}024}$; geometric, common ratio is $\frac{1}{4}$

6. 2, 6, 12, 20, 30; neither

7. 8, 8, 8, 8, 8; arithmetic or geometric, common difference is 0, common ratio is 1

8. $\frac{7}{4}, \frac{5}{2}, \frac{13}{4}, 4, \frac{19}{4}$; arithmetic, common difference is $\frac{3}{4}$

9. 3, 8, 13, 18, 23

10. 2, –1, –4, –7, –10

11. $-14\frac{1}{2}, -14, -13\frac{1}{2}, -13, -12\frac{1}{2}$

12. 5, 8, 11, 14, 17

13. 5, 8, 11, 14, 17

14. 5, 2, –1, –4, –7

15. –3, 3, 9, 15, 21

16. $\frac{1}{3}, \frac{5}{6}, 1\frac{1}{3}, 1\frac{5}{6}, 2\frac{1}{3}$

17. 148

18. 0

19. 48

20. 22

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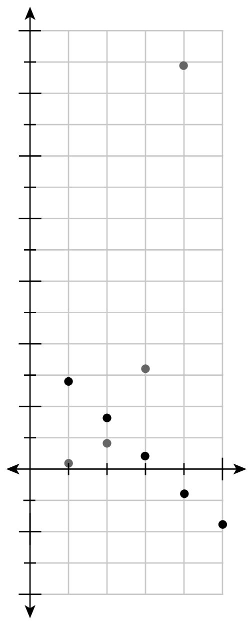
21. explicit: $f(n) = 4n$; recursive: $f(1) = 4$, $f(n + 1) = f(n) + 4$

22. explicit: $f(n) = 7n - 9$; recursive: $f(1) = -2$, $f(n + 1) = f(n) + 7$

23. explicit: $f(n) = -12n + 39$; recursive: $f(1) = 27$, $f(n + 1) = f(n) - 12$

24. explicit: $f(n) = \frac{1}{3}n + 2\frac{2}{3}$; recursive: $f(1) = 3$, $f(n + 1) = f(n) + \frac{1}{3}$

25.



26. This is a function; it represents an arithmetic sequence. The graph is discrete, but the points are linear. The domain is the positive integers: 1, 2, 3, … The range is the sequence itself: 14, 10, 6, 2, −2, … There are no asymptotes.

27. 19 and 8; $f(n) = 23 - 4n$

28. $\frac{50}{3}, \frac{10}{3}, \frac{2}{3}, \frac{2}{15}, \frac{2}{75}$

29. $42 - 29(4) = -74$

30. $\approx 12.38, \approx 12.44, 12.5, \approx 12.56, \approx 12.63, \ldots$

31. $f(n) = 5.5n + 94$, so solve $5.5n + 94 \leq 125$. $n \approx 5.64$. At $\approx 4 + 5.64 = 9.64$, he will be too tall. Davis can continue to ride until he is about $9\frac{1}{2}$ years old.

32. $f(n) = 0.4n + 5.4$, so solve $0.4n + 5.4 \geq 24.99$. $n = 48.975$. In 49 weeks, he will have \$25. If he must also cover tax, he will need another 3 or 4 weeks.

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Patterns of Growth in Tables and Graphs

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Overview

To recognize whether a sequence has a pattern of growth that is linear, exponential, or neither, look at the differences of the $f(n)$-values between successive n-values. If the difference is constant, the sequence exhibits a linear pattern of growth. If the difference is not constant, look at the pattern in the $f(n)$-values. If a constant multiplier can be used to move from one $f(n)$-value to the next, then the sequence has a pattern of growth that is exponential.

Examples

Based on each table, identify the shape of the graph.

1.

x	–3	–2	–1	0	1	2	3
y	–7	–5	–3	–1	1	3	5

x	–3	–2	–1	0	1	2	3
y	–7	–5	–3	–1	1	3	5

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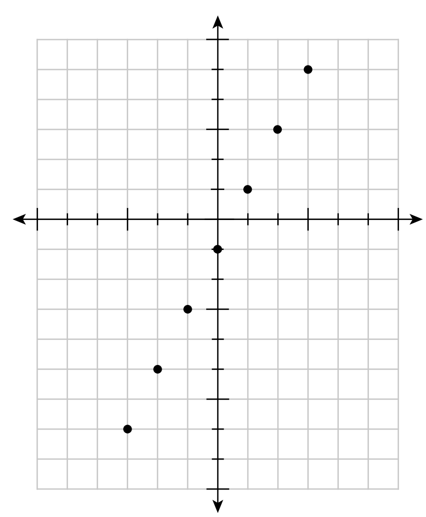
The difference in y-values is always 2, a constant.
The function is linear; the graph shown confirms this.



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2.

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

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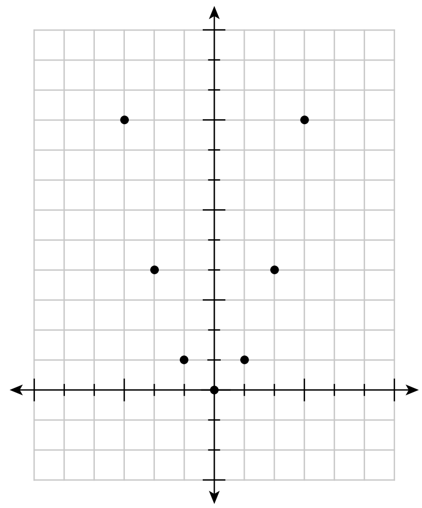
The difference in *y*-values is not constant, and there is not a constant multiplier in moving from one *y*-value to the next. The function is neither linear nor exponential; the graph shown confirms this.



3.

x	−3	−2	−1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

x	−3	−2	−1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

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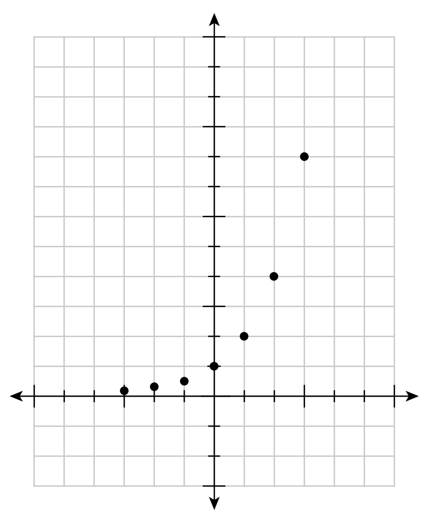
The *y*-values have a constant multiplier of 2. The function is exponential; the graph shown confirms this.



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Practice

Using the tables, identify whether each relationship is linear, exponential, or neither.

1.

x	−3	−2	−1	0	1	2	3
y	14	10	6	2	−2	−6	−10

2.

x	−3	−2	−1	0	1	2	3
y	$\frac{1}{2}$	1	2	4	8	16	32

3.

x	−3	−2	−1	0	1	2	3
y	21	12	5	0	−3	−4	−3

4.

x	−3	−2	−1	0	1	2	3
y	−16	−13	−10	−7	−4	−1	2

5.

x	−3	−2	−1	0	1	2	3
y	−18	−12	−6	0	6	12	18

6.

x	−3	−2	−1	0	1	2	3
y	−14	−9	−4	1	6	11	16

7.

x	−3	−2	−1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

8.

x	−3	−2	−1	0	1	2	3
y	4	8	16	32	64	128	256

9.

x	−3	−2	−1	0	1	2	3
y	11	9	7	5	3	1	−1

10.

x	−3	−2	−1	0	1	2	3
y	30	20	12	6	2	0	0

11.

x	−3	−2	−1	0	1	2	3
y	−27	−9	−3	0	3	9	27

12.

x	−3	−2	−1	0	1	2	3
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81

13.

x	−3	−2	−1	0	1	2	3
y	3	0	−1	0	3	8	15

14.

x	−3	−2	−1	0	1	2	3
y	0	5	8	9	8	5	0

15.

x	−3	−2	−1	0	1	2	3
y	$\frac{9}{8}$	$\frac{9}{4}$	$\frac{9}{2}$	9	18	36	72

16.

x	−3	−2	−1	0	1	2	3
y	1	0	−1	−2	−1	0	1

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Answers

1.	linear	**2.**	exponential	**3.**	neither	**4.**	linear
5.	linear	**6.**	neither	**7.**	exponential	**8.**	exponential
9.	neither	**10.**	linear	**11.**	exponential	**12.**	neither
13.	neither	**14.**	neither	**15.**	exponential	**16.**	neither

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Writing Functions for Geometric Sequences

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Overview

In Lesson 5.3.2, students determine the multiplier in a situation and use it to write the function of a geometric sequence.

In addition to the ways to write explicit functions for sequences, functions for sequences can also be written recursively. An explicit equation tells exactly how to calculate any specific term in the sequence. A recursive equation names the first term (or any other term) and how to get from one term to the next.

For an explanation of recursive sequences and functions for sequences, see the Methods & Meanings box in Lesson 5.3.2.

Examples

1. Determine the multiplier in each situation.

a. A 64% increase

> Any amount you start with is 100%. Since it is an increase, add 100% + 64% = 164%, which is a multiplier of 1.64.

b. A 34% decrease

> Any amount you start with is 100%. Since it is a decrease, we subtract 100% − 34% = 66%, which is a multiplier of 0.66.

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In examples 2 and 3, list the first five terms of each geometric sequence.

2. $f(n) = 3 \cdot 2^n$ (an explicit equation)

$f(1) = 3 \cdot 2^1 = 6$

$f(2) = 3 \cdot 2^2 = 12$

$f(3) = 3 \cdot 2^3 = 24$

$f(4) = 3 \cdot 2^4 = 48$

The sequence is: 6, 12, 24, 48, ...

3. $f(1) = 8$, $f(n+1) = f(n) \cdot \frac{1}{2}$ (a recursive equation)

$f(1) = 8$

$f(2) = f(1) \cdot \frac{1}{2} = 8 \cdot \frac{1}{2} = 4$

$f(3) = f(2) \cdot \frac{1}{2} = 4 \cdot \frac{1}{2} = 2$

$f(4) = f(3) \cdot \frac{1}{2} = 2 \cdot \frac{1}{2} = 1$

$f(5) = f(4) \cdot \frac{1}{2} = 1 \cdot \frac{1}{2} = \frac{1}{2}$

The sequence is: 8, 4, 2, 1, $\frac{1}{2}$, ...

4. Write an explicit and a recursive equation for the geometric sequence: 81, 27, 9, 3, ...

Explicit: $f(1) = 81$, $b = \frac{1}{3}$; $f(n) = a \cdot b^n$; so a (the zeroth term) is calculated by $a = 81 \div \frac{1}{3} = 243$ and the answer is: $f(n) = 243 \cdot \left(\frac{1}{3}\right)^n$.

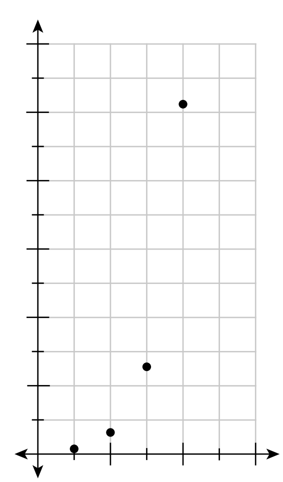
Recursive: $f(1) = 81$, $f(n+1) = f(n) \cdot \frac{1}{3}$

5. Rosa is running for class president. She decided to use word of mouth to campaign. During lunch, she told two friends that she is running for president and asked them to spread the word. Within 10 minutes, the two each told four people. Within the next 10 minutes, each of those eight students told four more people. Rosa knew this would continue until everyone in the entire school was talking about her campaign for president. Write a sequence for the number of people who knew about Rosa's campaign in 10-minute intervals, then write a function for the sequence. Fully describe a graph of this sequence.

Use the general equation for a geometric sequence: $f(n) = ab^n$. The multiplier is $b = 4$, and the zeroth term is $a = 2$. The function can be written $f(n) = ab^n = 2 \cdot 4^n$.

The sequence is: 8, 32, 128, 512, ... Note that the sequence is written starting with $n = 1$.

The graph of the sequence is shown. There are no intercepts. There is no point at (0, 2) because sequences are written starting with the first term, where $n = 1$. The domain consists of integers greater than or equal to 1. The range consists of the y-values of the points that follow the rule $f(n) = 2(4)^n$ when $n \geq 1$. The graph is exponential and is shown. There is no symmetry. This graph is discrete.



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Practice

Determine the multiplier in each of the situations below.

1. A 44% increase

2. A 93% decrease

3. A 32% increase

4. A 60% decrease

List the first five terms of each geometric sequence.

5. $f(n) = 5 \cdot 2^n$

6. $f(n) = -3 \cdot 3^n$

7. $f(n) = 40\left(\frac{1}{2}\right)^{n-1}$

8. $f(n) = 6\left(-\frac{1}{2}\right)^{n-1}$

9. $f(1) = 5$, $f(n + 1) = f(n) \cdot 3$

10. $f(1) = 100$, $f(n + 1) = f(n) \cdot \frac{1}{2}$

11. $f(1) = -3$, $f(n + 1) = f(n) \cdot (-2)$

12. $f(1) = \frac{1}{3}$, $f(n + 1) = f(n) \cdot \frac{1}{2}$

If r is the multiplier, calculate the 15th term of each geometric sequence.

13. $f(14) = 232$, $r = 2$

14. $f(16) = 32$, $r = 2$

15. $f(14) = 9$, $r = \frac{2}{3}$

16. $f(16) = 9$, $r = \frac{2}{3}$

Write an explicit and a recursive function for each geometric sequence.

17. 2, 10, 50, 250, 1,250, ...

18. 16, 4, 1, $\frac{1}{4}$, $\frac{1}{16}$, ...

19. 5, 15, 45, 135, 405, ...

20. 3, −6, 12, −24, 48, ...

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Answers

1. 1.44

2. 0.07

3. 1.32

4. 0.40

5. 10, 20, 40, 80, 160

6. −9, −27, −81, −243, −729

7. 40, 20, 10, 5, $\frac{5}{2}$

8. 6, −3, $\frac{3}{2}$, $-\frac{3}{4}$, $\frac{3}{8}$

9. 5, 15, 45, 135, 405

10. 100, 50, 25, $\frac{25}{2}$, $\frac{25}{4}$

11. −3, 6, −12, 24, −48

12. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{24}$, $\frac{1}{48}$

13. 464

14. 16

15. 6

16. $\frac{27}{2}$

17. explicit: $f(n) = \frac{2}{5} \cdot 5^n$; recursive: $f(1) = 2$, $f(n+1) = f(n) \cdot 5$

18. explicit: $f(n) = 64 \cdot \left(\frac{1}{4}\right)^n$; recursive: $f(1) = 16$, $f(n+1) = f(n) \cdot \frac{1}{4}$

19. explicit: $f(n) = \frac{5}{3} \cdot 3^n$; recursive: $f(1) = 5$, $f(n+1) = f(n) \cdot 3$

20. explicit: $f(n) = -\frac{3}{2} \cdot (-2)^n$; recursive: $f(1) = 3$, $f(n+1) = f(n) \cdot (-2)$

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