Describing Graphs of Functions

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Overview

A function can be described using various features of its graph, and graph investigation questions can be useful when examining the graph of a function. The following sample summary statements are for the square root function, $y = \sqrt{x}$.

Graph Investigation Question	Sample Summary Statement
What does the graph look like?	The graph looks like half of a parabola on its side.
Is the graph increasing or decreasing?	As x increases, y increases.
What are the x- and y-intercepts?	The graph intersects both the x- and y-axes at (0, 0).
What x-values (inputs) are possible?	The possible inputs are $x \geq 0$.
What y-values (outputs) are possible?	The possible outputs are $y \geq 0$.
Are there any special points?	The graph has a "starting" point at (0, 0).
Does the graph have any symmetry? If so, where?	The graph has no symmetry.
Are the points on the graph discrete or connected (continuous)?	The points on the graph are connected.

The more formal concepts of function, domain, and range are addressed in Lessons 1.2.2 and 1.2.3.

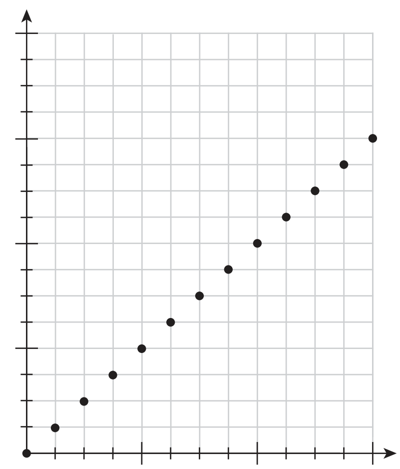
For more information, see the Methods & Meanings box in Lesson 1.1.3, and the Reflection Journal in Lesson 1.1.4.

Examples

1. At the farmer's market, apples cost \$0.50 each. Make a table of the price for different numbers of apples, draw a graph, and fully describe the graph for this situation.

Number of Apples	0	1	2	4	6	10
Cost (\$)	0	0.50	1.00	2.00	3.00	5.00

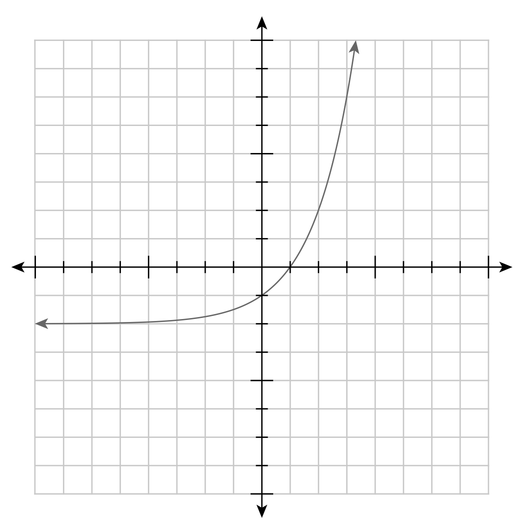
The graph starts at (0, 0) and increases linearly. The inputs are all whole numbers, and the outputs are 0 and multiples of \$0.50. The graph is discrete because you can only buy whole numbers of apples. There is no symmetry.



2. For the function $y = 2^x - 2$, make a table, draw a graph, and fully describe the features of the graph.

Create a table, adding more x-values as necessary.

x	−2	−1	0	1	2	3
y	−1.75	−1.5	−1	0	2	6



Be careful with your calculations when substituting negative exponents. Recall the properties of exponents as you work. For example, if $x = -2$, $y = 2^{-2} - 2 = \frac{1}{2^2} - 2 = -1.75$.

The graph is an increasing curve. The points are connected (the graph is continuous). The x-intercept is (1, 0). The y-intercept is (0, −1). All inputs are possible, and the outputs are greater than −2.

Practice

For each problem, make a table, draw a graph, and describe the features of the graph.

1. $y = \frac{x+4}{2}$

2. Gasoline costs $4.00 per gallon. How much does it cost to buy x gallons of gas?

3. $y = 2^x + 1$

4. A science experiment starts with 5 bacteria, and each hour the number of bacteria doubles. How many bacteria are there after x hours?

5. $y = 5 - 2x$

6. The product of two numbers is 12.

7. $y = (0.5)^x$

8. A tomato plant was 5 cm tall when planted and grows 2 cm per week. How tall is the tomato plant after x weeks?

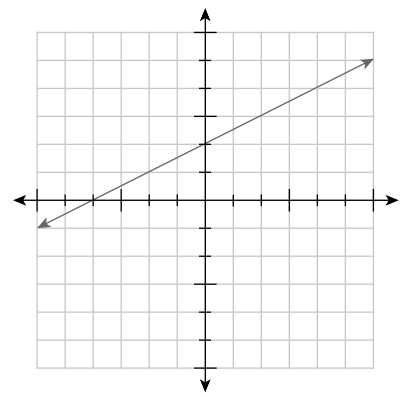
9. $y = x^2 - 4$

Answers

1.

x	–2	–1	0	1	2
y	1	1.5	2	2.5	3

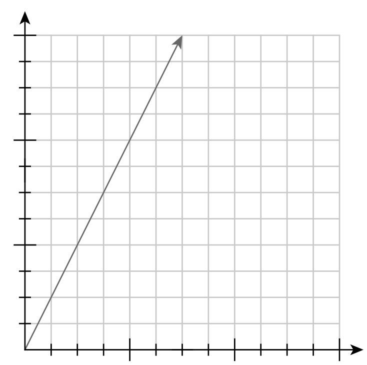
Line; intercepts (–4, 0) and (0, 2); increasing function; Inputs can be any real number. Outputs are any real number. The points are connected.



2.

x	0	1	2	3	4
y	0	4	8	12	16

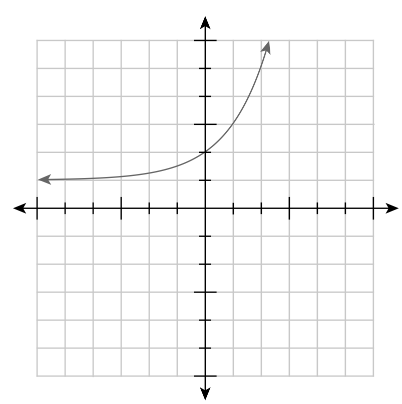
Ray (proportional graph); intercept and starting point (0, 0); increasing function; Inputs can be any non-negative number. Outputs are greater than or equal to 0. The points are connected.



3.

x	–2	–1	0	1	2
y	1.25	1.5	2	3	5

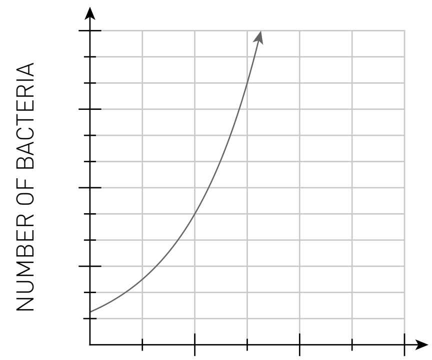
Curve; intercept (0, 2); increasing function; Inputs can be any number. Outputs are greater than 1. The points are connected.



4.

x	0	1	2	3	4
y	5	10	20	40	80

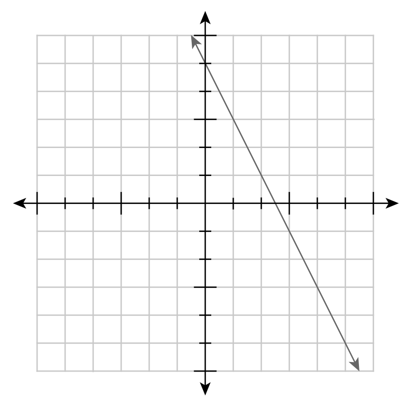
Curve; intercept and starting point (0, 5); increasing function; Inputs can be any non-negative number. Outputs are greater than or equal to 5.



5.

x	–2	–1	0	1	2
y	9	7	5	3	1

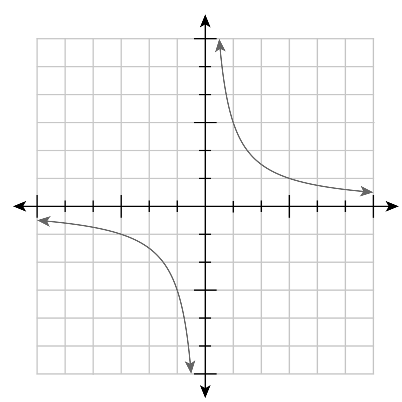
Line; intercepts (2.5, 0) and (0, 5); decreasing function; Inputs can be any real number. Outputs are any real number. The points are connected.



6.

x	–2	–1	0	1	2
y	–6	–12	undefined	12	6

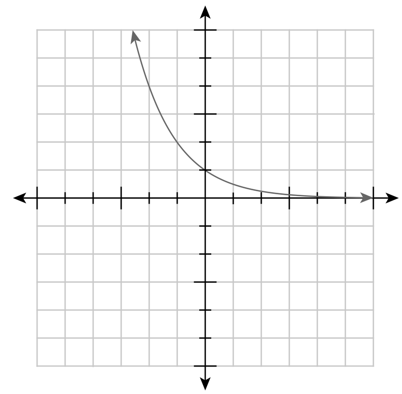
Inverse variation; no intercepts, decreasing function; Inputs can be any number except 0. Outputs are any number except 0. The points are connected except at $x = 0$.



7.

x	–2	–1	0	1	2
y	4	2	1	0.5	0.25

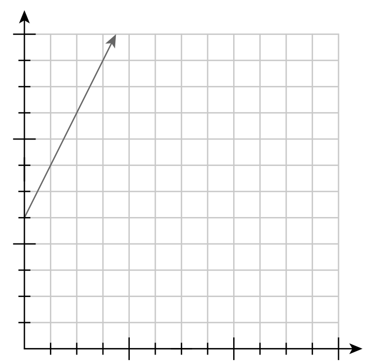
Curve; intercept (0, 1); decreasing function; Inputs can be any real number. Outputs are greater than 0. The points are connected.



8.

x	0	1	2	3	4
y	5	7	9	11	13

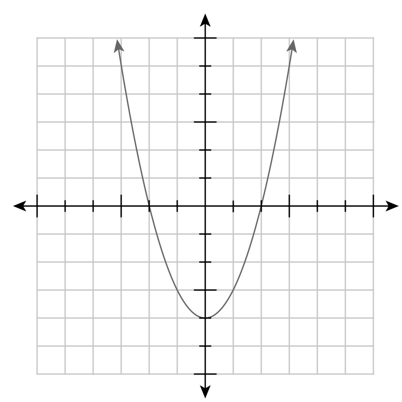
Ray; intercept and starting point (0, 5); increasing function; Inputs can be any non-negative number. Outputs are greater than or equal to 5 (but probably also less than or equal to 180 when the plant is mature).



9.

x	–2	–1	0	1	2
y	0	–3	–4	3	0

U-shape; intercepts (2, 0), (–2, 0), and (0, –4); decreasing for $x < 0$, increasing for $x > 0$; Minimum value at (0, –4). Inputs can be any real number. Outputs are greater than or equal to –4. The points are connected.



Functions

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Overview

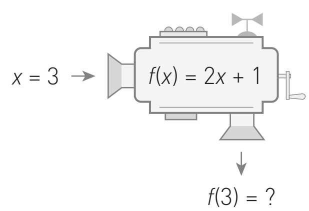
A relationship between input and output values is called a **function** if, for each input value, there is only one corresponding output value.

The set of all possible inputs of a relation is called the **domain**, while the set of all possible outputs of a relation is called the **range**.

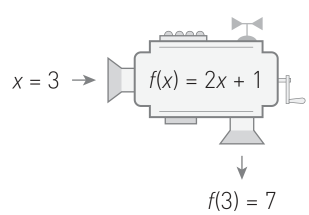
For additional information about functions, function notation, and domain and range, see the Methods & Meanings box in Lesson 1.2.3.

Examples

1. Numbers, represented by a letter or symbol such as x, are input into the function machine labeled f one at a time. The function performs the operations shown on each input to determine an output, $f(x)$. What is $f(3)$?



When $x = 3$ is input into the function f, the function machine multiplies 3 by 2 and adds 1 to get the output, 7. The notation $f(3) = 7$ shows that the function named f connects the input $x = 3$ with the output 7. This also means the point (3, 7) lies on the graph of the function.



2. a. If $f(x) = \sqrt{x-2}$, then what is $f(11)$?

$f(11) = \sqrt{11-2} = \sqrt{9} = 3$

b. If $g(x) = 3 - x^2$, then what is $g(5)$?

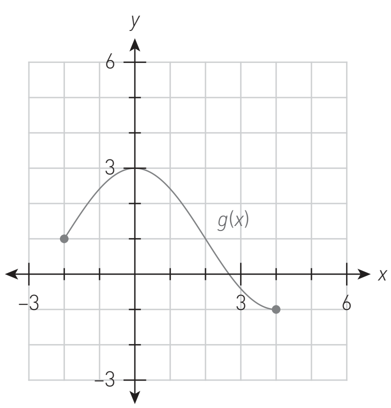
$g(5) = 3 - (5)^2 = 3 - 25 = -22$

c. If $f(x) = \frac{x+3}{2x-5}$, then what is $f(2)$?

$f(2) = \frac{2+3}{2(2)-5} = \frac{5}{-1} = -5$

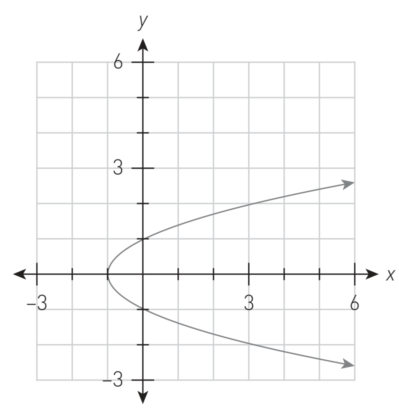
3. A relation in which each input has only one output is called a **function**.

a. Is $g(x)$ a function?



$g(x)$ is a function because each input (x) has only one output (y).

b. Is the graph shown a function?



It is not a function because each input greater than −1 has two outputs. For example, at $x = 0$, $y = 1$ and −1, and at $x = 3$, $y = 2$ and −2.

4. The set of all possible inputs of a relation is called the **domain**, while the set of all possible outputs of a relation is called the **range**.

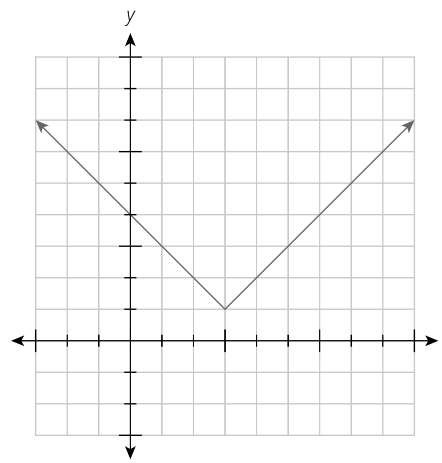
a. What are the domain and range of $g(x)$ from part (a) of Example 3?

The domain of $g(x)$ is $-2 \le x \le 4$. The range of $g(x)$ is $-1 \le y \le 3$.

b. What are the domain and range of the graph from part (b) of Example 3?

The domain is $x \ge -1$. The range is all real numbers.

5. What are the domain and range of the graph shown?

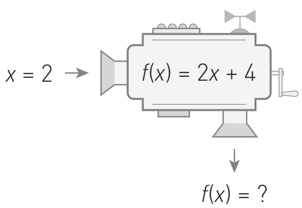


Since the x-values extend forever in both directions, the domain is all real numbers. The y-values start at 1 and go higher, so the range is $y \geq 1$ or "all numbers greater than or equal to 1."

Practice

For problems 1 through 9, determine the outputs for the given inputs for each function, if possible.

1.



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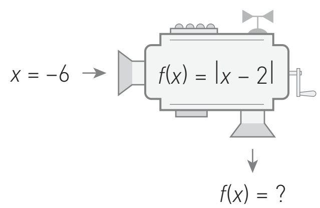
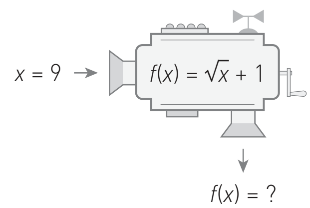


3.



4. $f(x) = (5 - x)^2$
$f(8) = ?$

5. $g(x) = x^2 - 5$
$g(-3) = ?$

6. $f(x) = \frac{2x + 7}{x^2 - 9}$
$f(3) = ?$

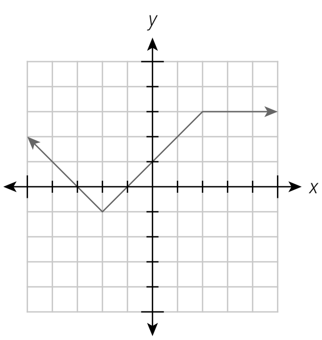
7. $h(x) = 5 - \sqrt{x}$
$h(9) = ?$

8. $h(x) = \sqrt{5 - x}$
$h(9) = ?$

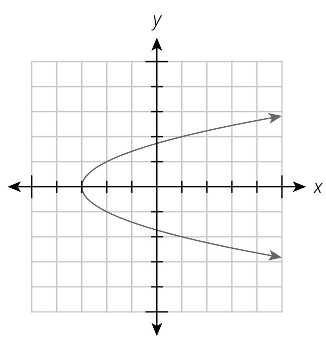
9. $f(x) = -x^2$
$f(4) = ?$

For problems 10 through 15, determine whether each relation is a function. Then state its domain and range.

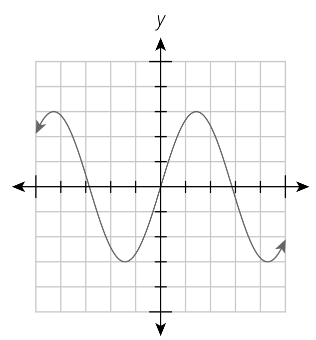
10.



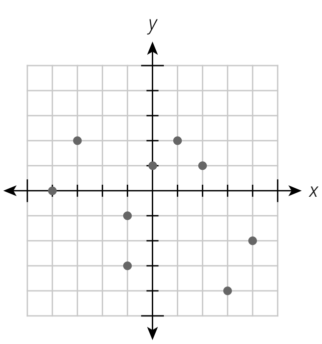
11.



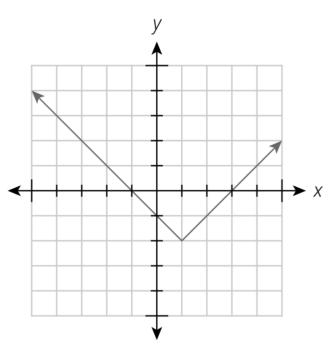
12.



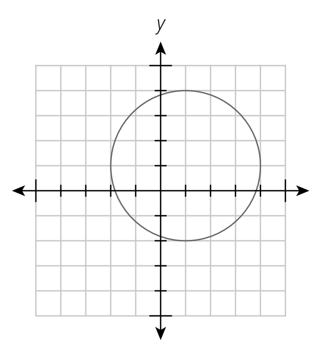
13.



14.



15.



Answers

1. $f(2) = 8$

2. $f(-6) = 8$

3. $f(9) = 4$

4. $f(8) = 9$

5. $g(-3) = 4$

6. not possible

7. $h(9) = 2$

8. not possible

9. $f(4) = -16$

10. function; domain is all real numbers, range is $-1 \le y \le 3$

11. not a function; domain is $x \ge -3$, range is all real numbers

12. function; domain is all real numbers, range is $-3 \le y \le 3$

13. not a function; domain is –4, –3, –1, 0, 1, 2, 3, 4; range is –4, –3, –2, –1, 0, 1, 2

14. function; domain is all real numbers, range is $y \ge -2$

15. not a function; domain is $-2 \le x \le 4$, range is $-2 \le y \le 4$

Properties of Exponents

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Overview

In general, to rewrite an expression that contains exponents means to eliminate parentheses and negative exponents, if possible. The basic **properties of exponents** are shown.

Property of Exponents	Algebraic Rule	Examples
Product	$x^a \cdot x^b = x^{a+b}$	$x^3 \cdot x^4 = x^7$ $2^7 \cdot 2^4 = 2^{11}$
Quotient	$\frac{x^a}{x^b} = x^{a-b}$	$\frac{x^{10}}{x^4} = x^6$ $\frac{2^4}{2^7} = 2^{-3}$
Power of a Power	$(x^a)^b = x^{ab}$	$(x^4)^3 = x^{12}$ $(2x^3)^5 = 2^5 \cdot x^{15} = 32x^{15}$
Zero Exponent	$x^0 = 1$	$2^0 = 1$ $(-3)^0 = 1$ $\left(\frac{1}{4}\right)^0 = 1$
Negative Exponent	$x^{-n} = \frac{1}{x^n}$	$x^{-3} = \frac{1}{x^3}$ $y^{-4} = \frac{1}{y^4}$ $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
	$\frac{1}{x^{-n}} = x^n$	$\frac{1}{x^{-5}} = x^5$ $\frac{1}{x^{-2}} = x^2$ $\frac{1}{3^{-2}} = 3^2 = 9$
Fractional Exponent	$x^{\frac{m}{n}} = \sqrt[n]{x^m}$	$x^{\frac{2}{3}} = \sqrt[3]{x^2}$ $y^{\frac{1}{2}} = \sqrt{y}$

In any expression with fractions, assume the denominator does not equal zero.

For additional information, see the Methods & Meanings box in Lesson 1.3.2. For additional examples and practice, see the Checkpoint 4 materials.

Examples

For examples 1 through 4, rewrite each expression with as few exponents as possible. The final expressions should contain no parentheses or negative exponents, and the exponents should be as small as possible. Assume the variables are not equal to 0.

1. $(2xy^3)(5x^2y^4)$

$(2xy^3)(5x^2y^4)$

$= 2 \cdot 5 \cdot x \cdot x^2 \cdot y^3 \cdot y^4$ — Reorder the factors.

$= 10x^3y^7$ — Use the Product Property.

2. $\frac{14x^2y^{12}}{7x^5y^7}$

$\frac{14x^2y^{12}}{7x^5y^7}$

$= \left(\frac{14}{7}\right) \cdot \left(\frac{x^2}{x^5}\right) \cdot \left(\frac{y^{12}}{y^7}\right)$ — Rewrite and separate coefficients and variables.

$= 2x^{-3}y^5$ — Use the Quotient Property.

$= \frac{2y^5}{x^3}$ — Use the Negative Exponents Property.

3. $(3x^2y^4)^3$

$(3x^2y^4)^3$

$= 3^3 \cdot (x^2)^3 \cdot (y^4)^3$ — Rewrite and separate coefficients and variables.

$= 27x^6y^{12}$ — Use the Power of a Power Property.

4. $(2x^3)^{-2}$

$(2x^3)^{-2}$

$= \frac{1}{(2x^3)^2}$ — Use the Negative Exponents Property.

$= \frac{1}{2^2 \cdot (x^3)^2}$ — Rewrite and separate coefficients and variables.

$= \frac{1}{4x^6}$ — Use the Power of a Power Property.

5. $\frac{10x^7y^3}{15x^{-2}y^3}$

$\frac{10x^7y^3}{15x^{-2}y^3}$

$= \left(\frac{10}{15}\right) \cdot \left(\frac{x^7}{x^{-2}}\right) \cdot \left(\frac{y^3}{y^3}\right)$ Rewrite and separate coefficients and variables.

$= \frac{2}{3}x^9y^0$ Use the Quotient Property.

$= \frac{2}{3}x^9 \cdot 1$ Use the Zero Exponent Property.

$= \frac{2}{3}x^9$

$= \frac{2x^9}{3}$

Practice

Rewrite each expression using the properties of exponents. The final expressions should contain no parentheses or negative exponents.

1. $y^5 \cdot y^7$

2. $b^4 \cdot b^3 \cdot b^2$

3. $8^6 \cdot 8^{-2}$

4. $(y^5)^2$

5. $(3a)^4$

6. $\frac{m^8}{m^3}$

7. $\frac{12m^8}{6m^{-3}}$

8. $(x^3y^2)^3$

9. $\frac{(y^4)^2}{(y^3)^2}$

10. $\frac{15x^2y^5}{3x^4y^5}$

11. $(4c^4)(ac^3)(3a^5c)$

12. $(7x^3y^5)^2$

13. $(4xy^2)(2y)^3$

14. $\left(\frac{4}{x^2}\right)^3$

15. $\frac{(2a^7)(3a^2)}{6m^3}$

16. $\left(\frac{5m^3n}{m^5}\right)^3$

17. $(3a^2x^3)^2(2ax^4)^3$

18. $\left(\frac{x^3y}{y^4}\right)^4$

19. $\left(\frac{6x^8y^2}{12x^3y^7}\right)^2$

20. $\frac{(2x^5y^3)^3(4xy^4)^2}{8x^7y^{12}}$

21. x^{-3}

22. $2x^{-3}$

23. $(2x)^{-3}$

24. $(2x^3)^0$

25. $5^{\frac{1}{2}}$

26. $\left(\frac{2x}{3}\right)^{-2}$

Answers

1.	y^{12}	**2.**	b^9	**3.**	8^4	**4.**	y^{10}
5.	$81a^4$	**6.**	m^5	**7.**	$2m^{11}$	**8.**	x^9y^6
9.	y^2	**10.**	$\frac{5}{x^2}$	**11.**	$12a^6c^8$	**12.**	$49x^6y^{10}$
13.	$32xy^5$	**14.**	$\frac{64}{x^6}$	**15.**	a^6	**16.**	$\frac{125n^3}{m^6}$
17.	$72a^7x^{18}$	**18.**	$\frac{x^{12}}{y^{12}}$	**19.**	$\frac{x^{10}}{4y^{10}}$	**20.**	$16x^{10}y^5$
21.	$\frac{1}{x^3}$	**22.**	$\frac{2}{x^3}$	**23.**	$\frac{1}{8x^3}$	**24.**	1
25.	$\sqrt{5}$	**26.**	$\frac{9}{4x^2}$				

Scientific Notation

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Overview

Scientific notation is a way of compactly writing very large and very small numbers. A number is said to be in scientific notation when it is written as a product of two factors, so that:

- The first factor is less than 10 and greater than or equal to 1.
- The second factor has a base of 10 and an integer exponent.
- A multiplication sign separates the factors.

The following table compares numbers written in scientific notation and standard form.

Scientific Notation	Standard Form
5.32×10^{6}	5,320,000
3.07×10^{-4}	0.000307
2.61×10^{-15}	0.00000000000000261

It is important to note that the exponent does not necessarily mean to use that number of zeros.

The number 5.32×10^{11} means $5.32 \times 100{,}000{,}000{,}000$. Thus, two of the eleven decimal places in the standard form of the number are the 3 and the 2 in 5.32. Standard form in this case is 532,000,000,000. In this example, the decimal point is moved eleven places to the right to write the number in standard form.

The number 2.61×10^{-15} means $2.61 \times 0.000000000000001$.

The decimal point is moved 15 places to the left to write the standard form.

The standard form is 0.00000000000000261.

Note: Some calculators and applications display scientific notation in other ways. In particular, “E notation” uses the letter “E” (or “e”, for “exponent”) to represent “times ten raised to the power of.” For example, 2.56×10^{5} can be displayed as 2.56E5 or 2.56e5.

For additional information, see the Methods & Meanings box in Lesson 1.3.1.

Examples

1. Write each number in standard form.

a. 7.84×10^{8}

784,000,000

b. 3.72×10^{-3}

0.00372

2. Write each number in scientific notation.

a. 52,050,000

5.205×10^{7}

b. 0.000372

3.72×10^{-4}

Practice

Write each number in standard form.

1. 7.85×10^{11}

2. 1.235×10^{9}

3. 1.2305×10^{3}

4. 3.89×10^{-7}

5. 5.28×10^{-4}

Write each number in scientific notation.

6. 391,000,000,000

7. 0.0000842

8. 123056.7

9. 0.000000502

10. 25.7

11. 0.035

12. 5,600,000

13. 1,346.8

14. 0.000000000006

15. 634,700,000,000,000

Answers

1.	785,000,000,000	**2.**	1,235,000,000	**3.**	1,230.5
4.	0.000000389	**5.**	0.000528	**6.**	3.91×10^{11}
7.	8.42×10^{-5}	**8.**	1.230567×10^{5}	**9.**	5.02×10^{-7}
10.	2.57×10^{1}	**11.**	3.5×10^{-2}	**12.**	5.6×10^{6}
13.	1.3468×10^{3}	**14.**	6.0×10^{-12}	**15.**	6.347×10^{14}