Using the Quadratic Formula

GUIDED SKILL SUPPORT

Overview

The **Quadratic Formula** can be used to calculate the roots of a quadratic function, that is, the x-intercepts of the parabola. The Quadratic Formula can be applied to any quadratic equation, whether factorable or not. There may be two roots, one root, or no roots, depending on whether the parabola intersects the x-axis twice, once, or not at all.

The Quadratic Formula is shown. To use the formula, the equation must be written in standard form, $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The ± symbol is read as "plus or minus." It is a shorthand notation that signifies the need to calculate the formula twice, once with + and once with –, to get both possible x-values.

This is necessary to correctly identify the values of a, b, and c. Once the equation is in standard form and equal to 0, a is the coefficient of the x^2-term, b is the coefficient of the x-term, and c is the constant term.

For additional information, see the Methods & Meanings boxes in Lessons 9.1.1 through 9.1.3.

Examples

1. Solve $2x^2 - 5x - 3 = 0$ using the Quadratic Formula.

Identify a, b, and c. Check the signs.

$$a = 2,\ b = -5,\ c = -3$$

Write the Quadratic Formula. Substitute a, b, and c into the formula and use the Order of Operations to rewrite the equation.

$$a = \boxed{2} \qquad b = \boxed{-5} \qquad c = \boxed{-3}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 - (-24)}}{4}$$

$$x = \frac{5 \pm \sqrt{49}}{4}$$

Calculate both values of x.

$$x = \frac{5 \pm \sqrt{49}}{4}$$

$$x = \frac{5 + \sqrt{49}}{4} \qquad x = \frac{5 - \sqrt{49}}{4}$$

$$x = \frac{5 + 7}{4} \qquad x = \frac{5 - 7}{4}$$

$$x = \frac{12}{4} \qquad x = \frac{-2}{4}$$

$$x = 3 \qquad x = -\frac{1}{2}$$

The solutions are $x = 3$ or $x = -\frac{1}{2}$.

2. Solve $3x^2 + 5x + 1 = 0$ using the Quadratic Formula.

Identify a, b, and c. Check the signs.

$$a = 3,\ b = 5,\ c = 1$$

Write the Quadratic Formula. Substitute a, b, and c into the formula and use the Order of Operations to rewrite the equation.

$$a = \boxed{3} \qquad b = \boxed{5} \qquad c = \boxed{1}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(1)}}{2(3)}$$

$$x = \frac{-5 \pm \sqrt{25 - 12}}{6}$$

$$x = \frac{-5 \pm \sqrt{13}}{6}$$

Calculate both values of x. Round to the nearest hundredth.

$$x = \frac{-5 \pm \sqrt{13}}{6}$$

$$x = \frac{-5 + \sqrt{13}}{6} \qquad x = \frac{-5 - \sqrt{13}}{6}$$

$$x \approx \frac{-5 + 3.6}{6} \qquad x \approx \frac{-5 - 3.6}{6}$$

$$x \approx \frac{-8.6}{6} \qquad x \approx \frac{-1.4}{6}$$

$$x \approx -0.23 \qquad x \approx -1.43$$

The solutions are $x \approx -0.23$ or $x \approx -1.43$.

3. Solve $25x^2 - 20x + 4 = 0$ using the Quadratic Formula.

Identify a, b, and c. Check the signs.

$$a = 25,\ b = -20,\ c = 4$$

Write the Quadratic Formula. Substitute a, b, and c into the formula and use the Order of Operations to rewrite the equation.

$$a = \boxed{25} \qquad b = \boxed{-20} \qquad c = \boxed{4}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(25)(4)}}{2(25)}$$

$$x = \frac{20 \pm \sqrt{400 - 400}}{50}$$

$$x = \frac{20 \pm \sqrt{0}}{50}$$

Since the second term in the numerator is zero, there is only one solution to this quadratic equation.

$$x = \frac{20}{50} = \frac{2}{5}$$

4. Solve $x^2 + 4x + 10 = 0$ using the Quadratic Formula.

Identify a, b, and c. Check the signs.

$$a = 1,\ b = 4,\ c = 10$$

Write the Quadratic Formula. Substitute a, b, and c into the formula and use the Order of Operations to rewrite the equation.

$$a = \boxed{1} \qquad b = \boxed{4} \qquad c = \boxed{10}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 40}}{2}$$

$$x = \frac{-4 \pm \sqrt{-24}}{2}$$

It is not possible to take the square root of a negative number and have a real number solution. Therefore, there are no real solutions for this quadratic equation.

5. Solve $(3x + 1)(x + 2) = 1$ using the Quadratic Formula.

Rewrite the equation in standard form, $ax^2 + bx + c = 0$, by applying the Distributive Property.

$$(3x + 1)(x + 2) = 1$$
$$3x^2 + 7x + 2 = 1$$
$$3x^2 + 7x + 1 = 0$$

Identify a, b, and c using the equation written in standard form. Check the signs.

$$a = 3,\ b = 7,\ c = 1$$

Write the Quadratic Formula. Substitute a, b, and c into the formula and use the Order of Operations to rewrite the equation.

$$a = \boxed{3} \qquad b = \boxed{7} \qquad c = \boxed{1}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(3)(1)}}{2(3)}$$

$$x = \frac{-7 \pm \sqrt{49 - 12}}{6}$$

$$x = \frac{-7 \pm \sqrt{37}}{6}$$

The two exact solutions are $x = \frac{-7 \pm \sqrt{37}}{6}$. Or, the approximate solutions are $x = \frac{-7 + \sqrt{37}}{6} \approx -0.15$ or $x = \frac{-7 - \sqrt{37}}{6} \approx -2.18$.

Practice

Use the Quadratic Formula to solve each equation. Write answers using decimal approximations as needed.

1. $x^2 - x - 2 = 0$

2. $x^2 - x - 3 = 0$

3. $-3x^2 + 2x + 1 = 0$

4. $-2 - 2x^2 = 4x$

5. $7x = 10 - 2x^2$

6. $-6x^2 - x + 6 = 0$

7. $6 - 4x + 3x^2 = 8$

8. $4x^2 + x - 1 = 0$

9. $x^2 - 5x + 3 = 0$

10. $0 = 10x^2 - 2x + 3$

11. $x(-3x + 5) = 7x - 10$

12. $(5x + 5)(x - 5) = 7x$

Answers

1. $x = 2$ or -1

2. $x \approx 2.30$ or -1.30

3. $x = -\frac{1}{3}$ or 1

4. $x = -1$

5. $x \approx 1.09$ or -4.59

6. $x \approx -1.09$ or 0.92

7. $x \approx 1.72$ or -0.39

8. $x \approx 0.39$ or -0.64

9. $x \approx 4.30$ or 0.70

10. no solution

11. $x \approx -2.19$ or 1.52

12. $x \approx 6.21$ or -0.81

Solving One-Variable Inequalities

GUIDED SKILL SUPPORT

Overview

One-variable inequalities can be solved by changing an inequality to an equation, then solving the equation. The solution to the equation is called a “boundary point” and is plotted on a number line. This point separates the number line into two regions. For situations involving $\geq$ or $\leq$, the boundary point is included in the solution and is represented with a solid circle. For situations involving strictly $>$ or $<$, the boundary point is excluded from the solution and is represented with an open circle.

Next, to determine the solution region on the number line, choose a test point from within each region separated by the boundary point. Substitute each value into the inequality and check whether the resulting inequality is true or false. If the inequality is true, then every number in that region is a solution to the inequality. Note that there are an infinite number of solutions to an inequality. If it is false, then no number in that region is a solution to the inequality.

For additional information, see the Methods & Meanings boxes in Lessons 9.2.1 and 9.3.2.

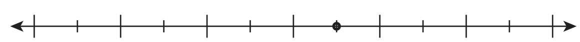
Examples

1. Solve the inequality $3x - (x + 2) \geq 0$.

Change the inequality to an equation and solve.

$$\begin{aligned} 3x - (x + 2) &= 0 \\ 3x - x - 2 &= 0 \\ 2x - 2 &= 0 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

Place the boundary point on the number line as a solid circle. The inequality symbol, $\geq$, indicates that $x = 1$ is included in the solution.



Choose two test points, one from each side of the boundary point, to substitute into the original inequality. In this example, the test points $x = 0$ and $x = 3$ are used.

$x = 0$

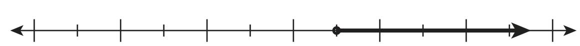
$$\begin{aligned} 3x - (x + 2) &\geq 0 \\ 3(0) - (0 + 2) &\geq 0 \\ 0 - 2 &\geq 0 \\ -2 &\geq 0 \end{aligned}$$

false

$x = 3$

$$\begin{aligned} 3x - (x + 2) &\geq 0 \\ 3(3) - (3 + 2) &\geq 0 \\ 9 - 5 &\geq 0 \\ 4 &\geq 0 \end{aligned}$$

true



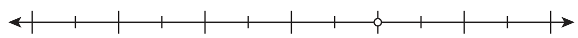
The solution is $x \geq 1$.

2. Solve the inequality $-x + 6 > x + 2$.

Change the inequality to an equation and solve.

$$\begin{aligned} -x + 6 &= x + 2 \\ -2x + 6 &= 2 \\ -2x &= -4 \\ x &= 2 \end{aligned}$$

Place the boundary point on the number line as an open circle. The inequality symbol, >, indicates that $x = 2$ is *not* included in the solution.



Choose two test points, one from each side of the boundary point, to substitute into the original inequality. In this example, the test points $x = 0$ and $x = 3$ are used.

$x = \boxed{0}$

$$\begin{aligned} -x + 6 &> x + 2 \\ -0 + 6 &> 0 + 2 \\ 6 &> 2 \end{aligned}$$

true

$x = \boxed{4}$

$$\begin{aligned} -x + 6 &> x + 2 \\ -4 + 6 &> 4 + 2 \\ 2 &> 6 \end{aligned}$$

false

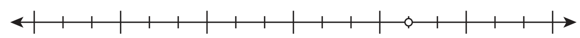
The solution is $x < 2$.

3. Solve the inequality $x < \frac{1}{4}x + 1$.

Change the inequality to an equation and solve.

$$x = \frac{1}{4}x + 1$$
$$\frac{3}{4}x = 1$$
$$x = \frac{4}{3}$$

Place the boundary point on the number line as an open circle. The inequality symbol, <, indicates that $x = \frac{4}{3}$ is *not* included in the solution.



Choose two test points, one from each side of the boundary point, to substitute into the original inequality. In this example, the test points $x = 0$ and $x = 2$ are used.

$x = 0$	$x = 2$
$x < \frac{1}{4}x + 1$	$x < \frac{1}{4}x + 1$
$0 < \frac{1}{4}(0) + 1$	$2 < \frac{1}{4}(2) + 1$
$0 < 1$	$2 < 1\frac{1}{2}$
true	false

The solution is $x < \frac{4}{3}$.

Practice

Solve each inequality.

1. $4x - 1 \geq 7$

2. $2(x - 5) \leq 8$

3. $3 - 2x < x + 6$

4. $\frac{1}{2}x > 5$

5. $3(x + 4) > 12$

6. $2x - 7 \leq 5 - 4x$

7. $3x + 2 < 11$

8. $4(x - 6) \geq 20$

9. $\frac{1}{4}x < 2$

10. $12 - 3x > 2x + 1$

11. $\frac{1}{7}x - 5 \leq -3$

12. $3(5 - x) \geq 7x - 1$

13. $3x - (2x + 2) \leq 7$

14. $x + \frac{2}{5} < \frac{2}{3}x$

15. $\frac{1}{2}x - \frac{2}{3} \geq 2x + 1$

Answers

1. $x \geq 2$

2. $x \leq 9$

3. $x > -1$

4. $x > 10$

5. $x > 0$

6. $x \leq 2$

7. $x < 3$

8. $x \geq 11$

9. $x < 8$

10. $x < \frac{11}{5}$

11. $x \leq 14$

12. $x \leq 1.6$

13. $x \leq 9$

14. $x < -\frac{6}{5}$

15. $x \leq -\frac{10}{9}$

Graphing Two-Variable Inequalities

GUIDED SKILL SUPPORT

Overview

Two-variable inequalities are graphed on a coordinate plane. To graph the solution region to a two-variable inequality, first graph the corresponding equation. The graph of the equation is the boundary line (or curve), since all points that make the inequality true lie on only one side of the line. If the inequality symbol is either $\le$ or $\ge$, then the boundary line is included in the solution to the inequality and is drawn as a solid line. If the inequality symbol is either $<$ or $>$, then the boundary line is *not* included in the solution to the inequality and it is drawn as a dashed line.

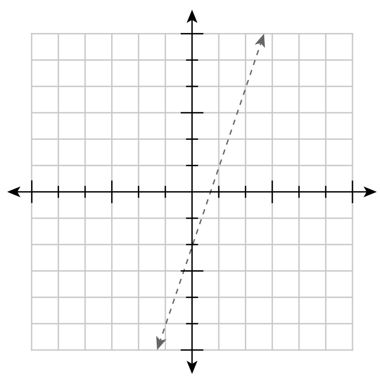
Next, as when solving one-variable inequalities, test points are used to determine which side of the boundary is the solution region. The point (0, 0) is a convenient point to use, but any point can be used as the test point as long as it is not on the boundary. If the inequality is true for the test point, then shade the graph on this side of the boundary. If the inequality is false for the test point, then shade the opposite side of the boundary. The solution region represents all values that make the inequality true. There are an infinite number of solution points.

For additional information, see the Methods & Meanings box in Lesson 9.4.1.

Examples

1. Graph the solutions to the inequality $y > 3x - 2$.

First, graph the line $y = 3x - 2$ using a dashed line since the inequality symbol, >, indicates that the boundary line is not part of the solution. For example, the point (0, –2) is on the boundary line, and it is not a solution to the inequality. When the point is substituted into the original inequality, it results in a false inequality statement: $-2 > 3(0) - 2$.



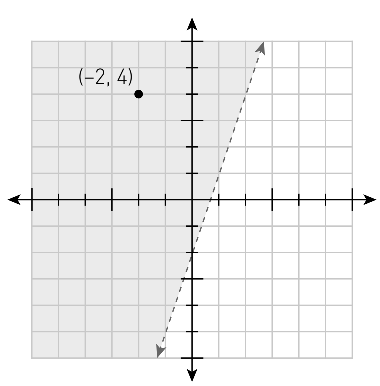
Next, test a point that is not on the boundary line by substituting the point into the original inequality. For this example, the point (–2, 4) is used.

$(-2, 4)$

$$y > 3x - 2$$
$$4 > 3(-2) - 2$$
$$4 > -8$$

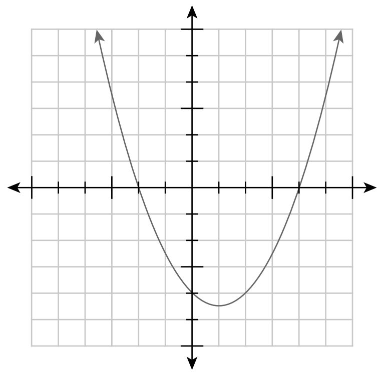
true

This is a true statement. Since the inequality is true for the test point, shade the region containing the point (–2, 4). The shaded region represents all possible solutions to the inequality.



2. Graph the solutions to the inequality $y \le x^2 - 2x - 8$.

Graph the equation $y = x^2 - 2x - 8$ using a solid line since the inequality symbol, $\le$, indicates that the boundary is part of the solution.

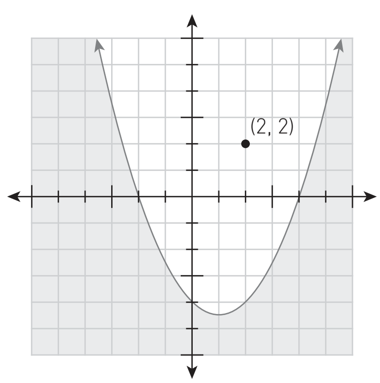


Next, test a point that is not on the boundary by substituting the point into the original inequality. For example, use the point (2, 2).

(2, 2)

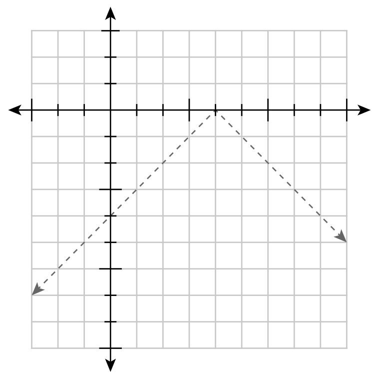
$y \le x^2 - 2x - 8$
$2 \le (2)^2 - 2(2) - 8$
$2 \le -8$
false

The inequality is false for the test point, which lies above the boundary. Therefore, the solution region is the area below the boundary. Shade below the boundary.



3. Graph the solutions to the inequality $y < -|x - 4|$.

Graph the equation $y = -|x - 4|$ using a dashed line since the inequality symbol, <, indicates that the boundary is not part of the solution.



Next, test a point that is not on the boundary by substituting the point into the original inequality. For example, use the point (0, 0).

(0, 0)

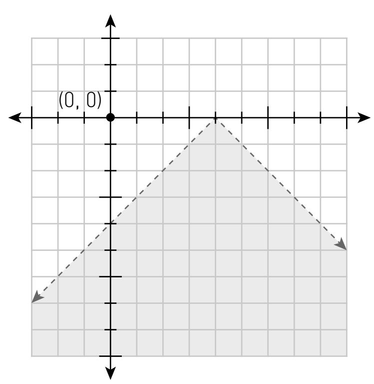
$y < -|x - 4|$

$0 < -|0 - 4|$

$0 \leq -4$

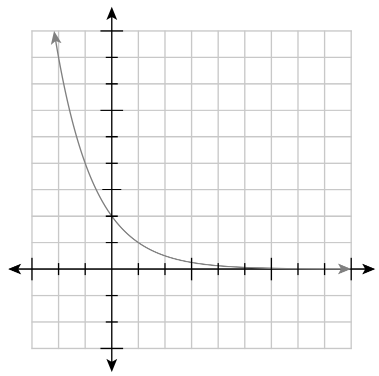
false

The inequality is false for the test point, which lies above the boundary. Therefore, the solution region is the area below the boundary. Shade below the boundary.



4. Graph the solutions to the inequality $y \geq 2(0.5)^x$.

Graph the equation $y = 2(0.5)^x$ using a solid line since the inequality symbol, $\geq$, indicates that the boundary is part of the solution.

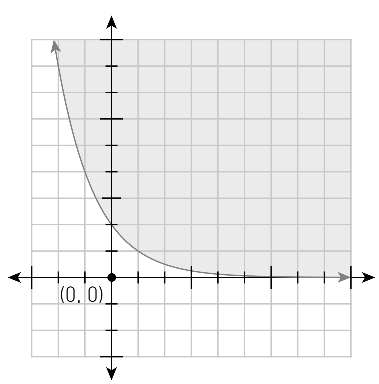


Next, test a point that is not on the boundary by substituting the point into the original inequality. For example, use the point (0, 0).

(0, 0)

$y \geq 2(0.5)^x$
$0 \geq 2(0.5)^0$
$0 \geq 2(1)$
$0 \geq 2$
false

The inequality is false for the test point, which lies below the boundary. Therefore, the solution region is the area above the boundary. Shade above the boundary.



Practice

Graph the solutions to each of the following inequalities on a separate set of axes.

1. $y \le 3x + 1$

2. $y \ge -2x + 3$

3. $y > 4x - 2$

4. $y < -3x - 5$

5. $y \le 3$

6. $y > 0.2(5)^x$

7. $y > \frac{2}{3}x + 8$

8. $y < -\frac{3}{5}x - 7$

9. $3x + 2y \ge 7$

10. $-4x + 2y < 3$

11. $y \ge x^2 - 3$

12. $y \le 0.4(2)^x$

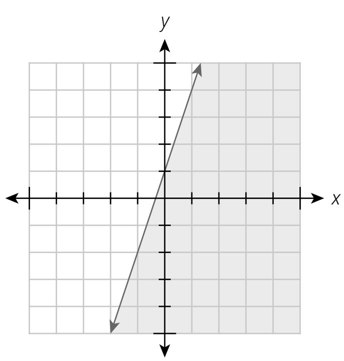
13. $y < 4 - x^2$

14. $y \le |x + 2|$

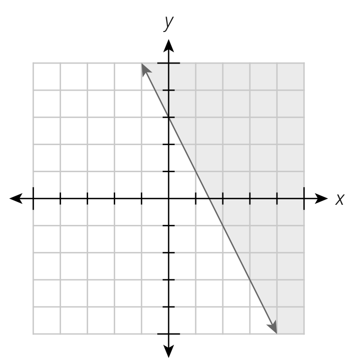
15. $y \ge -|x| + 3$

Answers

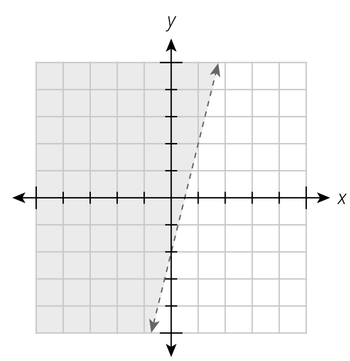
1.



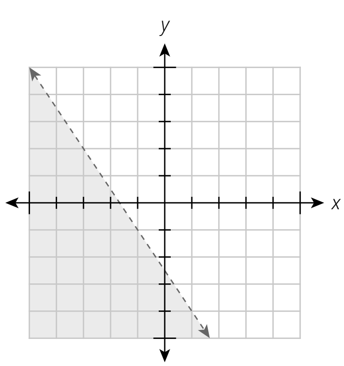
2.



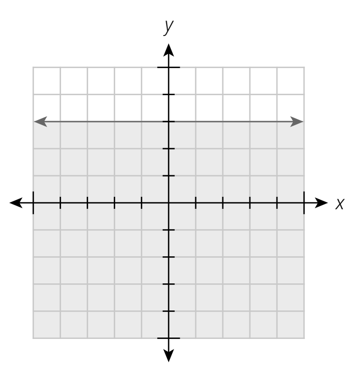
3.



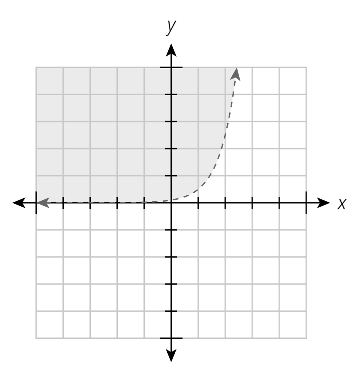
4.



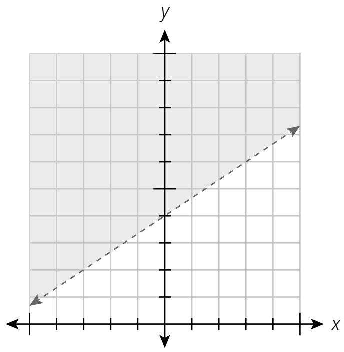
5.



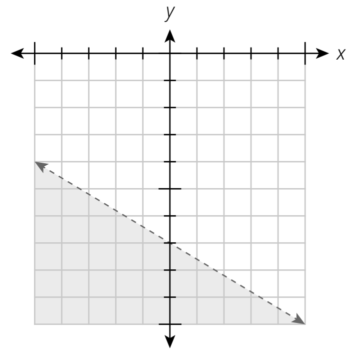
6.



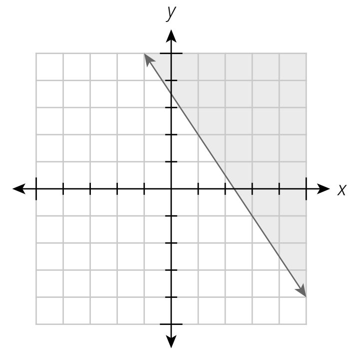
7.



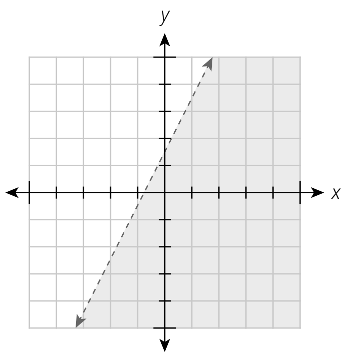
8.



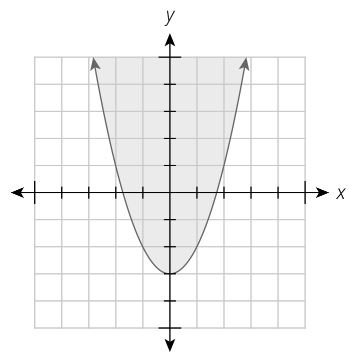
9.



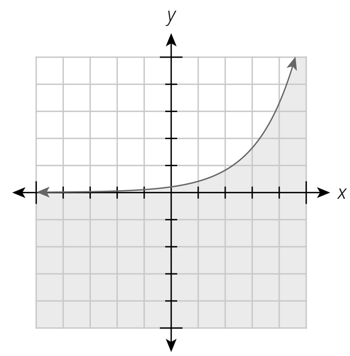
10.



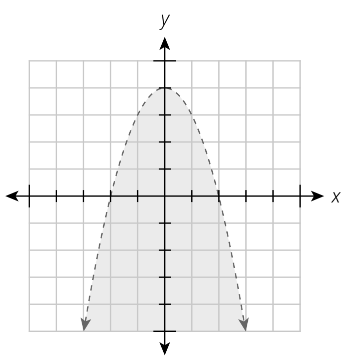
11.



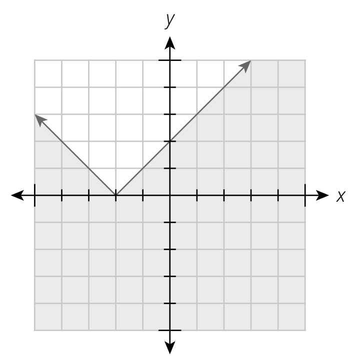
12.



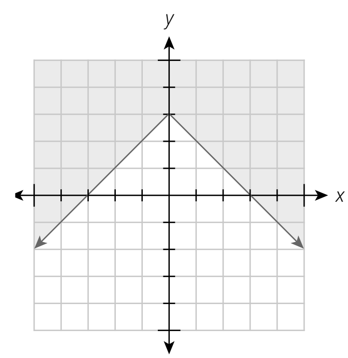
13.



14.



15.



Systems of Inequalities

GUIDED SKILL SUPPORT

Overview

A system of inequalities is similar to a system of equations. They are graphed on a coordinate plane with two lines. In a system of inequalities these lines are referred to as boundary lines, and the solution includes an infinite number of solutions, or points. A system of equations has one solution, an infinite number of solutions, or no solutions.

Systems of inequalities can be graphed in a similar way to two-variable inequalities. First, graph the line of the corresponding equation for each inequality, then use a test point from each region formed to identify and shade the solution region. The overlap of the shaded regions from both inequalities represents the solution to the system of inequalities.

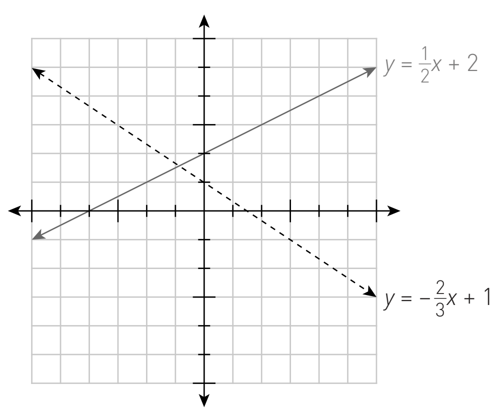
Additional information can be found in the Lesson 9.4.2 Reflection Journal.

Examples

1. Graph the system of inequalities.

$$y \le \frac{1}{2}x + 2$$
$$y > -\frac{2}{3}x + 1$$

Graph the lines $y = \frac{1}{2}x + 2$ and $y = -\frac{2}{3}x + 1$. The first line is solid and the second is dashed.



Example continues on next page.

1. Example continued from previous page.

Choose test points from each of the four regions created by the two lines. In this example, the points (–5, 0), (–4, 5), (3, 0), and (0, 0) are used. Substitute each point into both of the original inequalities. In order for a point to be a solution, both inequalities must result in a true statement.

(–5, 0)

$y \le \frac{1}{2}x + 2$

$0 \le \frac{1}{2}(-5) + 2$

$0 \le -\frac{1}{2}$

false

$y > -\frac{2}{3}x + 1$

$0 > -\frac{2}{3}(-5) + 1$

$0 > \frac{13}{3}$

false

(–4, 5)

$y \le \frac{1}{2}x + 2$

$5 \le \frac{1}{2}(-4) + 2$

$5 \le 0$

false

$y > -\frac{2}{3}x + 1$

$5 > -\frac{2}{3}(-4) + 1$

$5 > \frac{11}{3}$

true

(3, 0)

$y \le \frac{1}{2}x + 2$

$0 \le \frac{1}{2}(3) + 2$

$0 \le \frac{7}{2}$

true

$y > -\frac{2}{3}x + 1$

$0 > -\frac{2}{3}(3) + 1$

$0 > -1$

true

(0, 0)

$y \le \frac{1}{2}x + 2$

$0 \le \frac{1}{2}(0) + 2$

$0 \le 2$

true

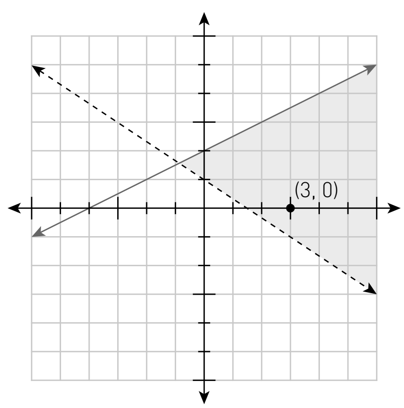
$y > -\frac{2}{3}x + 1$

$0 > -\frac{2}{3}(0) + 1$

$0 > 1$

false

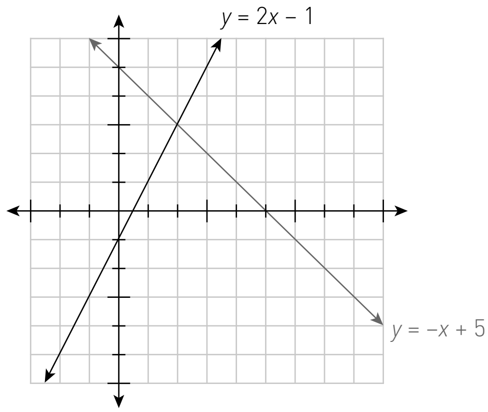
The point (3, 0) gave a true statement for both inequalities. Therefore, the region that includes this point is the solution.



2. Graph the system of inequalities.

$$y \leq -x + 5$$
$$y \geq 2x - 1$$

Graph the lines $y = -x + 5$ and $y = 2x - 1$ with solid lines.



Choose test points from each of the four regions created by the two lines. In this example, the points (0, 0), (0, 6), (6, 0), and (0, −2) are used. Substitute each point into both of the original inequalities. In order for a point to be a solution, both inequalities must result in a true statement.

(0, 0)

$y \leq -x + 5$	$y \geq 2x - 1$
$0 \leq -(0) + 5$	$0 \geq 2(0) - 1$
$0 \leq 5$	$0 \geq -1$
true	true

(0, 6)

$y \leq -x + 5$	$y \geq 2x - 1$
$6 \leq -(0) + 5$	$6 \geq 2(0) - 1$
$6 \leq 5$	$6 \geq -1$
false	true

(6, 0)

$y \leq -x + 5$	$y \geq 2x - 1$
$0 \leq -(6) + 5$	$0 \geq 2(6) - 1$
$0 \leq -1$	$0 \geq 11$
false	false

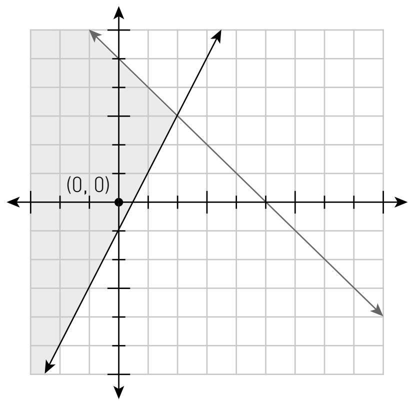
(0, −2)

$y \leq -x + 5$	$y \geq 2x - 1$
$-2 \leq -(0) + 5$	$-2 \geq 2(0) - 1$
$-2 \leq 5$	$-2 \geq -1$
true	false

Example continues on next page.

2. Example continued from previous page.

The point (0, 0) gave a true statement for both inequalities. Therefore, the region that includes this point is the solution.



Practice

Graph the solutions to each of the following pairs of inequalities on the same set of axes.

1. $y > 3x - 4$
$y \le -2x + 5$

2. $y \ge -3x - 6$
$y > 4x - 4$

3. $y < -\frac{3}{5}x + 4$
$y < \frac{1}{3}x + 3$

4. $y < -\frac{3}{7}x - 1$
$y > \frac{4}{5}x + 1$

5. $y < 3$
$y > \frac{1}{2}x + 2$

6. $x \le 3$
$y < \frac{1}{3}x - 4$

7. $y \le 2x + 1$
$y \ge 3x - 4$

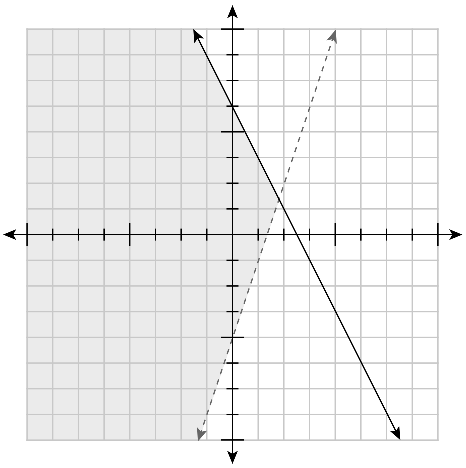
8. $y < -\frac{1}{2}x + 5$
$y \ge x + 1$

9. $y < -x + 6$
$y \le \frac{1}{4}x - 2$

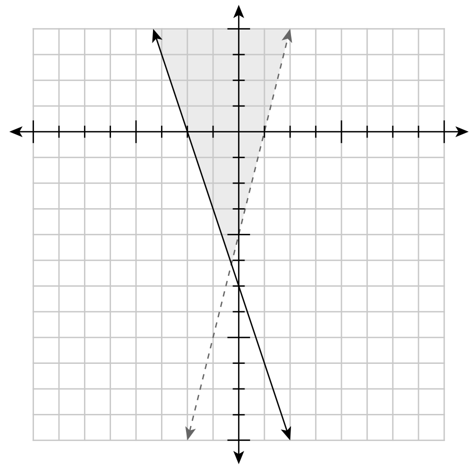
10. $y > -x + 4$
$y < x - 1$

Answers

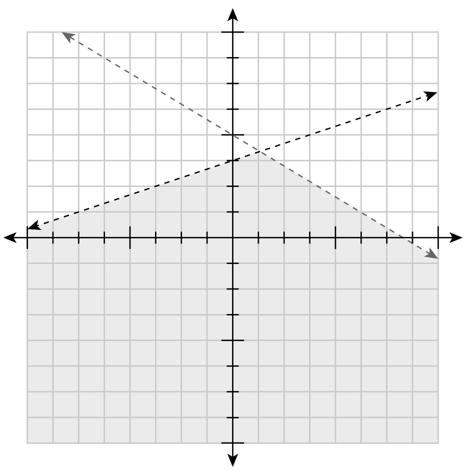
1.



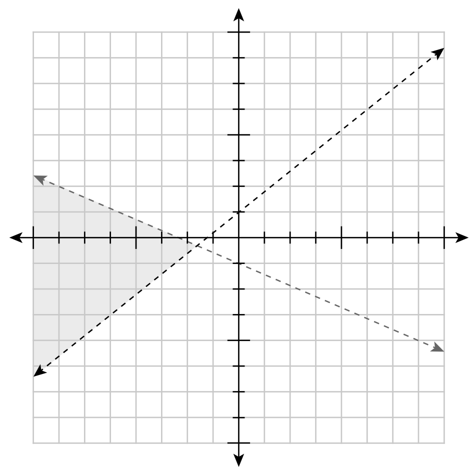
2.



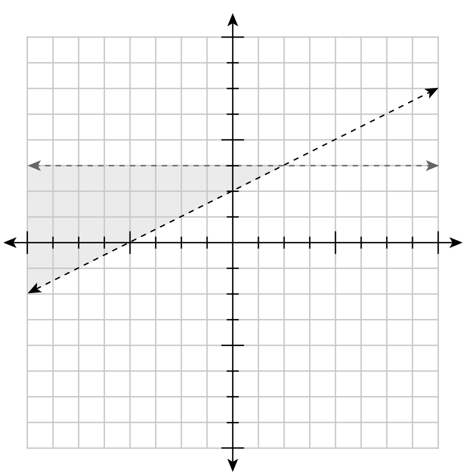
3.



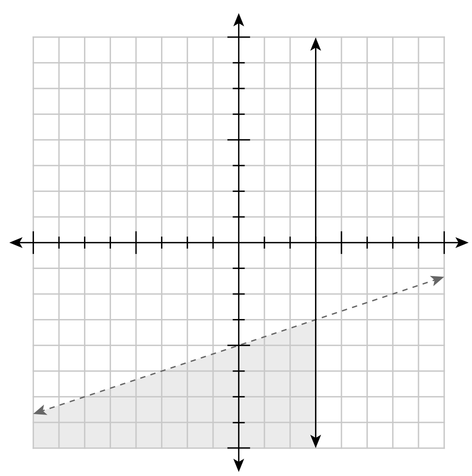
4.



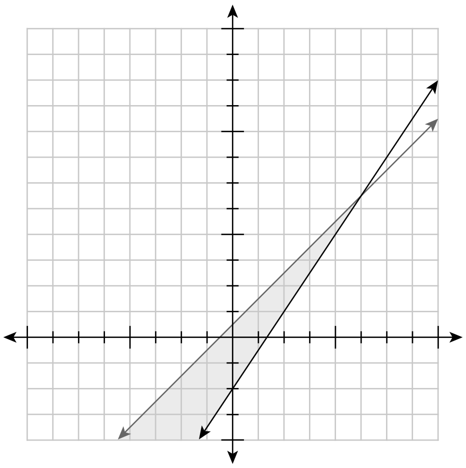
5.



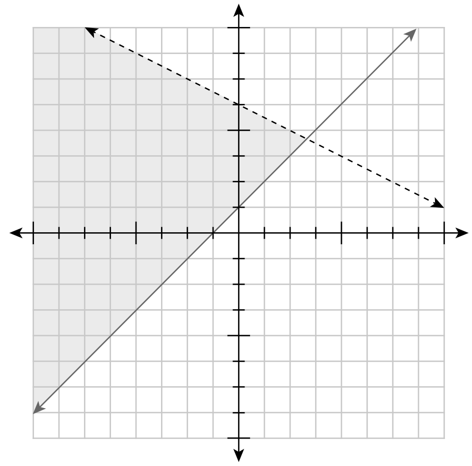
6.



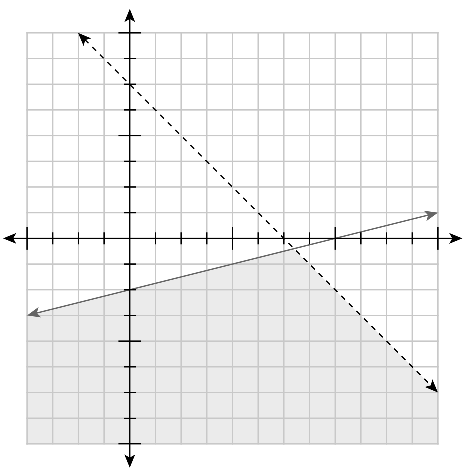
7.



8.



9.



10.