

# Rewriting Multivariable Equations

## GUIDED SKILL SUPPORT

### Overview

Some equations have more than one variable. These equations can be solved for any of the given variables. Rewriting equations with more than one variable uses the same process as solving an equation with one variable. The end result is often not a single number, but rather an algebraic expression that contains numbers and variables.

For additional examples and practice, refer to Checkpoint 8A.

### Examples

1. Solve  $3x - 2y = 6$  for  $y$ .

$$\begin{aligned} 3x - 2y &= 6 \\ -2y &= -3x + 6 \\ y &= \frac{-3x + 6}{-2} \\ y &= -\frac{3}{2}x + 3 \end{aligned}$$

Subtract  $3x$  from both sides of the equal sign.  
Divide by  $-2$  on both sides of the equal sign.  
Rewrite.

2. Solve  $7 + 2(x + y) = 11$  for  $y$ .

$$\begin{aligned} 7 + 2(x + y) &= 11 \\ 2(x + y) &= 4 \\ 2x + 2y &= 4 \\ 2y &= -2x + 4 \\ y &= \frac{-2x + 4}{2} \\ y &= -x + 2 \end{aligned}$$

Subtract 7 from both sides of the equal sign.  
Multiply by 2 using the Distributive Property.  
Subtract  $2x$  from both sides of the equal sign.  
Divide by 2 on both sides of the equal sign.  
Rewrite.

3. Solve  $y = 3x - 4$  for  $x$ .

$$y = 3x - 4$$

$$y + 4 = 3x$$

$$\frac{y+4}{3} = x$$

Add 4 to both sides of the equal sign.

Divide by 3 on both sides of the equal sign.

4. Solve  $l = prt$  for  $t$ .

$$l = prt$$

$$\frac{l}{pr} = t$$

Divide by  $pr$  on both sides of the equal sign.

## Practice

Solve each equation for the specified variable.

1. Solve for  $y$ :  $5x + 3y = 15$
2. Solve for  $x$ :  $5x + 3y = 15$
3. Solve for  $w$ :  $2l + 2w = P$
4. Solve for  $m$ :  $4n = 3m - 1$
5. Solve for  $a$ :  $2a + b = c$
6. Solve for  $a$ :  $b - 2a = c$
7. Solve for  $p$ :  $6 - 2(q - 3p) = 4p$
8. Solve for  $x$ :  $y = \frac{1}{4}x + 1$
9. Solve for  $r$ :  $4(r - 3s) = r - 5s$

## Answers

Note: other equivalent forms are possible.

1.  $y = -\frac{5}{3}x + 5$
2.  $x = -\frac{3}{5}y + 3$
3.  $w = -l + \frac{P}{2}$
4.  $m = \frac{4n+1}{3}$
5.  $a = \frac{c-b}{2}$
6.  $a = \frac{c-b}{-2}$  or  $\frac{b-c}{2}$
7.  $p = q - 3$
8.  $x = 4y - 4$
9.  $r = \frac{7s}{3}$

# Writing Equations for Word Problems

---

## GUIDED SKILL SUPPORT

### Overview

---

One equation or a system of equations can be used to model many everyday situations. First identify the question or problem to be solved. Then, identify the unknowns, using the questions as a guide. Write “let” statements to define each variable used in the equation or system of equations. In “let” statements, the units of measurement are included, often using parentheses. Note that counts do not have units (as in “the number of zebras”).

A **system of equations** is two or more equations that use the same set of variables to represent a situation.

For additional examples, reference the Lesson 6.1.4 Reflection Journal and Checkpoint 8B materials.

## Examples

1. The perimeter of a rectangle is 60 cm. The length is 4 times its width. Write one or more equations that model the relationship between the length and width.

Identify the unknown(s) in the situation. The length and width are unknown, so there are two variables.

Then, define variables using “let” statements to represent the unknowns. Let  $w$  represent the width (cm) of the rectangle. Let  $l$  represent the length (cm). Note how the units of measurement were defined.

Two equations can be written to represent this situation. From the first sentence, the perimeter of a rectangle can be represented by the equation  $P = 2l + 2w$ , or in this example as  $60 = 2l + 2w$ . The equation  $l = 4w$  represents the sentence “the length is 4 times the width.”

This situation can also be represented by a single equation:  $60 = 2(4w) + 2w$ .

2. Mike spent \$21.19 on a bag containing cashews and peanuts. The bag weighed 1.3 pounds. The cashews cost \$13.29 per pound and the peanuts cost \$10.79 per pound. Write one or more equations to represent the amount of cashews Mike bought.

Identify the unknown(s) in the situation. In this problem, the amounts of cashews and peanuts are unknown.

Then, define variables using “let” statements to represent the unknowns. Let  $c$  represent the amount of cashews (lb), and  $p$  represent the amount of peanuts (lb). Note how the units of measurement are defined. If the “let” statement read “ $c$  = cashews,” it would not be clear whether  $c$  represents the *weight* or the *cost* of the cashews.

Two equations can be written. From the statement “the bag weighed 1.3 pounds,” the weights can be represented by  $c + p = 1.3$ . A second equation can be written using the costs given in the problem:  $13.29c + 10.79p = 21.19$ .

## Practice

---

Write an equation or a system of equations that models each situation. Do not solve your equations.

1. A rectangle is three times as long as it is wide. Its perimeter is 36 units. Find the length of each side.
2. A rectangle is twice as long as it is wide. Its area is 72 square units. Find the length of each side.
3. The sum of two consecutive odd integers is 76. What are the numbers?
4. Nancy started the year with \$425 in the bank and is saving \$25 a week. Seamus started the year with \$875 and is spending \$15 a week. When will they have the same amount of money in the bank?
5. Oliver earns \$50 a day and \$7.50 for each package he processes at Company A. His paycheck on his first day was \$140. How many packages did he process?
6. Dustin has a collection of quarters and pennies. The total value is \$4.65. There are 33 coins. How many quarters and pennies does he have?
7. A one-pound mixture of raisins and peanuts costs \$7.50. The raisins cost \$3.25 a pound and the peanuts cost \$5.75 a pound. How much of each ingredient is in the mixture?
8. An adult ticket at an amusement park costs \$24.95 and a child's ticket costs \$15.95. A group of 10 people paid \$186.50 to enter the park. How many were adults?
9. Katy weighs 105 pounds and is gaining 2 pounds a month. James weighs 175 pounds and is losing 3 pounds a month. When will they weigh the same amount?
10. Harper Middle School has 125 fewer students than Holmes Junior High. When the two schools are merged, there are 809 students. How many students attend each school?

## Answers

Other equivalent forms are possible.

1. Let  $l$  = length and  $w$  = width;  
one-variable equation:  $2w + 2(3w) = 36$ ;  
system of equations:  $l = 3w$  and  $2w + 2l = 36$
2. Let  $l$  = length and  $w$  = width;  
one-variable equation:  $w(2w) = 72$ ;  
system of equations:  $l = 2w$  and  $lw = 72$
3. Let  $m$  = the first odd integer and  $n$  = the next consecutive odd integer;  
one-variable equation:  $m + (m + 2) = 76$ ;  
system of equations:  $m + n = 76$  and  $n = m + 2$
4. Let  $x$  = the number of weeks and  $y$  = the total money in the bank;  
one-variable equation:  $425 + 25x = 875 - 15x$ ;  
system of equations:  $y = 425 + 25x$  and  $y = 875 - 15x$
5. Let  $p$  = the number of packages Oliver processed;  
one-variable equation:  $50 + 7.50p = 140$
6. Let  $q$  = number of quarters and  $p$  = number of pennies;  
one-variable equation:  $0.25q + 0.01(33 - q) = 4.65$ ;  
system of equations:  $q + p = 33$  and  $0.25q + 0.01p = 4.65$
7. Let  $r$  = weight of raisins and  $p$  = weight of peanuts;  
one-variable equation:  $3.25r + 5.75(1 - r) = 7.5$ ;  
system of equations:  $r + p = 1$  and  $3.25r + 5.75p = 7.5(1)$
8. Let  $a$  = number of adult tickets and  $c$  = number of child's tickets;  
one-variable equation:  $24.95a + 15.95(10 - a) = 186.5$ ;  
system of equations:  $a + c = 10$  and  $24.95a + 15.95c = 186.5$
9. Let  $m$  = the number of months and  $w$  = the weight of each person;  
one-variable equation:  $105 + 2m = 175 - 3m$ ;  
system of equations:  $w = 105 + 2m$  and  $w = 175 - 3m$
10. Let  $x$  = number of Holmes students and  $y$  = number of Harper students;  
one-variable equation:  $x + (x - 125) = 809$ ;  
system of equations:  $x + y = 809$  and  $y = x - 125$

# Solving Systems of Equations Using the Equal Values Method

## GUIDED SKILL SUPPORT

### Overview

A system of equations is a set of equations that uses the same variables. The solution to a system of equations is usually a point  $(x, y)$ . This point is the intersection of the graphs of the equations.

A system can also have an infinite number of solutions (coinciding lines) or no solution (parallel lines).

The **Equal Values Method** is a way to solve a system of equations by taking two expressions that are each equal to the same variable and setting those expressions equal to each other.

$$\begin{array}{ccc} y = 23 + 4x & & y = 12 \\ \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} \\ \downarrow & & \downarrow \\ 23 + 4x = 12 \end{array}$$

This creates a single equation with one variable, such as  $x$ , making it more convenient to solve for the remaining variable. After you determine the value of the variable, it is substituted into either of the original equations to solve for the value of the other unknown variable.

Check the solution by substituting the point  $(x, y)$  into each equation.

For additional information, refer to the Lesson 6.2.1 Methods & Meanings box.

## Examples

1. Solve the system of equations using the Equal Values Method.

$$\begin{aligned}y &= 2x \\ y &= 9 - x\end{aligned}$$

Since both equations are written in slope-intercept form ( $y = mx + b$ ),  $y$  represents equal values in both equations. Therefore, using the Equal Values Method,  $2x = 9 - x$ . Solve for  $x$ .

$$\begin{array}{rcl}y = 2x & & y = 9 - x \\ \downarrow & & \downarrow \\ 2x = 9 - x & & \\ + x & + x & \\ \hline 3x = 9 & & \\ \div 3 & \div 3 & \\ x = 3 & & \end{array}$$

Use one of the original equations to solve for  $y$  by substituting in the  $x$ -value. Here, the first equation,  $y = 2x$ , is used to solve for  $y$ .

$$\begin{aligned}x &= 3 \\ \downarrow \\ y &= 2x \\ y &= 2(3) \\ y &= 6\end{aligned}$$

The solution to this system of equations is  $(3, 6)$  because the values  $x = 3$  and  $y = 6$  make both of the original equations true. When graphing,  $(3, 6)$ , is the point of intersection of the two lines.

A solution can be checked by substituting the  $x$ - and  $y$ -values back into *both* of the original equations to verify that the resulting equations are true statements.

$$\begin{array}{rcl}x = 3 & y = 6 & \\ \downarrow & \downarrow & \\ y = 2x & & \\ 6 = 2(3) & & \\ 6 = 6 & & \\ \text{true} & & \end{array}$$

$$\begin{array}{rcl}x = 3 & y = 6 & \\ \downarrow & \downarrow & \\ y = 9 - x & & \\ 6 = 9 - 3 & & \\ 6 = 6 & & \\ \text{true} & & \end{array}$$

2. Solve the system of equations using the Equal Values Method.

$$y = 5 + 4x$$

$$y = 4x - 1$$

Since both equations are written in slope-intercept form ( $y = mx + b$ ),  $y$  represents equal values in both equations.

Therefore, using the Equal Values Method,  $5 + 4x = 4x - 1$ . Solve for  $x$ .

$$\begin{array}{rcl} y = 5 + 4x & & y = 4x - 1 \\ \downarrow & & \downarrow \\ 5 + 4x = 4x - 1 \\ -4x & -4x & \\ \hline 5 = -1 \\ \text{false} \end{array}$$

This system of equations has no solution because 5 will never equal  $-1$ .

Alternatively, after using the Equal Values Method, reorganize the terms in the following way.

$$\begin{array}{rcl} y = 5 + 4x & & y = 4x - 1 \\ \downarrow & & \downarrow \\ 5 + 4x = 4x - 1 \\ 4x + 5 = 4x - 1 \\ \text{false} \end{array}$$

There are no cases when adding five to  $4x$  is the same as adding  $-1$  to  $4x$ .

## Practice

Solve each system of equations using the Equal Values Method.

1.  $y = 2x + 3$   
 $y = -x + 6$

2.  $y = -3x - 1$   
 $y = \frac{1}{2}x + 4$

3.  $y = 4x + 5$   
 $y = 4x - 2$

4.  $y = \frac{2}{3}x + 2$   
 $y = \frac{1}{6}x - 1$

5.  $y = 5x$   
 $y = -2x - 3$

6.  $y = 3x + 1$   
 $y = -\frac{1}{3}x - 5$

7.  $y = \frac{3}{2}x + 2$   
 $y = \frac{1}{2}x + 5$

8.  $y = x + 4$   
 $y = -x + 4$

## Answers

---

1.  $(1, 5)$

2.  $(-\frac{10}{7}, \frac{23}{7})$

3. No solution

4.  $(-6, -2)$

5.  $(-\frac{3}{7}, -\frac{15}{7})$

6.  $(-\frac{9}{5}, -\frac{22}{5})$

7.  $(3, \frac{13}{2})$

8.  $(0, 4)$

# Solving Systems of Equations Using the Substitution Method

---

## GUIDED SKILL SUPPORT

### Overview

---

A system of equations has two or more equations with two or more variables. One method for solving a system of equations is by graphing each equation and identifying the point of intersection of the graphs. Another method for solving a system of equations is the Equal Values Method. Graphing and the Equal Values Method are convenient when both equations are in  $y = mx + b$  form (or can easily be rewritten in  $y = mx + b$  form).

The **Substitution Method** is used to change a two-variable system of equations to a one-variable equation. This method is useful when one of the variables is isolated in one of the equations in the system. Two substitutions must be made to determine both the  $x$ - and  $y$ -values. If the substitution results in an equation that is not true, then there is no solution, and there is no point of intersection. There may also be an infinite number of solutions. This graph would appear as a single line for the two equations, which means any possible point on the line is a solution.

Additional information can be found in the Methods & Meanings boxes in Lessons 6.2.3. For additional examples, reference Checkpoint 9 Materials.

## Examples

Solve the following systems of equations using the Substitution Method.

1.  $y = 2x$   
 $x + y = 9$

In the equation  $y = 2x$ ,  $y$  is isolated on one side of the equation, and it is equivalent to  $2x$ . Therefore,  $2x$  can be substituted for  $y$  in the other equation.

$$\begin{array}{l} y = 2x \\ \downarrow \\ x + y = 9 \\ x + (2x) = 9 \\ 3x = 9 \\ x = 3 \end{array}$$

Then, substitute the value of  $x$  into either of the original equations to solve for  $y$ . In this example, the first equation is used.

$$\begin{array}{l} x = 3 \\ \downarrow \\ y = 2x \\ y = 2(3) \\ y = 6 \end{array}$$

The solution to this system of equations is  $(3, 6)$ . That means the values  $x = 3$  and  $y = 6$  make both of the original equations true. When graphing the system of equations,  $(3, 6)$  is the point of intersection of the two lines.

Check the solution by substituting the  $x$ - and  $y$ -values back into *both* of the original equations to verify that the resulting equations are true.

$$\begin{array}{l} (3, 6) \\ \swarrow \searrow \\ y = 2x \\ 6 = 2(3) \\ 6 = 6 \\ \text{true} \end{array}$$

$$\begin{array}{l} (3, 6) \\ \swarrow \searrow \\ x + y = 9 \\ 3 + 6 = 9 \\ 9 = 9 \\ \text{true} \end{array}$$

2.  $4x + y = 8$   
 $x = 5 - y$

In the equation  $x = 5 - y$ , the  $x$  is isolated and equal to  $5 - y$ . Therefore,  $5 - y$  can be substituted for  $x$  in the other equation. Substitute, then solve for  $y$ .

$$\begin{aligned} x &= 5 - y \\ \downarrow \\ 4x + y &= 8 \\ 4(5 - y) + y &= 8 \\ 20 - 4y + y &= 8 \\ 20 - 3y &= 8 \\ -3y &= -12 \\ y &= 4 \end{aligned}$$

Substitute the value of  $y$  into either of the original two equations to solve for  $x$ . In this example, the second equation is used.

$$\begin{aligned} y &= 4 \\ \downarrow \\ x &= 5 - y \\ x &= 5 - (4) \\ x &= 1 \end{aligned}$$

The solution is  $(1, 4)$ . Check the solution by substituting the  $x$ - and  $y$ -values back into *both* of the original equations to verify the resulting equations are true.

$$\begin{aligned} (1, 4) \\ \downarrow \quad \downarrow \\ 4x + y &= 8 \\ 4(1) + 4 &= 8 \\ 8 &= 8 \\ \text{true} \end{aligned}$$

$$\begin{aligned} (1, 4) \\ \downarrow \quad \downarrow \\ x &= 5 - y \\ 1 &= 5 - 4 \\ 1 &= 1 \\ \text{true} \end{aligned}$$

3.  $y = 7 - 3x$   
 $3x + y = 10$

In the equation  $y = 7 - 3x$ , the variable  $y$  is isolated and equal to  $7 - 3x$ . Therefore,  $7 - 3x$  can be substituted for  $y$  in the other equation.

$$\begin{aligned} y &= 7 - 3x \\ \downarrow \\ 3x - y &= 10 \\ 3x + (7 - 3x) &= 10 \\ 3x - 3x + 7 &= 10 \\ 7 &= 10 \\ \text{false} \end{aligned}$$

The resulting equation,  $7 = 10$ , is never true, and therefore, there is no solution to this system of equations.

4.  $y = 4 - 2x$   
 $-4x - 2y = -8$

In the equation  $y = 4 - 2x$ , the variable  $y$  is isolated and equal to  $4 - 2x$ . Substitute  $4 - 2x$  into the other equation and solve for  $y$ .

$$\begin{aligned} y &= 4 - 2x \\ \downarrow \\ -4x - 2y &= -8 \\ -4x - 2(4 - 2x) &= -8 \\ -4x - 8 + 4x &= -8 \\ -8 &= -8 \\ \text{true} \end{aligned}$$

The resulting equation,  $-8 = -8$ , is always true. This means there are an infinite number of solutions to this system of equations. All ordered pairs that make the equation  $y = 4 - 2x$  true, such as  $(1, 2)$ , also make the equation  $-4x - 2y = -8$  true.

5.  $y - 2x = -7$   
 $3x - 4y = 8$

Sometimes one of the equations needs to be solved for  $x$  or  $y$  to use the Substitution Method. In this example, the equation  $y - 2x = -7$  can be solved for  $y$  in one step.

$$\begin{array}{r} y - 2x = -7 \\ + 2x \quad + 2x \\ \hline y = 2x - 7 \end{array}$$

Substitute  $2x - 7$  into the other equation for  $y$ , and solve for  $x$ .

$$\begin{array}{r} y = 2x - 7 \\ \downarrow \\ 3x - 4y = 8 \\ 3x - 4(2x - 7) = 8 \\ 3x - 8x + 28 = 8 \\ -5x + 28 = 8 \\ -5x = -20 \\ x = 4 \end{array}$$

Solve for  $y$  by substituting the value of  $x$  into either of the original equations. The first equation is used in this example.

$$\begin{array}{r} x = 4 \\ \downarrow \\ y = 2x - 7 \\ y = 2(4) - 7 \\ y = 1 \end{array}$$

The solution is  $(4, 1)$ . Check the solution.

$$\begin{array}{r} (4, 1) \\ \swarrow \quad \searrow \\ y - 2x = -7 \\ 1 - 2(4) = -7 \\ 1 - 8 = -7 \\ -7 = -7 \\ \text{true} \end{array}$$

$$\begin{array}{r} (4, 1) \\ \swarrow \quad \searrow \\ 3x - 4y = 8 \\ 3(4) - 4(1) = 8 \\ 12 - 4 = 8 \\ 8 = 8 \\ \text{true} \end{array}$$

## Practice

Solve the following systems of equations using the Substitution Method.

1.  $y = -3x$   
 $4x + y = 2$

2.  $y = 7x - 5$   
 $2x + y = 13$

3.  $x = -5y - 4$   
 $x - 4y = 23$

4.  $x + y = 10$   
 $y = x - 4$

5.  $y = 5 - x$   
 $4x + 2y = 10$

6.  $3x + 5y = 23$   
 $y = x + 3$

7.  $y = -x - 2$   
 $2x + 3y = -9$

8.  $y = 2x - 3$   
 $-2x + y = 1$

9.  $x = \frac{1}{2}y + \frac{1}{2}$   
 $2x + y = -1$

10.  $x = 2y + 4$   
 $y - 2x = 16$

11.  $y = 3 - 2x$   
 $4x + 2y = 6$

12.  $y = x + 1$   
 $x - y = 1$

## Answers

1.  $(2, -6)$

2.  $(2, 9)$

3.  $(11, -3)$

4.  $(7, 3)$

5.  $(0, 5)$

6.  $(1, 4)$

7.  $(3, -5)$

8. No solution

9.  $(0, -1)$

10.  $(-12, -8)$

11. Infinite solutions

12. No solution

# Solving Systems of Equations Using the Elimination Method

## GUIDED SKILL SUPPORT

### Overview

Another method for solving systems of equations is the **Elimination Method**. This method is convenient when both equations are written in **standard form**,  $ax + by = c$ . To use the Elimination Method, look for variable terms with coefficients (the number multiplied by a variable) that have the opposite sign. If none exist, then one or both equations can be multiplied by a constant to create a pair of coefficients with opposite values. This is possible using the Addition Property of Equality.

Note, the special cases of “no solution” and “infinite solutions” can also occur.

The table below shows a summary of the methods to solve systems of equations.

| Method                                                          | Most Efficient When                                                                                               |
|-----------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| Equal Values                                                    | Both equations are in slope–intercept form, $y = mx + b$ .                                                        |
| Substitution                                                    | One equation has one variable isolated.                                                                           |
| Elimination: Add to eliminate one variable.                     | All equations are in standard form with opposite coefficients.                                                    |
| Elimination: Multiply one equation to eliminate one variable.   | All equations are in standard form. One equation can be multiplied to create opposite terms.                      |
| Elimination: Multiply both equations to eliminate one variable. | All equations are in standard form. Multiply each equation by a different factor to create opposite coefficients. |

For additional information, see the Methods & Meanings box in Lesson 6.3.3. For additional examples, see the Checkpoint 9 Materials.

## Examples

Solve the following systems of equations using the Elimination Method.

1.  $x - y = 2$   
 $2x + y = 1$

In the first equation,  $x - y$  is equivalent to 2. Therefore,  $x - y$  can be added to one side of the second equation and 2 to the other side of the equation. Then, combine like terms and solve for  $x$ .

$$\begin{array}{r} 2x + y = 1 \\ + (x - y) + 2 \\ \hline 3x = 3 \\ x = 1 \end{array}$$

Notice that the  $y$ -terms were eliminated and the resulting equation has only one variable,  $x$ . Substitute the value of  $x$  into either of the original equations to solve for  $y$ .

$$\begin{array}{r} x = 1 \\ \downarrow \\ 2x + y = 1 \\ 2(1) + y = 1 \\ 2 + y = 1 \\ y = -1 \end{array}$$

The solution is  $(1, -1)$ . Check the solution by substituting the solution into both of the original equations to verify that the resulting equations are true.

$$\begin{array}{r} (1, -1) \\ \downarrow \quad \downarrow \\ x - y = 2 \\ 1 - (-1) = 2 \\ 2 = 2 \\ \text{true} \end{array}$$

$$\begin{array}{r} (1, -1) \\ \downarrow \quad \downarrow \\ 2x + y = 1 \\ 2(1) + (-1) = 1 \\ 2 - 1 = 1 \\ 1 = 1 \\ \text{true} \end{array}$$

2.  $3x + 6y = 24$   
 $3x + y = -1$

Both equations are in standard form. Notice that both equations contain a  $3x$  term. The second equation can be rewritten by multiplying both sides by  $-1$ , resulting in  $-3x - y = 1$ . Now the two equations have terms that are opposites:  $3x$  and  $-3x$ . In the next step, these terms will be eliminated because  $-3x + 3x = 0$ .

Since  $-3x + (-y)$  is equivalent to  $1$ ,  $-3x + (-y)$  can be added to one side of the equation and  $1$  to the other side.

$$\begin{array}{r} 3x + 6y = 24 \\ -3x + (-y) = 1 \\ \hline 5y = 25 \\ y = 5 \end{array}$$

The resulting equation has only one variable,  $y$ , since  $3x$  and  $-3x$  eliminated each other. Now substitute the value of  $y$  into either of the original equations to solve for  $x$ .

$$\begin{array}{r} y = 5 \\ \downarrow \\ 3x + 6y = 24 \\ 3x + 6(5) = 24 \\ 3x + 30 = 24 \\ 3x = -6 \\ x = -2 \end{array}$$

The solution is  $(-2, 5)$ . Check that the solution makes both of the original equations true.

$$\begin{array}{r} (-2, 5) \\ \downarrow \quad \downarrow \\ 3x + 6y = 24 \\ 3(-2) + 6(5) = 24 \\ -6 + 30 = 24 \\ 24 = 24 \\ \text{true} \end{array}$$

$$\begin{array}{r} (-2, 5) \\ \downarrow \quad \downarrow \\ 3x + y = -1 \\ 3(-2) + 5 = -1 \\ -6 + 5 = -1 \\ -1 = -1 \\ \text{true} \end{array}$$

3.  $x + 3y = 7$   
 $4x - 7y = -10$

Both equations are in standard form. To use the Elimination Method, one of the terms in one of the equations needs to be the opposite of the corresponding term in the other equation. In this system, there are no variable terms with the same coefficient. Note that one of the  $x$ -terms has a coefficient of 1. Multiplying  $x$  by  $-4$  would make those terms opposites. Rewrite the equation  $x + 3y = 7$  by multiplying it by  $-4$ .

$$\begin{aligned} x + 3y &= 7 \\ -4(x + 3y) &= -4(7) \\ -4x - 12y &= -28 \end{aligned}$$

When multiplying an equation, be sure to multiply all the terms on both sides of the equation. With the first equation rewritten, the system of equations now looks like this:

$$\begin{aligned} -4x - 12y &= -28 \\ 4x - 7y &= -10 \end{aligned}$$

Since  $4x - 7y$  is equivalent to  $-10$ ,  $4x - 7y$  can be added to the left side of the first equation and  $-10$  on the right side. This is the same as adding the two equations together.

$$\begin{aligned} -4x + (-12y) &= -28 \\ + 4x &\quad - 7y \quad - 10 \\ \hline -19y &= -38 \\ y &= 2 \end{aligned}$$

Substitute the value of  $y$  into either of the original equations to solve for  $x$ .

$$\begin{aligned} y &= 2 \\ \downarrow \\ x + 3y &= 7 \\ x + 3(2) &= 7 \\ x + 6 &= 7 \\ x &= 1 \end{aligned}$$

The solution to the system of equations is  $(1, 2)$ . Check the solution.

$$\begin{aligned} (1, 2) \\ \downarrow \quad \downarrow \\ x + 3y &= 7 \\ 1 + 3(2) &= 7 \\ 7 &= 7 \\ \text{true} \end{aligned}$$

$$\begin{aligned} (1, 2) \\ \downarrow \quad \downarrow \\ 4x - 7y &= -10 \\ 4(1) - 7(2) &= -10 \\ 4 - 14 &= -10 \\ -10 &= -10 \\ \text{true} \end{aligned}$$

4.  $8x - 7y = 5$   
 $3x - 5y = 9$

Although both equations are written in standard form, it is not possible to multiply one equation by an integer constant to make opposite corresponding like terms for both equations. Instead, the two equations must be multiplied by different constants to get opposite coefficients.

One possible way to do this is to multiply the first equation by 3 and the second equation by  $-8$ . The resulting terms  $24x$  and  $-24x$  will be opposites.

$$\begin{array}{rcl} 3(8x - 7y = 5) & \longrightarrow & 24x - 21y = 15 \\ -8(3x - 5y = 9) & & -24x + 40y = -72 \end{array}$$

The two equations can be combined using addition. Combine like terms and solve for  $y$ .

$$\begin{array}{r} 24x - 21y = 15 \\ + -24x + 40y = -72 \\ \hline 19y = -57 \\ y = -3 \end{array}$$

Then, solve for  $x$  by substituting the value of  $y$  into either of the original equations. The solution is  $(-2, -3)$ . Check that  $(-2, -3)$  makes both of the original equations true.

$$\begin{array}{l} (-2, -3) \\ \downarrow \quad \downarrow \\ 8x - 7y = 5 \\ 8(-2) - 7(-3) = 5 \\ -16 + 21 = 5 \\ 5 = 5 \\ \text{true} \end{array}$$

$$\begin{array}{l} (-2, -3) \\ \downarrow \quad \downarrow \\ 3x - 5y = 9 \\ 3(-2) - 5(-3) = 9 \\ -6 + 15 = 9 \\ 9 = 9 \\ \text{true} \end{array}$$

5.  $4x + 2y = 6$   
 $2x + y = 3$

The second equation can be multiplied by 2 to create  $x$ -terms with the same coefficient. The resulting equation is  $4x + 2y = 6$ . The two equations are identical, which means the graph will be one line with *infinite* solutions.

Another way to think about this system of equations is to multiply the equation  $2x + y = 3$  by  $-2$ .

→

Notice that if either of these equations is multiplied by  $-1$ , the equations would be the same. Therefore there are infinitely many solutions. Any solution that makes the equation  $4x + 2y = 6$  true will also make  $2x + y = 3$  true.

6.  $2x + y = 3$   
 $4x + 2y = 8$

The first equation can be multiplied by 2 to create  $x$ - and  $y$ -terms with the same coefficient. The resulting equation is  $4x + 2y = 6$ . There are no values that could make  $4x + 2y$  equal both 6 and 8 at the same time. On a graph, these equations are parallel lines. This means there is no point of intersection, which means there is *no solution* to the system of equations.

## Practice

1.  $2x + y = 6$   
 $-2x + y = 2$

2.  $-4x + 5y = 0$   
 $-6x + 5y = -10$

3.  $2x - 3y = -9$   
 $x + y = -2$

4.  $y - x = 4$   
 $2y + x = 8$

5.  $2x - y = 4$   
 $\frac{1}{2}x + y = 1$

6.  $-4x + 6y = -20$   
 $2x - 3y = 10$

7.  $6x - 2y = -16$   
 $4x + y = 1$

8.  $6x - y = 4$   
 $6x + 3y = -16$

9.  $2x - 2y = 5$   
 $2x - 3y = 3$

10.  $y - 2x = 6$   
 $y - 2x = -4$

11.  $4x - 4y = 14$   
 $2x - 4y = 8$

12.  $3x + 2y = 12$   
 $5x - 3y = -37$

## Answers

---

**1.**  $(1, 4)$

**2.**  $(5, 4)$

**3.**  $(-3, 1)$

**4.**  $(0, 4)$

**5.**  $(2, 0)$

**6.** Infinite solutions

**7.**  $(-1, 5)$

**8.**  $(-\frac{1}{6}, -5)$

**9.**  $(4.5, 2)$

**10.** No solution

**11.**  $(3, -\frac{1}{2})$

**12.**  $(-2, 9)$