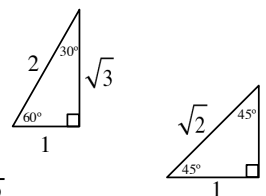


Glossary

30°-60°-90° triangle A special right triangle with acute angle measures of 30° and 60°. The side lengths are always in the ratio of $1:\sqrt{3}:2$. See the diagram at right. (p. 310)



45°-45°-90° triangle A special right triangle with acute angle measures of 45°. The side lengths are always in the ratio of $1:1:\sqrt{2}$. See the diagram at right. (p. 310)

AA ~ (Triangle Similarity) If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar. For example, given $\triangle ABC$ and $\triangle A'B'C'$ with $\angle A \cong \angle A'$ and $\angle B \cong \angle B'$, then $\triangle ABC \sim \triangle A'B'C'$. You can also show that two triangles are similar by showing that *three* pairs of corresponding angles are congruent (which would be called AAA ~), but two pairs of angles are sufficient to demonstrate similarity. (p. 170)

AAS $\cong$ (Triangle Congruence) If two angles and a non-included side of one triangle are congruent to the corresponding two angles and non-included side of another triangle, the two triangles are congruent. Note that $AAS \cong$ is equivalent to $ASA \cong$. (p. 358)

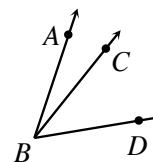
absolute value The absolute value of a number is the distance of that number from zero. Since absolute value represents a distance, without regard to direction, it is always non-negative. Thus the absolute value of a negative number is its opposite, while the absolute value of a non-negative number is just the number itself. The absolute value of x is usually written $|x|$. For example, $|-5| = 5$ and $|22| = 22$. (p. 768)

acute angle An angle with measure greater than 0° and less than 90°. One example is shown at right. See also *obtuse angle*. (p. 26)

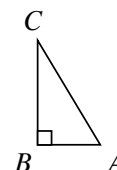


Addition Rule for Probability For any two events $\{A\}$ and $\{B\}$, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. See also *Multiplication Rule for Probability*. (p. 256)

adjacent angles For two angles to be adjacent, they must satisfy these three conditions: (1) The two angles must have a common side; (2) They must have a common vertex; and (3) They can have no interior points in common. This means that the common side must be between the two angles; no overlap between the angles is permitted. In the example at right, $\angle ABC$ and $\angle CBD$ are adjacent angles.

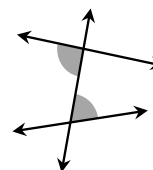


adjacent leg In a right triangle, the leg adjacent to an acute angle is the side of the angle that is not the hypotenuse. For example, in $\triangle ABC$ shown at right, $\overline{AB}$ is the leg adjacent to $\angle A$. (p. 285)



alpha (α) A Greek letter that is often used to represent the measure of an angle. Other Greek letters used to represent the measure of an angle include theta (θ) and beta (β). (p. 214)

alternate interior angles Angles between a pair of lines that switch sides of a third intersecting line (called a transversal). For example, in the diagram at right the shaded angles are alternate interior angles. If the lines intersected by the transversal are parallel, then the alternate interior angles are congruent. Conversely, if the alternate interior angles are congruent, then the two lines intersected by the transversal are parallel. See also *same-side interior angles* and *corresponding angles*. (p. 100)

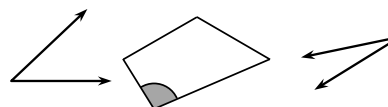


altitude See *height*.

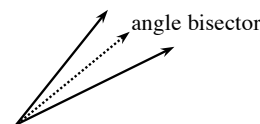
ambiguous Information is ambiguous when it has more than one interpretation or conclusion.

anagram A word game in which different arrangements of the letters of a word are created, using all the letters in the original word, one time each. The term anagram is also informally used in this course to refer to permutations in which there are duplicates of some items. For example, finding the number of ways to arrange one red, two blue, and three yellow flags in a row is the same as the number of ways to arrange the “word” RBBYYY. To compute this, find the number of permutations, and divide by the numbers of ways to arrange the repeated items, for example, $\frac{5!}{2!3!}$ for the flags. (p. 637)

angle In general, an angle is formed by two rays joined at a common endpoint. Angles in geometric figures are usually formed by two segments, with a common endpoint (such as the angle shaded in the figure at right). (p. 26)

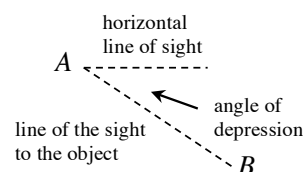


angle bisector A line segment or ray with an endpoint at the vertex of the angle that divides the angle into two equal pieces. (p. 559)

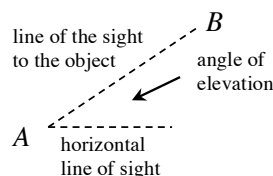


angle measure See *measurement*.

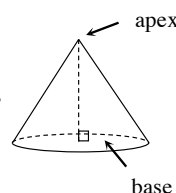
angle of depression When an object (*B*) is below the horizontal line of sight of the observer (*A*), the angle of depression is the angle formed by the line of sight to the object and the horizontal line of sight.



angle of elevation When an object (*B*) is above the horizontal line of sight of the observer (*A*), the angle of elevation is the angle formed by the line of sight to the object and the horizontal line of sight.

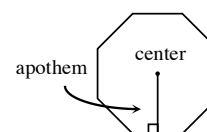


apex In a cone or pyramid, the apex is the point that is the farthest away from the flat surface (plane) that contains the base. In a pyramid, the apex is also the point at which the lateral faces meet. An apex is also sometimes called the vertex of a pyramid or cone. (p. 679)

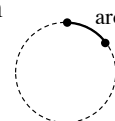


Core Connections Geometry

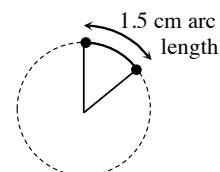
apothem A segment that connects the center of a regular polygon to a point on one of its sides, and is perpendicular to that side.



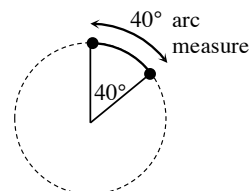
arc A connected part of a circle. Because a circle does not contain its interior, an arc is more like a portion of a bicycle tire than a slice of pizza. See also *major arc* and *minor arc* and *sector*. (p. 512)



arc length The length of an arc (in inches, centimeters, feet, etc.) is the distance from one of the arc's endpoints to the arc's other endpoint, measured around the circle. Note that arc length is different from arc measure. Arcs of two different circles may have the same arc measure (like 90°), but have different arc lengths if one circle has a larger radius than the other one. (p. 589)



arc measure The measure in degrees of an arc's central angle. Note that arc measure is measured in degrees and is different from arc length. (p. 589)



arccosine, arcsine, and arctangent. Other names for *inverse cosine* ($\cos^{-1} x$), *inverse sine* ($\sin^{-1} x$), and *inverse tangent* ($\tan^{-1} x$). (p. 294)

area On a flat surface (plane), the number of non-overlapping square units needed to cover the region. See also *surface area*. (p. 16)

area model A model that uses the area of rectangles to represent the probabilities of possible outcomes when considering two *independent* events. For example, suppose you are going to randomly select a student from a classroom. If the probability of selecting a female student from the room is $\frac{2}{3}$ and the probability of selecting a 9th grader from the room is $\frac{1}{4}$, the area of the shaded rectangle at right represents the probability that a randomly selected student is a female 9th grader ($\frac{2}{3} \cdot \frac{1}{4} = \frac{1}{6}$). A “generic” area model enables you to find the probabilities of outcomes without drawing a diagram to scale. See also *tree diagram*. (p. 249)

	male $\frac{1}{3}$	female $\frac{2}{3}$
9 th grade $\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{6}$
not 9 th grade $\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

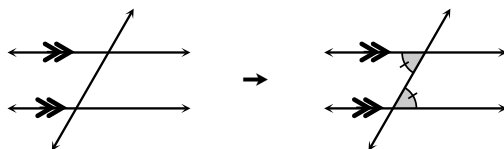
Area model

arithmetic sequence An ordered sequence of mathematical terms in which a constant number is added to each term to determine the next term in the sequence. The number added to each term to get the next term is called the sequence generator or common difference. The equation for an arithmetic sequence is $t(n) = mn + b$ or $a_n = mn + a_0$, where n is the term number, m is the common difference, and b or a_0 is the zeroth (or initial) term of the sequence. Sequences are usually written starting with the first term, where $n = 1$. For example in the sequence, 4, 7, 10, 13, ..., the common difference is 3, and the equation is $t(n) = 3n + 1$. See also *geometric sequence*. (p. 223)

Glossary

arrow diagram A pictorial representation of a conditional statement. The arrow points toward the conclusion of the conditional (the “then” part of the “If... then” statement). For example, the conditional statement “*If two lines cut by a transversal are parallel, then alternate interior angles are congruent*” can be represented by the arrow diagram below. (p. 92)

Lines cut by a transversal are parallel $\rightarrow$ *alternate interior angles are congruent*.



arrowheads To indicate a line on a diagram, we draw a segment with arrowheads on its ends. The arrowheads show that our diagram indicates a line, which extends indefinitely, as opposed to a segment, which has endpoints. Marks on pairs of lines or segments such as “ $\gg$ ” and “ $\ggg$ ” indicate that the lines or segments are parallel. Both types of marks are used in the diagrams above.

ASA $\cong$ (Triangle Congruence) If two angles and the included side of one triangle are congruent to the corresponding two angles and included side of another triangle, the triangles are congruent. Note that ASA $\cong$ is equivalent to AAS $\cong$. (p. 358)

association A relationship between two (or more) variables. An association between *numerical* variables can be displayed on a scatterplot, and described by its form, direction, strength, and outliers. Possible association between two *categorical* variables can be studied in a two-way table. (p. 609)

auxiliary lines Segments and lines added to existing figures. Auxiliary lines are usually added to a figure to allow us to prove something about the figure.

axioms Statements accepted as true without proof. Also known as *postulates*.

base (a) Triangle: Any side of a triangle to which a height is drawn. There are three possible bases in each triangle; (b) Trapezoid: the two parallel sides; (c) Parallelogram (and rectangle, rhombus, and square): Any side to which a height is drawn (there are four possible bases in each parallelogram); (d) Solid: See *cone*, *cylinder*, *prism*, and *pyramid*. (pp. 118, 537)

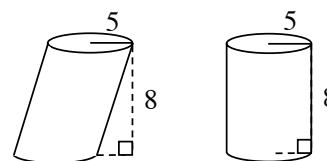
beta (β) A Greek letter that is often used to represent the measure of an angle. Other Greek letters used to represent the measure of an angle include alpha (α) and theta (θ). (p. 214)

binomial An expression that is the sum or difference of exactly two terms, each of which is a monomial. For example, $-2x + 3y^2$ is a binomial.

bisect To bisect a geometric object is to divide it into two congruent parts. (p. 558)

Core Connections Geometry

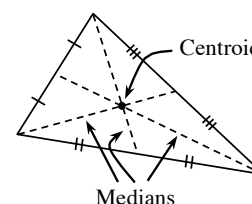
Cavalieri's Principle Two solids have the same volume if corresponding cross-sectional areas are the same. For example, an oblique solid has the same volume as the corresponding right solid with the same height, as long as all the cross-sections parallel to the base have the same area. See also *oblique prism (or cylinder)* and *oblique pyramid (or cone)*. (p. 542)



center of a circle On a flat surface, the fixed point from which all points on the circle are equidistant. See also *circle*. (p. 404)

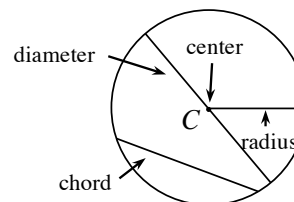
central angle An angle with its vertex at the center of a circle. (p. 589)

centroid The point at which the three medians of a triangle intersect. The centroid is also the center of balance of a triangle. The other points of concurrency studied in this course are the circumcenter and the incenter. (pp. 568, 604)

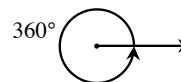


chord A line segment with its endpoints on a circle. A chord that passes through the center of a circle is called a diameter. See also *circle*. (p. 583)

circle The set of all points on a flat surface that are the same distance from a fixed point. If the fixed point (center) is O , the symbol $\odot O$ represents a circle with center O . If r is the length of a circle's radius and d is the length of its diameter, the circumference of the circle is $C = 2\pi r$ or $C = \pi d$. The area of the circle is $A = \pi r^2$. The graphing form (also called "center-radius form" or standard form) of the equation of a circle with radius length r and center (h, k) is $(x - h)^2 + (y - k)^2 = r^2$. The general form (also called expanded form) is $ax^2 + ay^2 + bx + cy + d = 0$. (p. 404)



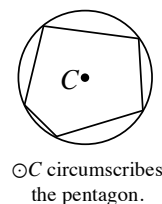
circular angle An angle with a measure of 360° . (p. 26)



circumcenter The center of the circle that passes through the vertices of a triangle. It can be found by locating the point of intersection of the perpendicular bisectors of the sides of the triangle. The other points of concurrency studied in this course are the centroid and the incenter. (p. 604)

circumference The perimeter of (distance around) a circle. (p. 512)

circumscribed circle A circle circumscribes a polygon when it passes through all of the vertices of the polygon. (pp. 603, 604)



clinometer A device used to measure angles of elevation and depression. (p. 231)

Glossary

clockwise Clockwise identifies the direction of rotation shown in the diagram at right. The word literally means “in the direction of the rotating hands of a clock.” The opposite of clockwise is *counter-clockwise*.



combination A combination is an arrangement of items that were selected without repeating items and without regard to order. The number of combinations that can be made by selecting r items from a set of n items can be found by dividing the total number of *permutations* by the number of arrangements of a single permutation, ${}_nC_r = \frac{{}_nP_r}{r!}$, which can be rewritten as ${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}$. For instance, choosing a committee of 3 students from a group of 5 volunteers is a combination since the order in which committee members are selected does not matter: ${}_5C_3 = \frac{5!}{3!(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = \frac{5 \cdot 4}{2} = 10$. See also *permutation*. (p. 643)

common difference The difference between consecutive terms of an arithmetic sequence, also known as the sequence generator of an arithmetic sequence. When the common difference is positive the sequence increases; when it is negative the sequence decreases. In the sequence 3, 7, 11, ..., the common difference is 4. See also *arithmetic sequence*. (p. 223)

common multiplier of a sequence Another name for *common ratio of a sequence*.

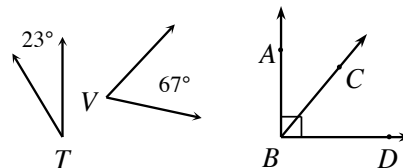
common ratio of corresponding sides Another name for the ratio of similarity. See *ratio of similarity*. (p. 154)

common ratio of a sequence Common ratio is another name for the multiplier or sequence generator of a geometric sequence. It is the number to multiply one term by to get the next one. In the sequence: 96, 48, 24, ..., the common ratio is $\frac{1}{2}$. See also *geometric sequence*. (p. 223)

compass (a) A tool used to draw circles; (b) A tool used to navigate the Earth. A compass uses the Earth’s magnetic field to determine which direction is north. (p. 554)

complement of an event The set of all the outcomes in the sample space that are not in the event. For example, if you randomly select one card from a deck of cards, the event {the card is a diamond} has a probability of $\frac{13}{52}$. The complement of the event is {the card is not a diamond}, or {the card is a heart, spade, or club}, and has a probability of $1 - \frac{13}{52}$. (p. 255)

complementary angles Two angles whose measures add up to 90° . Angles T and V are complementary because $m\angle T + m\angle V = 90^\circ$. Complementary angles may also be adjacent, like $\angle ABC$ and $\angle CBD$ in the diagram at far right. (p. 83)



completing the square A standard procedure for rewriting a quadratic equation from standard form into graphing form, or for rewriting the circle of an equation from general form into graphing form. For example, the first two terms in the equation $y = x^2 - 6x + 4$ look somewhat like $(x - 3)^2$. But $(x - 3)^2$ is 9 more than $x^2 - 6x$. That is, $x^2 - 6x = (x - 3)^2 - 9$. We can rewrite $y = x^2 - 6x + 4$ in graphing form as follows: (p. 736)

$$\begin{aligned} y &= x^2 - 6x + 4 \\ y &= (x - 3)^2 - 9 + 4 \\ y &= (x - 3)^2 - 5 \end{aligned}$$

concentric circles Circles that have the same center. For example, the circles shown in the diagram at right are concentric. (p. 554)

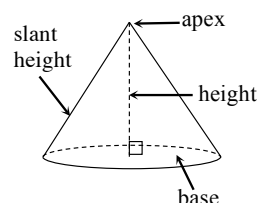


concurrency (point of) See *point of concurrency*.

conditional probability The probability of outcome A occurring, given that outcome B has already happened, is called the conditional probability of A given B . One way to calculate the conditional probability of A given B is to use the Multiplication Rule, $P(A \text{ given } B) = \frac{P(A \text{ and } B)}{P(B)}$. In many situations it is possible to calculate the conditional probability directly from the data by counting the number of outcomes (or computing the probabilities) of A that are within the outcomes (probabilities) for B . See also *Multiplication Rule for Probability*. (p. 624)

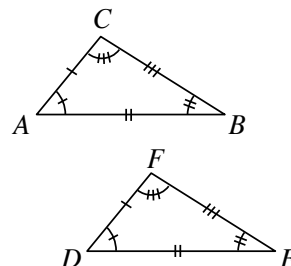
conditional statement A statement written in “If ..., then ...” form. For example, “*If a rectangle has four congruent sides, then it is a square*” is a conditional statement. (p. 119)

cone A three-dimensional figure that consists of a circular face, called the base, a point called the apex, that is not in the flat surface (plane) of the base, and the lateral surface that connects the apex to each point on the circular boundary of the base. The slant height of a cone is the distance from the circular boundary to the apex. See also *right pyramid (or cone)* and *oblique pyramid (or cone)*. (p. 690)



congruence conditions See *triangle congruence conditions*.

congruence statement A statement indicating that two figures are congruent. The order of the letters in the names of the shapes indicates which sides and angles are congruent to each other. For example, if $\triangle ABC \cong \triangle DEF$, then $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$.



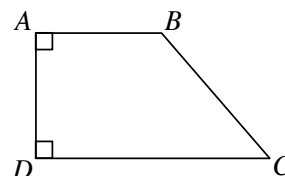
congruent Two shapes are congruent if there is a sequence of rigid transformations that carries one onto the other. The corresponding angles and sides of congruent polygons have equal measures. Congruent shapes are similar and have a scale factor of 1. The symbol for congruence is $\cong$. (pp. 175, 345)

Glossary

conic section A curve that is the intersection of a flat surface or plane with an infinite double cone. Conic sections include parabolas, ellipses, and hyperbolas. Circles, lines, and points are special cases of the conic sections. (p. 742)

conjecture An educated guess. Conjectures often result from noticing a pattern during an investigation. Conjectures are also often written in conditional (“If..., then...” form). Once a conjecture is proven, it becomes a theorem. (p. 11)

consecutive angles of a polygon The angles that occur at the two ends of any side of a polygon. For example, $\angle B$ and $\angle C$ are consecutive angles in the right trapezoid at right. (p. 443)



construction The process of using a straightedge and compass to solve a problem and/or create a geometric diagram. (p. 552)

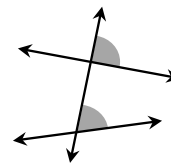
converse The converse of a conditional statement can be found by switching the hypothesis (the “if” part) and the conclusion (the “then” part). For example, the converse of “If P , then Q ” is “If Q , then P .” Knowing that a conditional statement is true does not tell you whether its converse is true. (p. 363)

convex polygon A polygon is convex if any pair of points inside the polygon can be connected by a segment without leaving the interior of the polygon. See also *non-convex polygon*. (p. 476)

coordinate geometry The study of geometry on a coordinate graph. (p. 454)

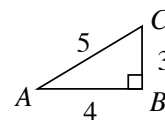
coordinate graph A method of specifying points (x, y) by their relationship to two perpendicular linear axes, often labeled the x -axis and y -axis. Also known as a rectangular coordinate graph or a Cartesian coordinate graph. See also *graph*.

corresponding angles (a) When two lines are intersected by a third line (called a transversal), angles on the same side of the two lines and on the same side of the transversal are called corresponding angles. For example, the shaded angles in the diagram at right are corresponding angles. Note that if the two lines cut by the transversal are parallel, the corresponding angles are congruent. Conversely, if the corresponding angles are congruent, then the two lines intersected by the transversal are parallel; (b) Angles in two figures may also correspond, as shown in the figure in *corresponding parts*. See also *alternate interior angles* and *same-side interior angles*. (p. 100)



corresponding parts Points, sides, edges, or angles in two or more figures that are images of each other with respect to a sequence of transformations. If two figures are congruent, their corresponding parts are congruent to each other. (p. 410)

cosine ratio In a right triangle, the cosine ratio of an acute angle, A , is $\cos A = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$. In the triangle at right, $\cos A = \frac{AB}{AC} = \frac{4}{5}$. See also *sine ratio*, *tangent ratio*, and *inverse cosine*. (p. 285)



counter-clockwise Counter-clockwise identifies the direction of rotation shown in the diagram at right. The opposite of counter-clockwise is clockwise.

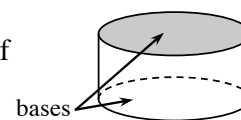


counterexample An example showing that a generalization has at least one exception; that is, a situation in which the statement is false. For example, the number 4 is a counterexample to the statement “*All even numbers are greater than 7.*” (p. 282)

cross-section The intersection of a three-dimensional solid and a plane. The cross-sections of an infinite double cone are called conic sections. (p. 686)

cube A polyhedron all of whose faces are squares. (p. 671)

cylinder (circular) A three-dimensional figure that consists of two parallel congruent circular regions (called bases) and a lateral surface containing segments connecting each point on the circular boundary of one base to the corresponding point on the circular boundary of the other. See also *oblique prism (or cylinder)* and *right prism (or cylinder)*. (p. 33)



decagon A polygon with ten sides. (p. 52)

decision chart A method for counting the number of outcomes (the size of the sample space) for a sequence of probabilistic situations, often in which the order of the items matters. A tree diagram could be used to count the number of outcomes, but if the number of outcomes is large a decision chart is more useful. For example, how many three-letter arrangements could be made by lining up any three blocks, chosen from a set of 26 alphabet blocks, if the first letter must be a vowel and the blocks are not reused? There are three decisions (three blocks to be chosen), with 5 choices for the first letter (a vowel), 25 for the second, and 24 for the third. According to the Fundamental Principle of Counting, the total number of possibilities is:

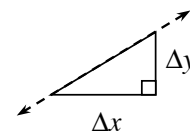
$$\frac{5}{\text{1st decision}} \cdot \frac{25}{\text{2nd decision}} \cdot \frac{24}{\text{3rd decision}} = 3000.$$

This decision chart is a short way to represent a tree with 5 branches for the first alphabet block, followed by 25 branches for each of those branches; each of those 125 branches would then have 24 branches representing the possibilities for the third alphabet block. See also *Fundamental Principle of Counting*. (p. 631)

deck of cards See *playing cards*.

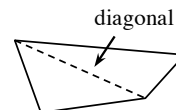
degree One degree is an angle measure that is $\frac{1}{360}$ of a full circle.

delta (Δ) A Greek letter that is often used to represent a difference. Its uses include Δx and Δy , which represent the lengths of the horizontal and vertical legs of a slope triangle, respectively. (p. 59)



density The quantity of something per unit measure, especially length, area, or volume. For example, the density of birds on a power line may be 7 birds/meter, the population density of Singapore is 7953 people per square kilometer, or the mass density of iron is $7.874 \frac{\text{g}}{\text{cm}^3}$. (p. 556)

diagonal In a polygon, it is a segment that connects any two vertices of the polygon but is not a side of the polygon. (p. 481)



diameter A line segment drawn through the center of a circle with both endpoints on the circle. The length of a diameter is usually denoted d . Note that the length of a circle's diameter is twice the length of its radius. See also *circle*. (p. 404)

dilation A transformation which produces a figure similar to the original by proportionally shrinking or stretching the figure. In a dilation, a shape is stretched (or compressed) proportionally from a point, called the point of dilation. See also *point of dilation*. (p. 148)

dimension (a) Flat figures have two dimensions (which can be labeled base and height); (b) Solids have three dimensions (such as “width,” “height,” and “depth”). (p. 107)

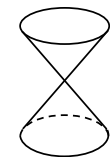
directrix A line that, along with a point (called a focus), defines a conic section (such as a parabola). For any point on a parabola, the length of the segment between the focus and the point is equal to the length of the segment perpendicular from the directrix to point. (p. 743)

disjoint Another name for *mutually exclusive*. (p. 610)

dissection The process of dividing a flat shape or solid into parts that have no interior points in common.

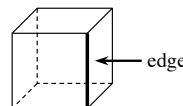
dodecahedron A polyhedron with twelve faces.

double cone Two cones placed apex to apex so that their bases are parallel. Generally we think of double cones as extending to infinity beyond their bases.



double cone

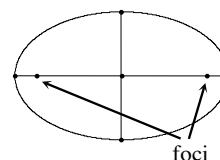
edge In three dimensions, a line segment formed by the intersection of two faces of a polyhedron. (p. 671)



ellipse A conic section created by slicing a cone with a plane. In a future course it will be defined using a focus and a directrix. A typical algebraic equation of an ellipse in graphing form is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \text{ and in general form it is}$$

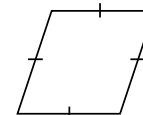
$$ax^2 + by^2 + cx + dy + e = 0.$$



endpoint See *line segment* and *ray*. (p. 749)

equator If we represent the Earth as a sphere, the equator is the great circle equally distant between the North and South poles. The equator divides the Earth into two halves called the Northern and Southern hemispheres. The equator also marks a latitude of 0° . See also *latitude*. (p. 699)

equilateral A polygon is equilateral if all its sides have equal length. The word equilateral comes from *equi* (meaning equal) and *lateral* (meaning side). Equilateral triangles not only have sides of equal length, but also angles of equal measure. However, a polygon with more than three sides can be equilateral without having congruent angles. For example, see the rhombus at right. (p. 416)



evaluate (an expression) Substitute one or more numbers for the variables in a mathematical expression. Compare evaluating an expression, in which you know the value of a variable and use it to rewrite an expression, to solving an equation, in which you determine the value of a variable. For example, to evaluate $x^2 + 3$ for $x = -4$ means to rewrite $x^2 + 3$ as $(-4)^2 + 3$ or 19. See also *solve (an equation)*.

event The set of one or more outcomes (or results) of a probabilistic situation. (p. 31)

exact answer An answer that is precisely accurate and not approximate. For example, if the length of a side of a triangle is exactly $\sqrt{10}$, the exact answer to the question “How long is that side of the triangle?” would be $\sqrt{10}$, while 3.162 would be a decimal approximation of the answer. (p. 133)

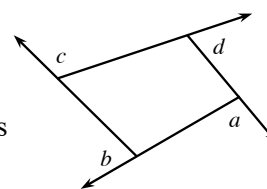
expected value For this course, the expected value of a game is the average amount expected to be won or lost on each play of the game if the game is played many times. Expected value can be found by summing the probability of each outcome multiplied by its value. For example, if you play a game where you roll a die and win one point for every dot on the face that comes up, the expected value of this game is $(\frac{1}{6})1 + (\frac{1}{6})2 + (\frac{1}{6})3 + (\frac{1}{6})4 + (\frac{1}{6})5 + (\frac{1}{6})6 = 3.5$ points for each play. The expected value need not be a value that a player could actually win on a single play of the game. (p. 304)

experimental probability The probability based on data collected in experiments. The experimental probability of an event is defined to be

$$\frac{\text{number of successful outcomes in the experiment}}{\text{total number of outcomes in the experiment}}. \text{ (p. 31)}$$

Glossary

exterior angle (of a polygon) When a side of a polygon is extended to form an angle with an adjacent side outside of the polygon, that angle is called an exterior angle. For example, the angles marked with letters in the diagram at right are exterior angles of the quadrilateral. Note that an exterior angle of a polygon is always adjacent and supplementary to an interior angle of that polygon. The sum of the exterior angles of a convex polygon is always 360° . See also *interior angle (of a polygon)*. (p. 484)



face of a polyhedron One of the flat surfaces of a polyhedron, including the base(s). (p. 671)

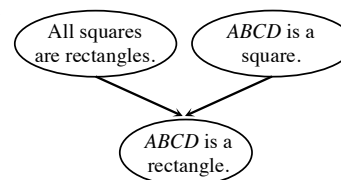
factor A factor of an expression is an integer or a polynomial, which when multiplied by one or more other factors, gives the expression. For example, 3, 6, $(2x + 1)$, and $(4x + 2)$ are each factors of $12x + 6$, and factors of $x^2 + 3x + 2$ are $(x + 1)$ and $(x + 2)$.

factorial A shorthand notation for the product of a list of consecutive positive integers from the given number down to 1: $n! = n(n - 1)(n - 2)(n - 3) \cdot \dots \cdot 3 \cdot 2 \cdot 1$. For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. (p. 638)

fair game For the purposes of this course, a fair game has an expected value of 0 for all players. Therefore, over many plays, any player would expect to neither win nor lose points or money by playing a fair game. (p. 304)

flip See *reflection*.

flowchart A diagram showing an argument for a conclusion from certain evidence. A flowchart uses ovals connected by arrows to show the logical structure of the argument. When each oval has a reason stated next to it showing how the evidence leads to that conclusion, the flowchart represents a proof. See the example at right. (p. 187)



focus A point that, along with a line (called a directrix), can be used to define a conic section (like a parabola). For any point on a parabola, the length of the segment between the focus and the point is equal to the length of the segment perpendicular from the directrix to point. (p. 743)

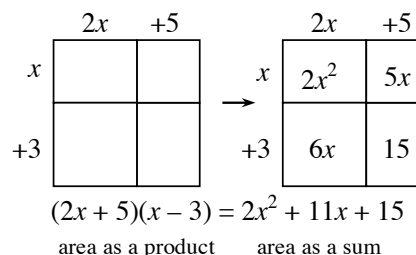
Fraction Buster Fraction Busting is a method of simplifying equations involving fractions by using the Multiplicative Property of Equality to alter the equation so that no fractions remain. To use this method, multiply both sides of an equation by the common denominator of all the fractions in the equation. The result will be an equivalent equation with no fractions. For example, when given the equation $\frac{x}{7} + 2 = \frac{x}{3}$, we can multiply both sides by the Fraction Buster 21. The resulting equation, $3x + 42 = 7x$, is equivalent to the original but contains no fractions.

frequency table A two-way table that contains entries that are counts (as opposed to fractions or percents). See also *two-way table*.

function A relation in which for each input value there is one and only one output value. For example, the relation $f(x) = x + 4$ is a function; for each input value (x) there is exactly one output value. In terms of ordered pairs (x, y) , no two ordered pairs of a function have the same first member (x). For example, a graph of a circle does not represent a function because for most x -values there are two y -values.

Fundamental Principle of Counting A method for counting the number of outcomes (the size of the sample space) of a probabilistic situation, often in which the order of the items matters. If event $\{A\}$ has m outcomes, and event $\{B\}$ has n outcomes after event $\{A\}$ has occurred, then event $\{A\}$ followed by event $\{B\}$ has $m \cdot n$ outcomes. For a sequence of events, a tree diagram could be used to count the number of outcomes, but if the number of outcomes is large a decision chart is more useful. See also *decision chart*. (p. 631)

generic rectangle A type of diagram used to visualize multiplying expressions without algebra tiles. Each expression to be multiplied forms a side length of the rectangle, and the product is the sum of the areas of the sections of the rectangle. For example, the generic rectangle at right can be used to multiply $(2x + 5)$ by $(x + 3)$.



geometric sequence An ordered sequence of mathematical terms in which each term is multiplied by a constant number to determine the next term in the sequence. The number each term is multiplied by to get the next term is called the multiplier, sequence generator, or common ratio. The equation for a geometric sequence is $t(n) = ab^n$ or $a_n = a_0 \cdot b^n$, where n is the term number, b is the multiplier, and a or a_0 is the zeroth (or initial) term of the sequence. Sequences are usually written starting with the first term, where $n = 1$. For example in the sequence, 5, 15, 45, ..., the multiplier is 3, and the equation is $t(n) = \frac{5}{3} \cdot 3^n$. See also *arithmetic sequence*. (p. 223)

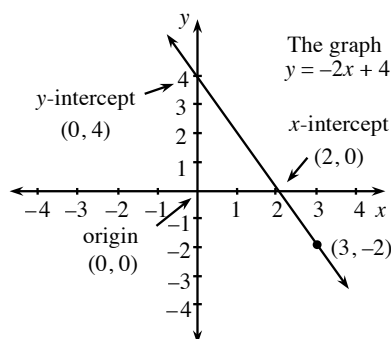
golden ratio The number $\frac{1+\sqrt{5}}{2}$, which is often labeled with the Greek letter phi (ϕ), pronounced “fee.” (p. 759)

golden rectangle A rectangle whose side lengths are in the proportion of the golden ratio. (p. 759)

graph A graph represents numerical information spatially. The numbers may come from a table, situation (pattern), rule (equation or inequality), or figure. Most of the graphs in this course show points, lines, figures, and/or curves on a two-dimensional coordinate graph like the one below right.

A complete graph includes all the necessary information about a line or a curve. To be complete, a graph must have the following components:

(1) the x -axis and y -axis labeled, clearly showing the scale; (2) the equation of the graph written near the line or curve; (3) the line or curve extended as far as possible on the graph with arrows if the line or curve continues beyond the axes; (4) the coordinates of all special points, such as x - and y -intercepts, shown in (x, y) form.



great circle A cross-section of a sphere that has the same radius as the sphere. If the Earth is represented with a sphere, the equator is an example of a great circle. (p. 700)

height (a) Triangle: the length of a segment that connects a vertex of the triangle to a line containing the opposite base (side) and is perpendicular to that line; (b) Trapezoid: the length of any segment that connects a point on one base of the trapezoid to the line containing the opposite base and is perpendicular to that line; (c) Parallelogram (includes rectangle, rhombus, and square): the length of any segment that connects a point on one base of the parallelogram to the line containing the opposite base and is perpendicular to that line; (d) Pyramid and cone: the length of the segment that connects the apex to a point in the plane containing the figure's base and is perpendicular to that plane; (e) Prism or cylinder: the length of a segment that connects one base of the figure to the plane containing the other base and is perpendicular to that plane. Some texts make a distinction between height and altitude, where the altitude is the segment described in the definition above and the height is its length. (p. 122)

hemisphere Half of a sphere. A great circle of a sphere divides it into two congruent parts, each of which is called a hemisphere. Hemispheres can also be created by slicing a sphere with a plane passing through the sphere's center. If the Earth is represented as a sphere, the Northern hemisphere is the portion of the Earth north of (and including) the equator. Likewise, the Southern hemisphere is the portion of the Earth south of (and including) the equator. (p. 699)

heptagon A polygon with seven sides. (p. 473)

heptahedron A polyhedron with seven faces. (p. 674)

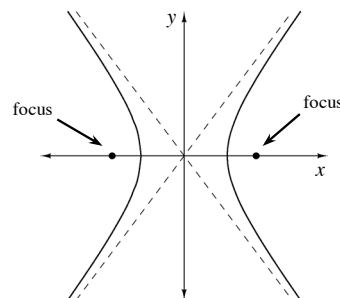
hexagon A polygon with six sides. (p. 52)

hexahedron A polyhedron with six faces. A regular hexahedron is a cube. (p. 674)

HL $\cong$ (Triangle Shortcut) If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, the two right triangles are congruent. Note that this congruence condition applies only to right triangles. (p. 358)

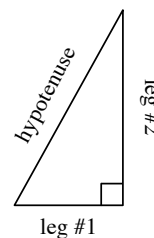
hyperbola A hyperbola looks like two curves (not parabolas) facing away from each other. A hyperbola is a conic section created by slicing a double cone with a plane. In a future course it will be defined using a focus and a directrix. A typical algebraic equation of a hyperbola is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1. \text{ (p. 749)}$$



hypothesis A conjecture (or educated guess) in science.

hypotenuse The side of a right triangle opposite the right angle. Note that legs of a right triangle are always shorter than its hypotenuse. See also *legs*. (p. 128)



icosahedron A polyhedron with twenty faces. (p. 674)

“If ..., then ...” statement A statement written in the form “If ..., then ...”. Also known as a conditional statement. (p. 119)

image The shape that results from a transformation, such as a translation, rotation, reflection, or dilation. (p. 37)

incenter The center of the circle inscribed in a triangle. It can be found by locating the point at which the angle bisectors of a triangle intersect. The other points of concurrency studied in this course are the centroid and the circumcenter. (p. 604)

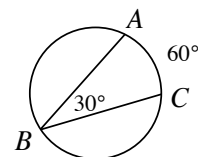
independent events If the outcome of a probabilistic event does not affect the probability of another event, the events are independent. For example, assume you plan to roll a normal six-sided die twice and want to know the probability of rolling a 1 twice. The result of the first roll does not affect the probability of rolling a 1 on the second roll; the events are independent. If two events $\{A\}$ and $\{B\}$ are independent, then $P(A \text{ given } B) = P(A)$. The converse is also true. Alternatively, by rewriting the Multiplication Rule, if two events are independent, then $P(A \text{ and } B) = P(A) \cdot P(B)$. Again, the converse is also true. (pp. 232, 624)

indirect proof See *proof by contradiction*. (p. 105)

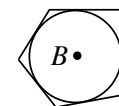
inequality symbols The symbols $<$ (less than), $>$ (greater than), $\leq$ (less than or equal to), and $\geq$ (greater than or equal to).

inscribed angle An angle with its vertex on the circle and sides intersecting the circle at two distinct points. In the figure at right $\angle ABC$ is an inscribed angle. (p. 589)

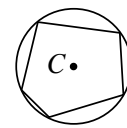
Inscribed Angle Theorem The measure of an inscribed angle is half the measure of its intercepted arc. Likewise, the measure of an intercepted arc is twice the measure of an inscribed angle whose sides pass through the endpoints of the arc. In the figure at right, if $m\widehat{AC} = 60^\circ$, then $m\angle ABC = 30^\circ$. (p. 595)



inscribed circle A circle is inscribed in a polygon if each side of the polygon intersects the circle at exactly one point. (p. 554)



$\odot B$ is inscribed in the pentagon.

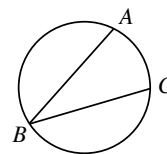


The pentagon is inscribed in $\odot C$.

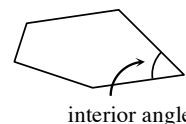
inscribed polygon A polygon is inscribed in a circle if each vertex of the polygon lies on the circle. (p. 555)

integers The set of numbers $\{ \dots -3, -2, -1, 0, 1, 2, 3, \dots \}$.

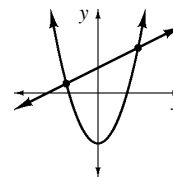
intercepted arc The arc of a circle bounded by the points where the two sides of an inscribed angle meet the circle. In the circle at right, $\angle ABC$ intercepts $\widehat{AC}$. (p. 588)



interior angle (of a polygon) An angle formed by two consecutive sides of the polygon. The vertex of the angle is a vertex (corner) of the polygon. The sum of the interior angles of a polygon with n -sides is $180^\circ(n - 2)$. (p. 479)



intersection A point of intersection is a point that the graphs of two or more equations have in common. Graphs may intersect in one, two, several, many or no points. The set of coordinates of a point of intersection are a solution to the equation for each graph. The functions at right have two points of intersection. (p. 95)



intersection of two events The intersection of the two events $\{A\}$ and $\{B\}$ is a new event that includes all the outcomes in which both A and B will occur. For example, if event $\{A\}$ is the thirteen diamond-suited cards in a deck of playing cards, and $\{B\}$ is the four Aces, then the event $\{A \text{ intersection } B\}$ contains 1 outcome: $\{A \spadesuit\}$. See also *union of two events*. (p. 255)

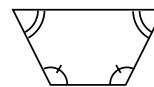
inverse cosine ($\cos^{-1}(x)$), inverse sine ($\sin^{-1}(x)$), and inverse tangent ($\tan^{-1}(x)$)

The inverse of the cosine, sine, and tangent functions (the functions that “undo” cosine, sine, and tangent). When the ratio between the lengths of two sides of a right triangle is known, $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ can be used to find the measure of one of the triangle’s acute angles. For example, if $\cos \theta = \frac{\sqrt{3}}{2}$, to find θ , find the inverse cosine of both sides: (p. 294)

$$\begin{aligned}\cos \theta &= \frac{\sqrt{3}}{2} \\ \cos^{-1}(\cos \theta) &= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ \theta &= 30^\circ\end{aligned}$$

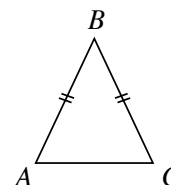
irrational numbers The set of numbers that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. For example, π and $\sqrt{2}$ are irrational numbers.

isosceles trapezoid A trapezoid with a pair of equal base angles (from the same base). The base of a trapezoid is always one of the two parallel sides. Note that the non-parallel sides on an isosceles triangle are congruent. (p. 444)



isosceles triangle A triangle with two sides of equal length. (p. 46)

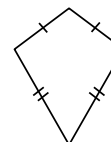
Isosceles Triangle Theorem If a triangle is isosceles, then the base angles (which are opposite the congruent sides) are congruent. For example, if $\triangle ABC$ is isosceles with $\overline{BA} \cong \overline{BC}$, then the angles opposite these sides are congruent; that is, $\angle A \cong \angle C$.



iteration The act of repeating an action over and over. (p. 760)

justify To give a logical reason supporting a statement or step in a proof. More generally, to use facts, definitions, rules, and/or previously proven conjectures in an organized sequence to convincingly demonstrate that your claim (or your answer) is valid (true).

kite A quadrilateral with two distinct pairs of consecutive congruent sides. (p. 431)



lateral edge See *pyramid*. (p. 680)

lateral face See *pyramid*. (pp. 537, 680)

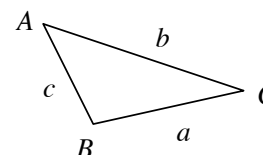
lateral surface (a) All the faces of a prism or pyramid, with the exception of the base(s); (b) On a cone, the surface not including the circular base. The lateral surface connects the apex to each point on the circular boundary of the base.

lateral surface area The sum of the areas of all the lateral surfaces of a polyhedron or solid. See *lateral surface*. (p. 680)

latitude An angular measure (in degrees) that indicates how far north or south of the equator a position on the Earth is. All points on the equator have a latitude of 0° . The North Pole has a latitude of 90° , while the South Pole has a latitude of -90° . (p. 699)

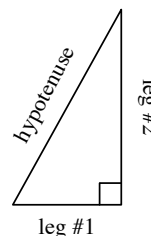


Law of Cosines For any $\triangle ABC$ with sides a , b , and c opposite $\angle A$, $\angle B$, and $\angle C$ respectively, it is always true that $c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$, $b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$, and $a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$. (p. 318)

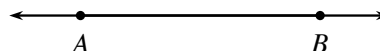


Law of Sines For any $\triangle ABC$ with sides a , b , and c opposite $\angle A$, $\angle B$, and $\angle C$ respectively, it is always true that $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. (p. 314)

legs The two sides of a right triangle that form the right angle. Note that the legs of a right triangle are always shorter than its hypotenuse. See also *hypotenuse*. (p. 128)

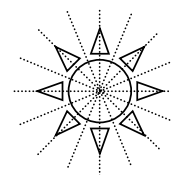


line A line is one-dimensional and extends without end in two directions. It is made up of points and has no thickness. A line can be named with a letter (such as l), but also can be labeled using two points on the line, such as $\overleftrightarrow{AB}$ below. A line is a mathematically undefined term in geometry. See also *ray* and *line segment*. (p. 88)

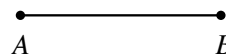


line of reflection Same as the *line of symmetry*. (pp. 37, 47)

line of symmetry A line that divides a shape into two congruent parts that are mirror images of each other. If you fold a shape over its line of symmetry, the shapes on both sides of the line will match each other perfectly. Some shapes have more than one line of symmetry, such as the example at right. Other shapes may only have one line of symmetry or no lines of symmetry. (p. 6)



line segment The portion of a line between two points. A line segment is named using its endpoints. For example, the line segment at right can be named either $\overline{AB}$ or $\overline{BA}$. See also *line*. (p. 46)



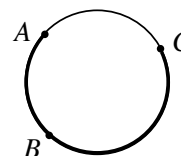
linear scale factor Same as *ratio of similarity*. (p. 499)

logical argument A logical sequence of statements and reasons that lead to a conclusion. A logical argument can be written in a paragraph, represented with a flowchart, or documented in a two-column proof. (p. 18)

longitude An angular measure (in degrees) that indicates how far west or east of the prime meridian a position on the Earth is. Lines of longitude (which are actually circles) are all great circles that pass through the North and South Poles. (p. 700)

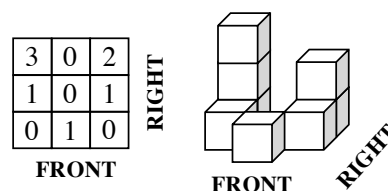


major arc An arc with measure greater than 180° . Each major arc has a corresponding minor arc that has a measure that is less than 180° . A major arc is named with three letters on the arc. For example, the highlighted arc at right is the major arc $\widehat{ABC}$. (p. 583)



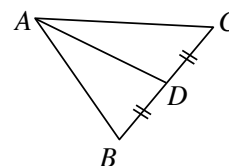
mapping In general, the term mapping is synonymous with function, but in this course we restrict its meaning to refer to functions that take points on a line or in the plane, to other points on a line or in the plane. (p. 37)

mat plan A top (or bottom) view of a multiple cube solid. The number in each square is the number of cubes in that stack. For example, the mat plan at right represents the solid at the far right. (p. 531)



measurement For the purposes of this course, a measurement is an indication of the size or magnitude of a geometric figure. For example, an appropriate measurement of a line segment would be its length. Appropriate measurements of a square would include not only the length of a side, but also its area and perimeter. The measure of an angle represents the number of degrees of rotation from one ray to the other about the vertex.

median A line segment that connects a vertex of a triangle with the midpoint of side opposite to the vertex. For example, since D is a midpoint of $\overline{BC}$, then $\overline{AD}$ is a median of $\triangle ABC$. The three medians of a triangle intersect at a point called the centroid. (p. 568)

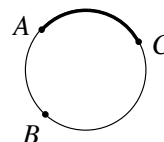


meridian A line of longitude. See *longitude* and *prime meridian*. (p. 700)

midpoint A point that divides a segment into two segments of equal length. For example, point D is the midpoint of $\overline{BC}$ in $\triangle ABC$ in the example for median. (p. 50)

midsegment A segment joining the midpoints of two sides of a triangle. (p. 444)

minor arc An arc with measure less than 180° . Each minor arc has a corresponding major arc that has a measure that is more than 180° . Minor arcs are named using the endpoints of the arc. For example, the highlighted arc at right is named $\widehat{AC}$. (p. 583)



Möbius strip A one-sided surface in a closed loop that can be formed by giving a rectangular strip of paper a half-twist and affixing its ends. (p. 9)



monomial An expression with only one term. It can be a number, a variable, or the product of a number and one or more variables. For example, 7 , $3x$, $-4ab$, and $3x^2y$ are each monomials.

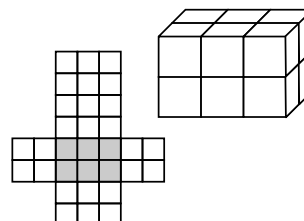
Multiplication Rule for Probability For two events, $\{A\}$ and $\{B\}$, $P(A \text{ and } B) = P(A \text{ given } B) \cdot P(B)$, or equivalently, $P(A \text{ given } B) = \frac{P(A \text{ and } B)}{P(B)}$. When the two events are independent, then $P(A \text{ given } B) = P(A)$, and the Multiplication Rule can be rewritten as $P(A \text{ and } B) = P(A) \cdot P(B)$. See also *Addition Rule for Probability*. (p. 624)

multiplier in a geometric sequence The number by which one term in a geometric sequence is multiplied to generate the next term. The multiplier is also known as the common ratio or the sequence generator. In the sequence $5, 15, 45, \dots$, the multiplier is 3 . See also *geometric sequence*. (p. 223)

mutually exclusive Two events $\{A\}$ and $\{B\}$ are mutually exclusive if they have no outcomes in common. That is, $P(A \text{ and } B) = 0$. This situation is also called disjoint. (p. 610)

n -gon A polygon with n sides. A polygon is often referred to as an n -gon when we do not yet know the value of n . (p. 473)

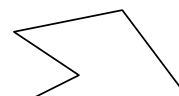
net A diagram that, when folded, forms the surface of a three-dimensional solid. A net is essentially the faces of a solid laid flat. It is one of several ways to represent a three-dimensional diagram. The diagram at right is a representation of the solid at the far right. Note that the shaded region of the net indicates the (bottom) base of the solid. (p. 535)



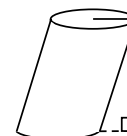
nonagon A polygon with nine sides. (p. 473)

nonahedron A polyhedron with nine faces. (p. 674)

non-convex polygon A polygon that is not convex. An example of a non-convex polygon is shown at right. See also *convex polygon*. (p. 476)

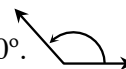


oblique prism (or cylinder) A prism (or cylinder) is oblique when its lateral surface(s) are not perpendicular to the base. See figure at right. See also *Cavalieri's Principle*. (p. 541)



oblique pyramid (or cone) A pyramid (or cone) is oblique when its apex is not directly above the center of the base. (p. 684)

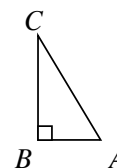
obtuse angle Any angle that measures between (but not including) 90° and 180° . (p. 26)



octagon A polygon with eight sides. (p. 52)

octahedron A polyhedron with eight faces. In a regular octahedron, all of the faces are equilateral triangles. (p. 674)

opposite In a figure, opposite means “across from.” For example, in a triangle, an opposite side is the side that is across from a particular angle and is not a side that makes the angle. For example, the side $\overline{AB}$ in $\triangle ABC$ at right is opposite $\angle C$, while $\overline{AC}$ is opposite the right angle. (p. 285)

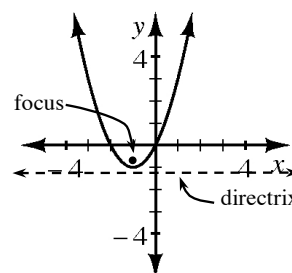


opposite interior angles See *remote interior angles*.

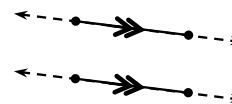
orientation In this course, orientation refers to the placement and alignment of a figure in relation to others or an object of reference (such as coordinate axes). Unless it has rotation symmetry, the orientation of a figure changes when it is rotated (turned) less than 360° . Also, the orientation of a shape changes when it is reflected, except when reflected across a line of symmetry.

outcome The result of a probabilistic situation. The set of all possible outcomes of a probabilistic situation is called the sample space. See also *event*.

parabola A parabola is a conic section created by slicing a cone with a plane parallel to a lateral edge of the cone. Parabolas may also be generated geometrically: the set of all points that are equidistant from a single point (the focus) and a line (the directrix). The graph of a quadratic equation is a parabola. A quadratic equation can be given in standard form $y = ax^2 + bx + c$ or in graphing form $y = a(x - h)^2 + k$, with vertex at (h, k) . (p. 749)



parallel lines Two lines on a flat surface are parallel if they never intersect. Two line segments on a flat surface are parallel if the lines they lie on never intersect. There is a constant distance between two parallel lines (or line segments). Identical arrow markings are used to note parallel lines or line segments as shown in the diagrams at right. (p. 59)



parallelogram A quadrilateral with two pairs of parallel sides. (p. 431)



pentagon A polygon with five sides. (p. 52)

pentagram A pentagram is created by connecting five equally-spaced points on the circumference of a circle. Note that all of the corresponding lengths and angles are equal to each other. (p. 738)



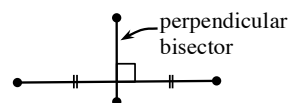
pentahedron A polyhedron with five faces. (p. 674)

perimeter The distance around the exterior of a figure on a flat surface. For a polygon, the perimeter is the sum of the lengths of its sides. The perimeter of a circle is also called the circumference. (p. 16)

permutation An arrangement of items in which items cannot be selected more than once and the order of selection matters. The number of permutations that can be made by selecting r items from a set of n total items can be represented with tree diagrams or decision charts, or calculated using the formula ${}_nP_r = \frac{n!}{(n-r)!}$. For example, if each of five letters, A, B, C, D, E, is printed on a card, the number of 3-letter sequences can you make by selecting three of the five cards is a permutation, ${}_5P_3 = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 5 \cdot 4 \cdot 3 = 60$. See also *combination*. (p. 638)

perpendicular Two rays, line segments, or lines that meet (intersect) to form a right angle (90°) are called perpendicular. A line and a flat surface can also be perpendicular if the line does not lie on the flat surface, but intersects it and forms a right angle with every line on the flat surface passing through the point of intersection. A small square at the point of intersection of two lines or segments indicates that the lines are perpendicular. (p. 59)

perpendicular bisector The perpendicular bisector of a line segment is a line segment perpendicular to the original segment, and passing through its midpoint. (p. 564)



phi (ϕ) A Greek letter, pronounced “fee,” that represents the golden ratio. $\phi = \frac{1+\sqrt{5}}{2}$ (p. 761)

pi (π) The ratio of the circumference (C) of the circle to its diameter (d). For every circle, $\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{C}{d}$. Numbers such as 3.14, 3.14159, or $\frac{22}{7}$ are approximations of π . (p. 512)

plane A plane is an undefined term in geometry. It is a two-dimensional flat surface that extends without end. It is made up of points and has no thickness.

Platonic solid A convex regular polyhedron. All faces of a platonic solid are congruent, regular polygons. There are only five possible Platonic solids: tetrahedron (with four faces of equilateral triangles), cube (with six faces of squares), octahedron (with eight faces of equilateral triangles), dodecahedron (with twelve faces of regular pentagons), and icosahedron (with twenty faces of equilateral triangles). (pp. 671, 674)

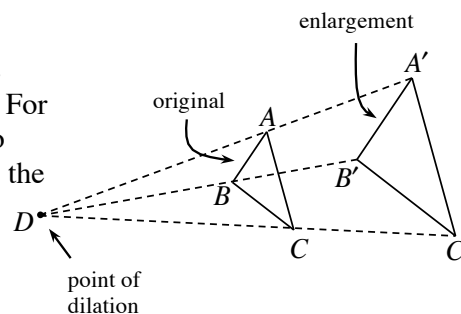
playing cards A standard deck of playing cards contains the following 52 cards:

- 13 cards with red hearts ($\heartsuit$), numbered 2 through 10, Jack, Queen, King, and Ace
- 13 cards with red diamonds ($\diamondsuit$), numbered 2 through 10, Jack, Queen, King, and Ace
- 13 cards with black spades ($\spadesuit$), numbered 2 through 10, Jack, Queen, King, and Ace
- 13 cards with black clubs ($\clubsuit$), numbered 2 through 10, Jack, Queen, King, and Ace

The face cards are the Jacks, Queens, and Kings (Aces are not considered face cards).

point of concurrency The single point where two or more lines intersect on a plane. For the points of concurrency of a triangle, see *circumcenter*, *centroid*, and *incenter*. (pp. 568, 604)

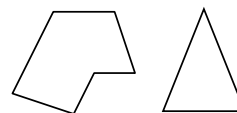
point of dilation The point from which a figure is stretched proportionally when the figure is dilated. For example, in the diagram at right, $\triangle ABC$ is dilated to form $\triangle A'B'C'$. Notice that while a dilation changes the size and location of the original figure, it does not rotate or reflect the original. While lengths can change, angles do not change under a dilation. (p. 148)



Core Connections Geometry

point of intersection See *intersection*. (p. 95)

polygon A two-dimensional closed figure of three or more line segments (sides) connected end to end. Each segment is a side and only intersects the endpoints of its two adjacent sides. Each point of intersection is a vertex. At right are two examples of polygons. See also *regular polygon*. (p. 52)



polyhedron (plural: polyhedra) A three-dimensional object with no holes that is bounded by polygons. The polygons are joined at their sides, forming the edges of the polyhedron. Each polygon is a face of the polyhedron. See also *solid*. (p. 674)

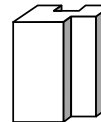
polynomial An algebraic expression that involves at most the operations of addition, subtraction, and/or multiplication. A polynomial in one variable is an expression that can be written as the sum of terms that have the form: (any number) $\cdot x^{(\text{whole number})}$. (p. 319)

postulates Statements accepted as true without proof. Also known as axioms.

preimage The original figure in a transformation.

prime meridian The line of longitude on the Earth chosen to represent 0° . It passes through Greenwich, England, on the eastern edge of London. (p. 700)

prism A three-dimensional figure that consists of two parallel congruent polygons (called bases) and a lateral surface containing segments connecting each point on each side of one base to the corresponding point on the other base. The lateral surface of a prism consists of parallelograms. See also *right prism* and *oblique prism*. (p. 537)



probabilistic situation A situation in which the outcomes are determined by random processes. See also *random*.

probability A number that represents how likely an event is to happen. When an event has a finite number of equally-likely outcomes, the probability that one of those outcomes, called A, will occur is expressed as a ratio and written as:

$$P(A) = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}.$$

For example, when flipping a coin, the probability of getting tails, $P(\text{tails})$, is $\frac{1}{2}$ because there is only one tail (successful outcome) out of the two possible equally-likely outcomes (a head and a tail). Probability can be written as a ratio, decimal, or percent. A probability of 0 (or 0%) indicates that it is impossible for the event to occur, while a probability of 1 (or 100%) indicates that the event must occur. Events that “might happen” will have values somewhere between 0 and 1 (or between 0% and 100%). (p. 31)

probability table Another name for *two-way table*.

proof A convincing logical argument that uses definitions and previously proven conjectures in an organized sequence to show that a conjecture is true. A proof can be written in a paragraph, represented with a flowchart, or documented in a formal two-column proof. (p. 11)

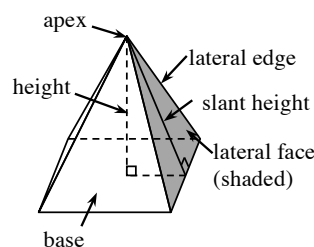
proof by contradiction A proof that begins by assuming that an assertion is true and then shows that this assumption leads to a contradiction of a known fact. This demonstrates that the assertion is false. Also known as an indirect proof. (p. 105)

proportional equation An equation stating that two ratios are equal. For example, the equation below is a proportion. A proportion is a useful type of equation to set up when solving problems involving proportional relationships. (p. 158)

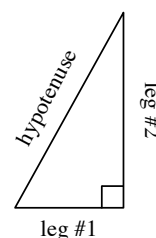
$$\frac{68 \text{ votes for Mr. Mears}}{100 \text{ people surveyed}} = \frac{34 \text{ votes for Mr. Mears}}{50 \text{ people surveyed}}$$

protractor A geometric tool used for measuring the number of degrees in an angle. (p. 24)

pyramid A polyhedron with a polygonal base formed by connecting each vertex of the base to a single point (the apex) that is above the flat surface containing the base. The lateral faces of a pyramid are the triangles formed by connecting consecutive vertices of the base to the apex. Lateral edges are two of the sides of these lateral triangular faces. The slant height is the height of a lateral triangular face. See also *right pyramid* and *oblique pyramid*. (p. 679)



Pythagorean Theorem The statement relating the lengths of the legs of a right triangle to the length of the hypotenuse: $(\text{leg \#1})^2 + (\text{leg \#2})^2 = \text{hypotenuse}^2$. The Pythagorean Theorem is powerful because if you know the lengths of any two sides of a right triangle, you can use this relationship to find the length of the third side. (p. 133)



Pythagorean Triple Any three positive integers a , b , and c that make the relationship $a^2 + b^2 = c^2$ true. Commonly used Pythagorean Triples include 3, 4, 5 and 5, 12, 13. (p. 310)

quadratic function (equation) An equation that can be written in the form $ax^2 + bx + c = 0$, where a , b , and c are real numbers and a is nonzero. A quadratic equation written in this form is said to be in standard form. For example, $3x^2 - 4x + 7.5 = 0$ is a quadratic equation in standard form. The graph of a quadratic function is a parabola. (p. 319)

Quadratic Formula The Quadratic Formula states that if $ax^2 + bx + c = 0$ and $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. For example, if $5x^2 + 9x + 3 = 0$, then $x = \frac{-9 \pm \sqrt{9^2 - 4(5)(3)}}{2(5)} = \frac{-9 \pm \sqrt{21}}{10}$. (p. 238)

quadrilateral A polygon with four sides. (pp. 52, 431)

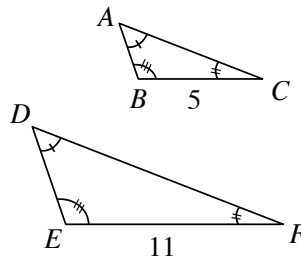
radian The ratio of the arc length to the radius for any sector of a circle. The ratio will be constant for a given central angle, regardless of the length of the radius of the circle. Along with degrees, radians are a very common method unit of measurement for the central angle of a circle. (p. 516)

radius (plural: radii) (1) Of a circle: A line segment drawn from the center of a circle to a point on the circle. Note that the length of a circle's radius is half the length of the circle's diameter. See also *circle*. (2) Of a regular polygon: A line segment that connects the center of a regular polygon with a vertex. The length of a radius is usually denoted r . (p. 404)

random An outcome is random if it happens by chance, that is, its result cannot be predicted from previous outcomes. A sequence of several random outcomes in a row will have no predictable order or pattern. For example, the outcome of rolling a die is random because the outcome cannot be predicted, and many rolls in a row will have no predictable pattern.

ratio A ratio compares two quantities by division. A ratio can be written using a colon, but is more often written as a fraction. For example, in the two similar triangles shown below, a ratio can be used to compare the length of $\overline{BC}$ in $\triangle ABC$ with the length of $\overline{EF}$ in $\triangle DEF$. This ratio can be written as 5:11 or as the fraction $\frac{5}{11}$. (p. 154)

ratio of similarity $r : r^2 : r^3$ The ratio of any pair of corresponding sides of two similar figures. This means that once it is determined that two figures are similar, all of their pairs of corresponding sides have the same ratio. For example, for the similar triangles $\triangle ABC$ and $\triangle DEF$ above, the ratio of similarity is $\frac{5}{11}$. The ratio of similarity can also be called the linear scale factor. When the ratio is comparing a figure and its image after a dilation, this ratio can also be called the zoom factor.



When a two-dimensional figure is enlarged (or reduced) proportionally, its lengths and area change. If the linear scale factor is r , then all lengths (such as sides, perimeter, and heights) of the original (preimage) figure is multiplied by a factor of r while the area is multiplied by a factor of r^2 .

When a solid is enlarged (or reduced) proportionally, its lengths, surface area, and volume also change. If the linear scale factor is r , then the surface area is multiplied by a factor of r^2 and the volume is multiplied by a factor of r^3 . Thus, if a solid is enlarged proportionally by a linear scale factor of r , then the new edge lengths, surface area, and volume can be found using the relationships below. (pp. 154, 499)

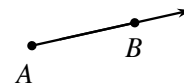
New edge length = $r \cdot$ (corresponding edge length of original polyhedron)

New surface area = $r^2 \cdot$ (original surface area)

New volume = $r^3 \cdot$ (original volume)

rationalizing the denominator Rewriting a fractional expression that has radicals in the denominator, in order to eliminate the radicals from the denominator. For example, $\frac{2}{\sqrt{3}}$ can be rationalized as follows: $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$. (p. 299)

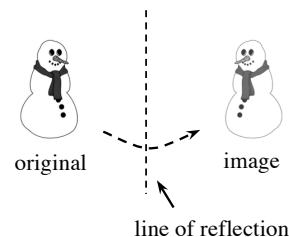
ray A ray is part of a line that starts at one endpoint and extends without end in one direction. In the example at right, ray $\overrightarrow{AB}$ is part of $\overline{AB}$ that starts at A and contains all of the points of $\overline{AB}$ that are on the same side of A as point B, including A. Point A is the endpoint of $\overrightarrow{AB}$. See also *line*.



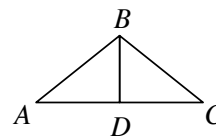
reasoning See *logical argument* and *proof*.

rectangle A quadrilateral with four right angles. (p. 431)

reflection A transformation across a line that produces a mirror image of the original (preimage) shape. The reflection is called the image of the original figure. The line is called a line of reflection. See the example at right. Note that a reflection is also sometimes referred to as a “flip.” (pp. 37, 47)



Reflexive Property The Reflexive Property states that any expression is always equal to itself. That is, $a = a$. This property is often useful when proving that two triangles that share a side or an angle are congruent. For example, in the diagram at right, since $\triangle ABD$ and $\triangle CBD$ share a side ($\overline{BD}$), the Reflexive Property justifies that $BD = BD$. (p. 421)



reflection symmetry See *symmetry*.

regular polygon A polygon is regular if it is a convex polygon with congruent angles and congruent sides. For example, the shape at right is a regular hexagon. (p. 416)



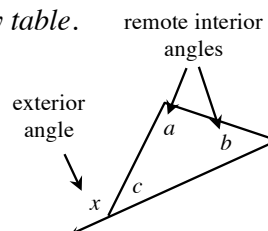
relation An equation that relates two or more variables. For example, $y = 3x - 2$ and $x^2 + y^2 = 9$ are both relations. See also *function*.

relationship For this course, a relationship is a way that two objects (such as two line segments or two triangles) are connected. When you know that the relationship holds between two objects, learning about one object can give you information about the other. Relationships can be described in two ways: a geometric relationship (such as a pair of vertical angles or two line segments that are parallel) and a relationship between the measures (such as two angles that are complementary or two sides of a triangle that have the same length). Common geometric relationships between two figures include being similar (when two figures have the shape, but not necessarily the same size) and being congruent (when two figures have the same shape and the same size).

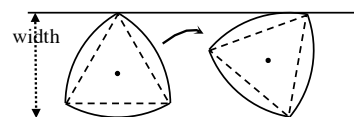
Core Connections Geometry

relative frequency table A two-way table that contains entries that are fractions or percents (as opposed to counts). See also *two-way table*.

remote interior angles If a triangle has an exterior angle, the remote interior angles are the two angles not adjacent to the exterior angle. Also called opposite interior angles. (p. 477)



Reuleaux curve The curve created by drawing arcs between two consecutive vertices (with a center at the opposite vertex) on a regular polygon with an odd number of sides. As the curve is “rolled,” the width of the figure is constant. (p. 400)

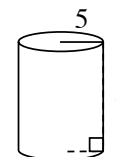


rhombus A quadrilateral with four congruent sides. (p. 560)

right angle An angle that measures 90° . A small square is used to note a right angle, as shown in the example at right. (p. 26)



right prism (or cylinder) A prism (or cylinder) with lateral surface(s) that are perpendicular to the base. See the figure at right.



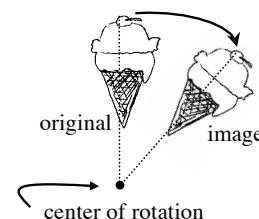
right pyramid (or cone) A pyramid (or cone) with its apex directly above the center of the base. (p. 684)

right triangle A triangle that has one right angle. The side of a right triangle opposite the right angle is called the hypotenuse, and the two sides adjacent to the right angle are called legs. Note that legs of a right triangle are always shorter than its hypotenuse. (p. 128)

rigid motions See *rigid transformations*.

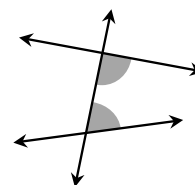
rigid transformations Movements of figures that preserve their shape and size. Examples of rigid transformations are reflections, rotations, and translations. Also called rigid motions or “isometries.” See also *similarity transformations*. (p. 37)

rotation A transformation that turns all of the points in the original (preimage) figure the same number of degrees around a fixed center point (such as the origin on a graph). The result is called the image of the original figure. The point that the shape is rotated about is called the center of rotation. To define a rotation, you need to state the measure of turn (in degrees), the direction the shape is turned (such as clockwise or counter-clockwise), and the center of rotation. See the example at right. Note that a rotation is also sometimes referred to as a “turn.” (pp. 37, 47)



rotation symmetry See *symmetry*. (p. 56)

same-side interior angles Two angles between two lines and on the same side of a third line that intersects them (called a transversal). The shaded angles in the diagram at right are an example of a pair of same-side interior angles. Note that if the two lines that are cut by the transversal are parallel, then the two angles are supplementary (add up to 180°). Conversely, if the two angles are supplementary, then the two lines that are cut by the transversal are parallel. See also *alternate interior angles* and *corresponding angles*. (p. 100)



sample space The set of all possible outcomes of a probabilistic situation. The sample space for rolling a standard 6-sided die is the set $\{1, 2, 3, 4, 5, 6\}$ because those are all the possible outcomes. (p. 31)

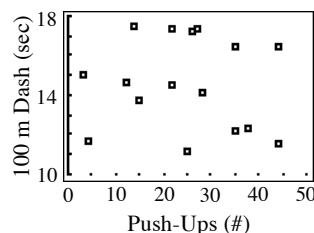
SAS $\cong$ (Triangle Congruence) Two triangles are congruent if two sides and their included angle of one triangle are congruent to the corresponding two sides and included angle of another triangle. (p. 358)

SAS $\sim$ (Triangle Similarity) If two triangles have two pairs of corresponding sides that are proportional and have congruent included angles, then the triangles are similar. (pp. 170, 192)

scale The ratio between a length of the representation (such as a map, model, or diagram) and the corresponding length of the actual object. For example, the map of a city may use one inch to represent one mile.

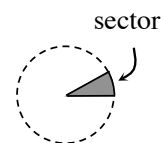
scalene triangle A triangle with no congruent sides. (p. 68)

scatterplot A way of displaying two-variable numerical data where two measurements are taken for each subject (like height and forearm length, or surface area of cardboard and volume of cereal held in a cereal box). To create a scatterplot, the two values for each subject are written as coordinate pairs and graphed on a pair of coordinate axes (each axis representing a variable). See also *association*.



secant A line that intersects a circle at two distinct points. (p. 598)

sector A region formed by two radii of a central angle and the arc between their endpoints on the circle. You can think of it as a portion of a circle and its interior, resembling a piece of pizza. (p. 512)



segment See *line segment*. (p. 46)

semicircle In a circle, a semicircle is an arc with endpoints that are endpoints of any diameter of the circle. It is a half circle and has a measure of 180° . (p. 592)

sequence An ordered sequence of mathematical terms. A sequence is a function in which the independent variable is a positive integer (sometimes called the term number). The dependent variable is the term value. A sequence is usually written as a list of numbers, starting with the first term where $n = 1$. See also *arithmetic sequence* and *geometric sequence*. (p. 329)

sequence generator The generator of a sequence tells what you do to each term to get the next term. Note that this is different from the function for the n^{th} term of the sequence. The generator only tells you how to find the following term, when you already know one term. In an arithmetic sequence the generator is the common difference; in a geometric sequence it is the multiplier or common ratio. See also *arithmetic sequence* and *geometric sequence*.

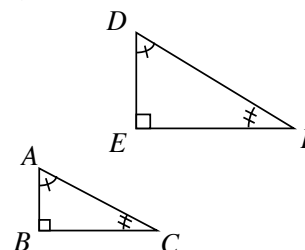
side of an angle One of the two rays that form an angle.

side of a polygon See *polygon*.

similar figures Two shapes are similar if there is a sequence of rigid motions, followed by a dilation, that carries one onto the other. The corresponding angles of similar polygons are congruent, and the corresponding sides are proportional. The symbol for similar is $\sim$. See also *ratio of similarity*. (p. 163)

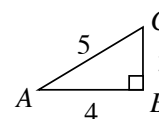
similarity conditions See *triangle similarity conditions*. (p. 166)

similarity statement A statement that indicates that two figures are similar. The order of the letters in the names of the shapes in the similarity statement indicates which sides and angles correspond to each other. For example, $\triangle ABC \sim \triangle DEF$ is a similarity statement. It indicates that $\angle A$ corresponds to $\angle D$, and $\overline{AB}$ corresponds to $\overline{DE}$. (p. 163)



similarity transformations Movements of figures that preserve their shape, but not necessarily their size. Examples of similarity transformations are reflections, rotations, translations, and dilations. See also *rigid transformations*. (p. 163)

sine ratio In a right triangle, the sine ratio of an acute angle ($\angle A$) is $\sin A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$. In the triangle at right, $\sin A = \frac{BC}{AC} = \frac{3}{5}$. See also *cosine ratio*, *tangent ratio*, and *inverse sine*. (p. 285)



skew lines Lines that do not lie in the same flat surface (or plane).

slant height See *pyramid* or *cone*. (p. 680)

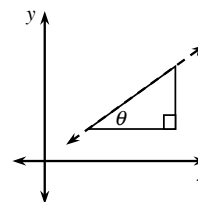
slide See *translation*.

slope A ratio that describes how steep (or flat) a line is. Slope can be positive, negative, or even zero, but a straight line has only one slope. Slope is the ratio $\frac{\text{vertical change}}{\text{horizontal change}}$ or $\frac{\text{change in } y\text{-value}}{\text{change in } x\text{-value}}$, sometimes written $\frac{\Delta y}{\Delta x}$. When the equation of a line is written in $y = mx + b$ form, m is the slope of the line. A line has positive slope if it slopes upward from left to right on a graph, negative slope if it slopes downward from left to right, zero slope if it is horizontal, and undefined slope if it is vertical. Parallel lines have equal slopes, and the slopes of perpendicular lines are opposite reciprocals of each other (e.g., $\frac{3}{5}$ and $-\frac{5}{3}$). (p. 59)

slope angle The acute angle a line forms with the x -axis on a coordinate graph. Also see *slope triangle*. (p. 214)

Slope-Intercept Form A form of a linear equation: $y = mx + b$. In this form, m is the slope and the point $(0, b)$ is the y -intercept. (p. 129)

slope triangle A right triangle with legs (parallel to the x - and y -axes) that meet the hypotenuse at two points on a given line or line segment. The lengths of the legs of the slope triangle can be used to find the slope of the line. The angle (θ) formed in a slope triangle by the hypotenuse and the horizontal leg is equivalent to the slope angle. See also *slope angle*.



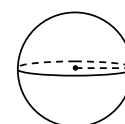
solid A closed three-dimensional shape and all of its interior points. Examples include regions bounded by pyramids, cylinders, and spheres. (p. 537)

solve (an equation) To find all the solutions to an equation or an inequality (or a system of equations or inequalities). Compare evaluating an expression, in which you know the value of a variable and use it to rewrite an expression, to solving an equation, in which you determine the value of a variable. For example, solving the equation $x^2 = 9$ gives the solutions $x = 3$ and $x = -3$. The solution(s) may be number(s), variable(s), or an expression.

space The set of all points in three dimensions.

special right triangle A right triangle with particular notable features that can be used to solve problems. Sometimes, these triangles can be recognized by the angles, such as the 45° - 45° - 90° triangle (also known as an isosceles right triangle) and a 30° - 60° - 90° triangle. Some people also categorize triangles for which the sides are Pythagorean Triples as special triangles. (p. 310)

sphere The set of all points in space that are the same distance from a fixed point. The fixed point is the center of the sphere and the distance is its radius. (p. 706)



Core Connections Geometry

square A quadrilateral with four right angles and four congruent sides. (pp. 416, 431)

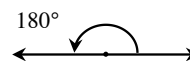


square root A number a is a square root of b if $a^2 = b$. For example, the number 9 has two square roots, 3 and -3 . A negative number has no real square roots; a positive number has two; and zero has just one square root, namely, itself. In a geometric context, the principal square root of a number x (written $\sqrt{x}$) represents the length of a side of a square with area x . For example, $\sqrt{16} = 4$. Therefore if the side of a square has a length of 4 units, then its area is 16 square units. (p. 299)

SSS $\cong$ (Triangle Congruence) Two triangles are congruent if all three pairs of corresponding sides are congruent. (p. 358)

SSS $\sim$ (Triangle Similarity) If two triangles have all three pairs of corresponding sides that are proportional (this means that the ratios of corresponding sides are equal), then the triangles are similar. (p. 192)

straight angle An angle that measures 180° . This occurs when the rays of the angle point in opposite directions, forming a line. (p. 26)



straightedge A tool used as a guide to draw lines, rays, and segments. A ruler has measurement markings while a straightedge does not. (p. 552)

star polygon A star polygon is created by connecting equally-spaced points on the circumference of a circle. The star polygon to the right is commonly called a pentagram. (p. 761)



statement A recording of fact to present evidence in a logical argument (proof). (p. 119)

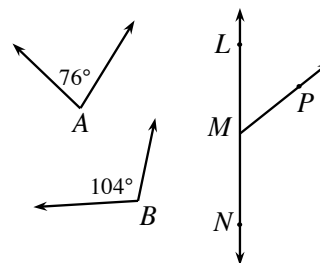
stretch point See *point of dilation*. (p. 148)

Substitution Method A method for solving a system of equations by replacing one variable with an expression involving the remaining variable(s). For example, in the system of equations at right the first equation tells you that y is equal to $-3x + 5$. We can substitute $-3x + 5$ in for y in the second equation to get $2(-3x + 5) + 10x = 18$, then solve this equation to find that $x = 2$. Once we have x , we substitute that value back into either of the original equations to find that $y = -1$. (p. 95)

$$\begin{aligned} y &= -3x + 5 \\ 2y + 10x &= 18 \end{aligned}$$

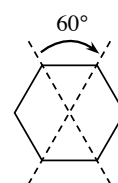
Substitution Property The Substitution Property states that in an expression, one can replace a variable, number, or expression with something equal to it without altering the value of the whole. For example, if $x = 3$, then $5x - 7$ can be evaluated by replacing x with 3. This results in $5(3) - 7 = 8$.

supplementary angles Two angles a and b for which $a + b = 180^\circ$. Each angle is called the supplement of the other. In the example at right, angles A and B are supplementary. Supplementary angles are often adjacent. For example, since $\angle LMN$ is a straight angle, then $\angle LMP$ and $\angle PMN$ are supplementary angles because $m\angle LMP + m\angle PMN = 180^\circ$. (p. 83)



surface area The sum of all the area(s) of the surface(s) of a three-dimensional solid. For example, the surface area of a cylinder is the sum of the areas of its top base, its bottom base, and its lateral surface. (p. 542)

symmetry (1) Rotation symmetry: A shape has rotation symmetry when it can be rotated for less than 360° and it appears not to change. For example the hexagon at right appears not to change if it is rotated 60° clockwise (or counter-clockwise); (2) Reflection symmetry: A shape has reflection symmetry when it appears not to change after being reflected across a line. The regular hexagon at right also has reflection symmetry across any line drawn through opposite vertices, such as those shown in the diagram. See *reflection, rotation, and line of symmetry*. (p. 55)

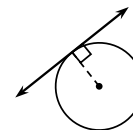


system of equations A system of equations is a set of equations with the same variables. Solving a system of equations means finding one or more solutions that make each of the equations in the system true. A solution to a system of equations gives a point of intersection of the graphs of the equations in the system. There may be zero, one, or several solutions to a system of equations. For example, $(1.5, -3)$ is a solution to the system of linear equations at right; $x = 1.5$ and $y = -3$ makes both of the equations true. Also, $(1.5, -3)$ is a point of intersection of the graphs of these two equations. (p. 95)

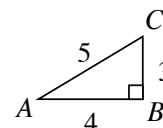
$$\begin{aligned} y &= 2x - 6 \\ y &= -2x \end{aligned}$$

systematic list A list created by following a system (an orderly process).

tangent A line on the same flat surface as a circle that intersects the circle in exactly one point. A tangent of a circle is perpendicular to a radius of the circle at their point of intersection (also called the point of tangency). (p. 712)



tangent ratio In a right triangle, the tangent ratio of an acute $\angle A$ is $\tan(A) = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$. In the triangle at right, $\tan(A) = \frac{BC}{AB} = \frac{3}{4}$. See also *cosine ratio, sine ratio, and inverse tangent*. (p. 228)



tetrahedron A polyhedron with four faces. The faces of a tetrahedron are triangles. In a regular tetrahedron, all of the faces are congruent equilateral triangles. Any of the faces of a tetrahedron can be considered the base. (p. 404)

theorem A conjecture that has been proven to be true. Some examples of theorems are the Pythagorean Theorem and the Triangle Angle Sum Theorem. (p. 82)

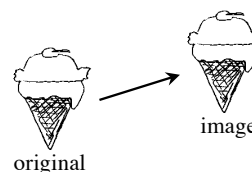
theoretical probability A calculated probability based on the possible outcomes when each outcome has the same chance of occurring: (number of successful outcomes)/(total number of possible outcomes). (p. 31)

theta (θ) A Greek letter that is often used to represent the measure of an angle. Other Greek letters used to represent the measure of an angle include alpha (α) and beta (β). (p. 214)

three-dimensional An object that has length, width, and depth. (p. 403)

transformation This course studies four transformations: reflection, rotation, translation, and dilation. All of them preserve shape, and the first three preserve size. See each term for its particular definition. (p. 50)

translation A transformation that preserves the size, shape, and orientation of a figure while sliding (moving) it to a new location. The result is called the image of the original figure (preimage). Note that a translation is sometimes referred to as a “slide.” (pp. 37, 47)



transversal A line that intersects two or more other lines on a flat surface (plane). In this course, we often work with a transversal that intersects two parallel lines. (p. 86)

trapezoid A quadrilateral with at least one pair of parallel sides. (p. 431)

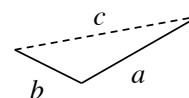
tree diagram A model used to organize the possible outcomes, and the respective probabilities, of two or more events. For an example of a tree diagram, see the Math Notes box *Probability Models* in Lesson 4.2.3. (p. 241)

triangle A polygon with three sides. (p. 52)

Triangle Angle Sum Theorem The sum of the measures of the interior angles in any triangle is 180° . (p. 109)

triangle congruence conditions Conditions that use the minimum number of congruent corresponding parts to prove that two triangles are congruent. They are: SSS $\cong$, SAS $\cong$, AAS $\cong$, ASA $\cong$, and HL $\cong$. (p. 358)

Triangle Inequality In a triangle with side lengths a , b , and c , c must be less than the sum of a and b and greater than the difference of a and b . In the example at right, a is greater than b (that is, $a > b$), so the possible values for c are all numbers such that $c > a - b$ and $c < a + b$. (p. 128)



Triangle Midsegment Theorem The segment that connects the midpoints of any two sides of a triangle measures half the length of the third side and is parallel to that side. (p. 444)

triangle similarity conditions Conditions that use the minimum number of congruent and/or proportional corresponding parts to prove that two triangles are similar. They are: SSS $\sim$, SAS $\sim$, and AA $\sim$. (p. 192)

trigonometry Literally, the “measure of triangles.” In this course, this word is used to refer to the development of triangle tools such as trigonometric ratios (sine, cosine, and tangent) and the Laws of Sines and Cosines. (p. 217)

turn See *rotation*.

two-column proof A form of proof in which statements are written in one column as a list and the reasons for the statements are written next to them in a second column. (p. 439)

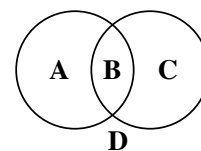
two-dimensional A figure that lies on a flat surface and that has length and width. See also *plane*. (p. 403)

two-way table A way to display categorical two-variable data. The categories of one of the variables is the header of the rows, the other variable is the header of the columns. The entries in the table can be counts (frequencies) or percents (relative frequencies). See also *relative frequency table*.

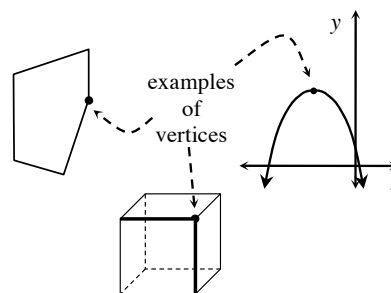
union of two events The union of the two events $\{A\}$ and $\{B\}$ is a new event that includes all the outcomes that either A will occur *or* B will occur. For example, if event $\{A\}$ is the thirteen diamond-suited cards in a deck of playing cards, and $\{B\}$ is the four Aces, then the event $\{A \text{ union } B\}$ contains 16 outcomes: $\{\text{all the diamonds (including the } A\spadesuit), A\heartsuit, A\clubsuit, \text{ and } A\spadesuit\}$. See also *intersection of two events*. (p. 255)

unit of measure A standard quantity (such as a centimeter, second, square foot, or gallon) that is used to measure and describe an object. A single object can be measured using different units of measure, which will usually yield different results. For example, a pencil may be 80 mm long, meaning that it is 80 times as long as a unit of 1 mm. However, the same pencil is 8 cm long, so that it is the same length as 8 cm laid end-to-end. (This is because 1 cm is the same length as 10 mm.) (p. 107)

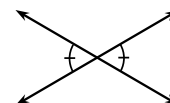
Venn diagram A type of diagram used to classify objects. It is usually composed of two or more overlapping circles representing different conditions. An item is placed or represented in the Venn diagram in the appropriate position based on the conditions it meets. In the example of the Venn diagram at right, if an object meets one of two conditions, it is placed in region A or C but outside region B. If an object meets both conditions, it is placed in the intersection (B) of both circles. If an object does not meet either condition, it is placed outside of both circles (region D). (p. 64)



vertex (plural: vertices) (a) For a two-dimensional geometric shape, a vertex is a point where two or more line segments or rays meet to form a “corner,” such as in a polygon or angle. (b) For a three-dimensional polyhedron, a vertex is a point where the edges of the solid meet. See also *apex*. (c) On a graph, a vertex can be used to describe the highest or lowest point on the graph of a parabola or absolute value function (depending on the graph’s orientation). (pp. 80, 671)



vertical angles The two opposite (that is, non-adjacent) angles formed by two intersecting lines. “Vertical” is a relationship between pairs of angles, so one angle cannot be called vertical. Angles that form a vertical pair are always congruent. (p. 100)



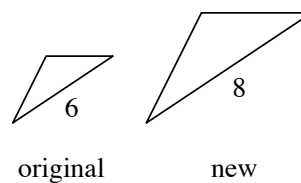
volume A measurement of the size of the three-dimensional region enclosed within an object. It is expressed as the number of $1 \times 1 \times 1$ unit cubes (or parts of cubes) that fit inside a solid. (p. 542)

x -intercept A point where a graph crosses the x -axis. A graph may have several x -intercepts, no x -intercepts, or just one.

y -intercept A point where a graph crosses the y -axis. A function has at most one y -intercept; a relation may have several.

Zero Product Property The Zero Product Property states that when the product of two or more factors is zero, one of these factors must equal zero. That is, if $a \cdot b = 0$, then either $a = 0$ or $b = 0$ (or both). For example, if $(x + 4)(2x - 3) = 0$, then either $x + 4 = 0$ or $2x - 3 = 0$ (or both). The Zero Product Property can be used to solve factorable quadratic equations. (p. 238)

zoom factor The amount each side of a figure is multiplied by when the figure is proportionally enlarged or reduced in size. It is written as the ratio of a length in the new figure (image) to a length in the original figure (preimage). For the triangles at right, the zoom factor is $\frac{8}{6}$ or $\frac{4}{3}$. See also *ratio of similarity*. (p. 154)



zero factorial Zero factorial is 1, $0! = 1$. (p. 658)