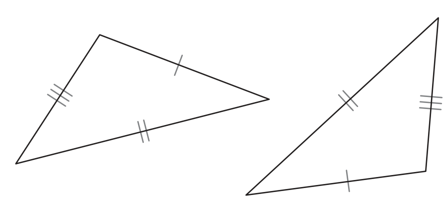
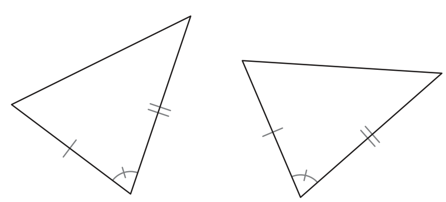
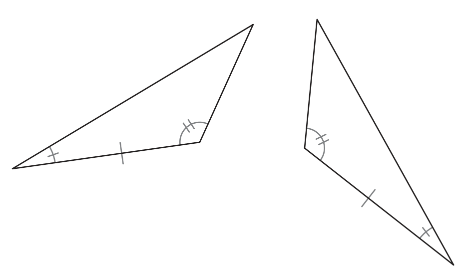
Triangle Congruence

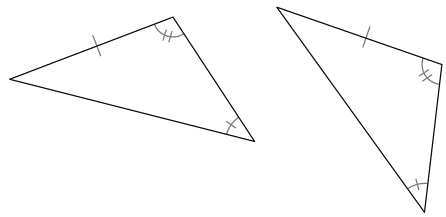
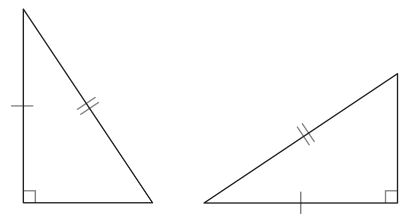
GUIDED SKILL SUPPORT

Overview

Two triangles are **congruent** if there is a sequence of rigid transformations (reflections, rotations, and translations) that maps one triangle onto the other. If all three corresponding angles and all three corresponding sides of two triangles are congruent, then the two triangles are congruent. It is not necessary to show that all three corresponding angle and side pairs are congruent. Specific combinations of corresponding parts of triangles are sufficient to prove triangle congruence. In this course, these are called triangle congruence conditions, and they are shown in the table.

Note: "S" stands for "side" and "A" stands for "angle." HL ≅ is only used with right triangles. The "H" stands for "hypotenuse" and the "L" stands for leg.

Triangle Congruence Condition	Abbreviation	Example	Description
Side–Side–Side	SSS ≅		If all three pairs of corresponding sides have equal lengths, then the triangles are congruent.
Side–Angle–Side	SAS ≅		If two pairs of corresponding sides have equal lengths *and* the corresponding angles between them (the included angles) have equal measure, then the triangles are congruent.
Angle–Side–Angle	ASA ≅		If two pairs of corresponding angles have equal measure *and* the pair of sides between them have equal measure, then the triangles are congruent.

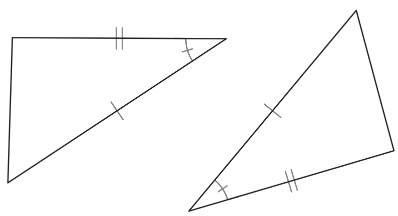
Triangle Congruence Condition	**Abbreviation**	**Example**	**Description**
Angle–Angle–Side	AAS ≅		If two pairs of corresponding angles *and* a pair of corresponding sides that are not between them have equal measure, then the triangles are congruent.
Hypotenuse–Leg	HL ≅		If the hypotenuse and a leg of one right triangle have the same lengths as the hypotenuse and a leg of another right triangle, then the triangles are congruent.

Additional information about triangle congruence can be found in the Lessons 7.1.5, 7.1.6, 7.1.7, and 7.1.9 Methods & Meanings boxes. For additional examples and more practice, refer to the Lesson 7.1.7 Reflection Journal and the Checkpoint 10 Materials.

Examples

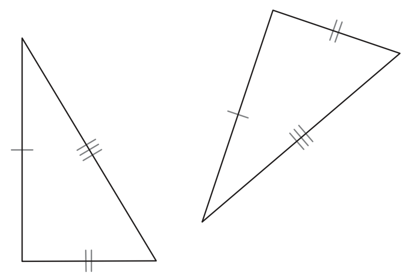
1. For each pair of triangles shown, determine whether the triangles are congruent. If the triangles are congruent, state the triangle congruence condition that justifies that conclusion. If you cannot conclude that the triangles are congruent, explain why not.

a.



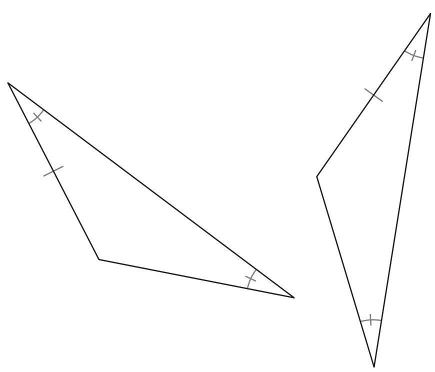
The triangles are congruent by SAS ≅.

b.



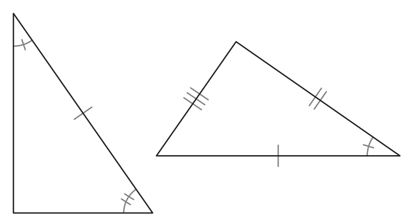
The triangles are congruent by SSS ≅.

c.



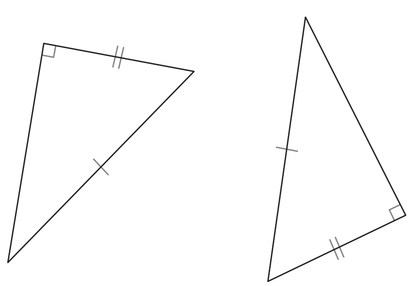
The triangles are congruent by AAS ≅.

d.



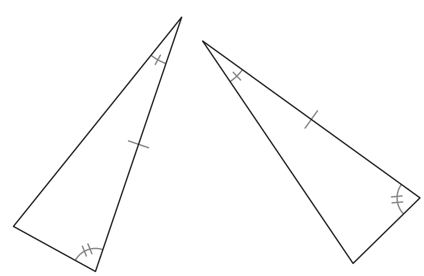
The triangles are not necessarily congruent. The first triangle displays an ASA arrangement, while the second triangle displays a SSS arrangement. The triangles could be congruent, but based on the markings, no conclusion can be determined.

e.



The triangles are right triangles and congruent by HL ≅.

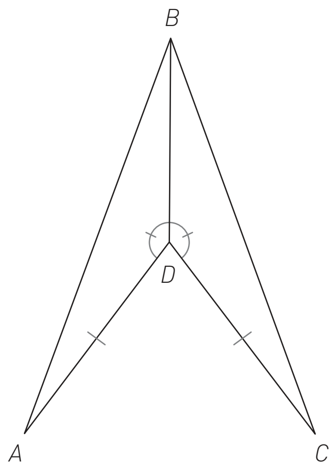
f.



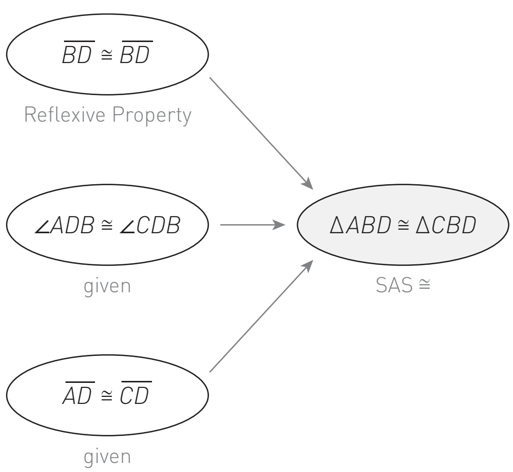
The triangles are congruent by ASA ≅.

2. Using the information in each diagram, determine whether each pair of triangles is congruent. If the triangles are congruent, create a flowchart to justify your answer.

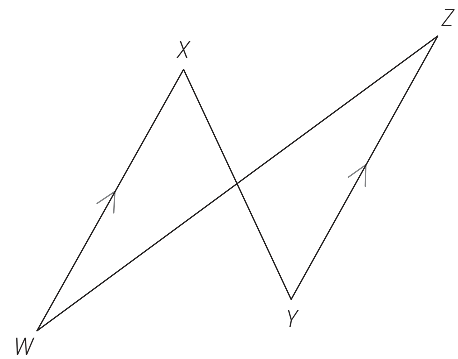
a.



$\Delta ABD \cong \Delta CBD$ by the SAS $\cong$ conjecture. Note: Only "SA" is marked. Observe that $\overline{BD}$ is congruent to itself by the Reflexive Property.



b.

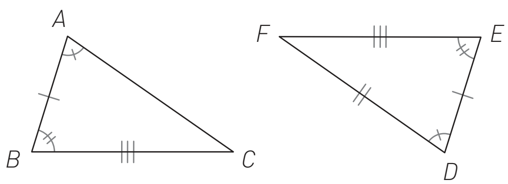


There is not enough information to confirm that the triangles are congruent. There is enough information to conclude that all of the corresponding angles are congruent, making the triangles similar. More information about the corresponding sides is needed to determine whether the triangles are congruent.

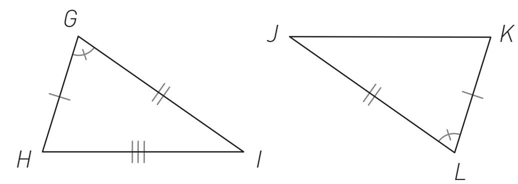
Practice

Determine whether each pair of triangles is congruent. If the triangles are congruent, state the triangle congruence condition that justifies the conclusion. If a conclusion about triangle congruence cannot be made, explain why not.

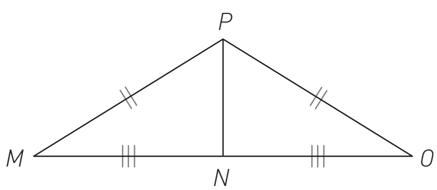
1.



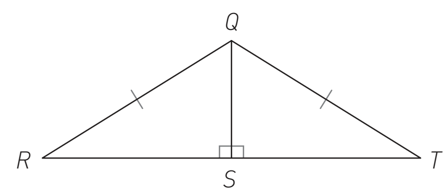
2.



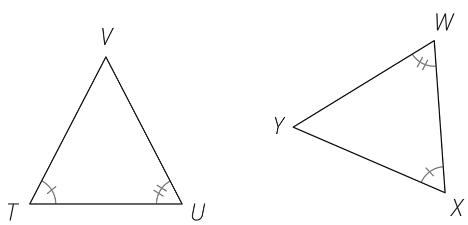
3.



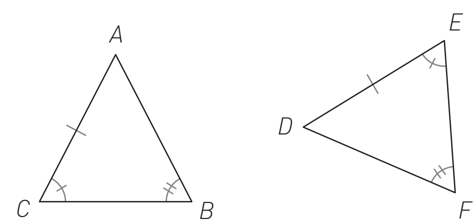
4.



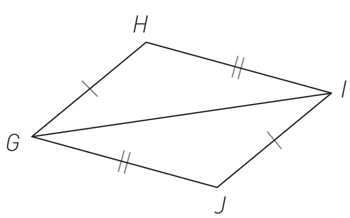
5.



6.



7.



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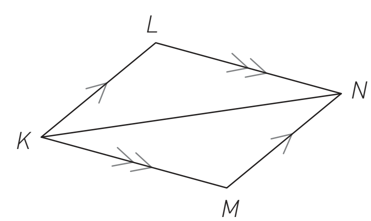

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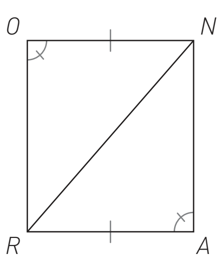
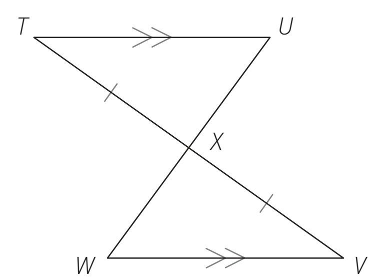


10.



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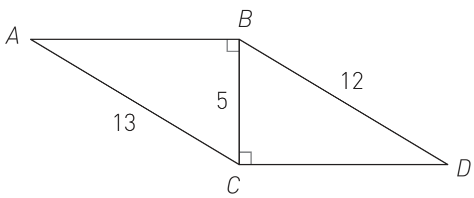
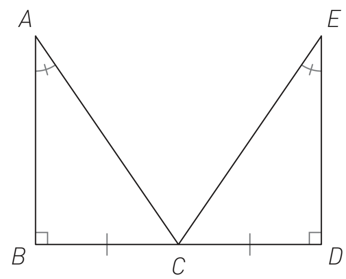
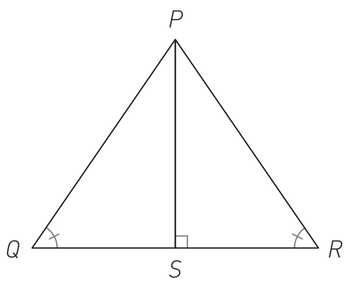


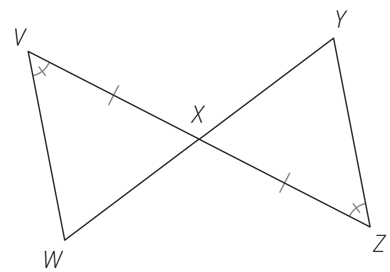
12.



13.

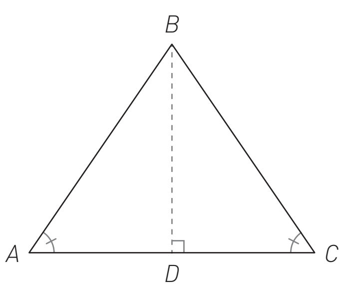


14.

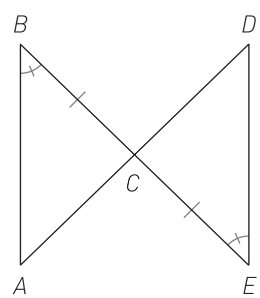


Using the information in each diagram, determine whether each pair of triangles is congruent. If the triangles are congruent, create a flowchart to justify your answer. If they are not congruent or cannot be determined to be congruent, justify why not.

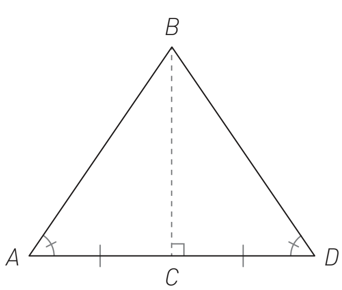
15.



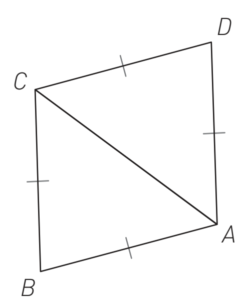
16.



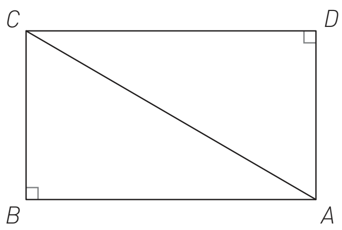
17.



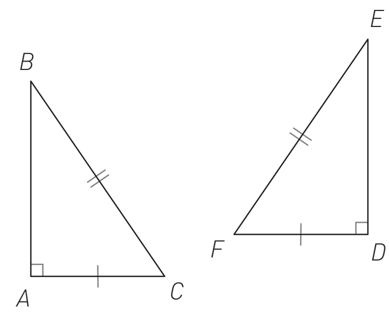
18.



19.



20.



Answers

1. $\Delta ABC \cong \Delta DEF$ by ASA ≅.

2. $\Delta GIH \cong \Delta LJK$ by SAS ≅.

3. $\Delta PNM \cong \Delta PNO$ by SSS ≅.

4. $\overline{QS} \cong \overline{QS}$, so $\Delta QRS \cong \Delta QTS$ by HL ≅.

5. These are not necessarily congruent without information about the sides.

6. $\Delta ABC \cong \Delta DFE$ by ASA ≅ or AAS ≅.

7. $\overline{GI} \cong \overline{GI}$, so $\Delta GHI \cong \Delta IJG$ by SSS ≅.

8. Two sets of alternate interior angles are congruent, so $\Delta KLN \cong \Delta NMK$ by ASA ≅.

9. Not necessarily congruent. Explanations may vary. For example, additional information about the other pair of opposite sides is needed for SAS ≅.

10. Vertical angles are congruent and alternate interior angles are congruent, so $\Delta TUX \cong \Delta VWX$ by ASA ≅.

11. No, the length of each hypotenuse is different.

12. $\Delta ABC \cong \Delta EDC$ by AAS ≅.

13. $\overline{PS} \cong \overline{PS}$, so $\Delta PQS \cong \Delta PRS$ by AAS ≅.

14. Vertical angles are congruent, so $\Delta VXW \cong \Delta ZXY$ by ASA ≅.

15.

$\angle BAD \cong \angle BCD$
given

$\overline{BD} \cong \overline{BD}$
Reflexive Property

$\angle BDC \cong \angle BDA$
right $\angle$s $\cong$

$\Delta ABD \cong \Delta CBD$
AAS $\cong$

16.

$\angle B \cong \angle E$
given

$\overline{BC} \cong \overline{CE}$
given

$\angle BCA \cong \angle ECD$
vertical $\angle$s $\cong$

$\Delta ABC \cong \Delta DEC$
ASA $\cong$

17.

$\overline{AC} \cong \overline{DC}$
given

$\overline{BC} \cong \overline{BC}$
Reflexive Property

$\angle BCD \cong \angle BCA$
right $\angle$s $\cong$

$\Delta ABC \cong \Delta DBC$
SAS $\cong$

18. Note that corresponding sides can also be $\overline{AD} \cong \overline{AB}$ and $\overline{BC} \cong \overline{CD}$.

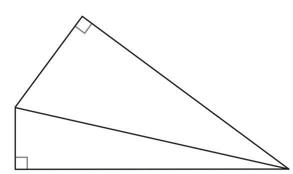
$\overline{AD} \cong \overline{BC}$
given

$\overline{CA} \cong \overline{CA}$
Reflexive Property

$\overline{BA} \cong \overline{CD}$
given

$\Delta ABC \cong \Delta CDA$
SSS $\cong$

19. Congruence cannot be determined. Justifications may vary. For example, with the given information, the easiest way to determine congruence is with HL≅, but the leg measurements are unknown. Alternatively, another pair of triangles with the same characteristics is clearly not congruent.



20.

$\overline{BC} \cong \overline{EF}$
given

$\overline{AC} \cong \overline{DF}$
given

$\Delta ABC \cong \Delta DEF$
HL $\cong$

Coordinate Geometry

GUIDED SKILL SUPPORT

Overview

Coordinate geometry is the study of geometry on a coordinate grid. Algebraic skills, knowledge of the coordinate plane, and properties of triangles and quadrilaterals are applied in coordinate geometry. The concepts of slope and the Right Triangle Theorem can determine whether a polygon plotted on a coordinate grid has special properties. In this course, coordinate geometry focuses on special triangles and quadrilaterals. The special quadrilaterals studied in this course are:

- rectangle;
- square;
- trapezoid, isosceles trapezoid, right trapezoid;
- parallelogram;
- rhombus; and
- kite.

Additional information about coordinate geometry and slopes of parallel and perpendicular lines can be found in the Lessons 7.2.2 and 7.2.3 Methods & Meanings boxes.

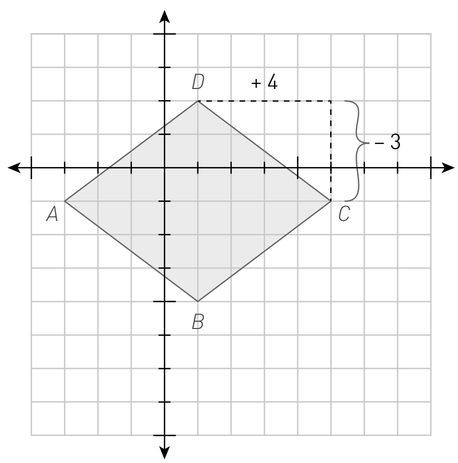
Examples

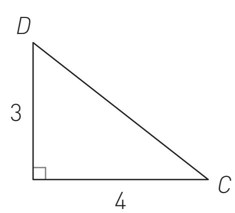
1. Is the quadrilateral formed by the points $A(-3, -1)$, $B(1, -4)$, $C(5, -1)$, and $D(1, 2)$ a rhombus? Justify your answer.

Plot the points on a coordinate plane and connect them in order. Although the shape appears to be a parallelogram or possibly a rhombus, no conclusion can be made without more information.

A rhombus has four sides of equal length (definition of a rhombus). Use the Right Triangle Theorem to determine the length of a segment on the coordinate graph.

Outline a **slope triangle**, creating a right triangle with $\overline{DC}$ as the hypotenuse. The lengths of the legs of this right triangle are 3 (vertical) and 4 (horizontal) units. Using the Right Triangle Theorem:

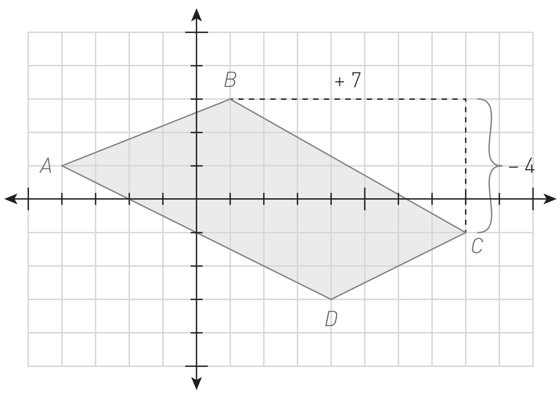




Similarly, slope triangles and the Right Triangle Theorem can be used for the other three sides of the quadrilateral. In each case, the length of each side of the quadrilateral is 5 units. Therefore, since all four sides have the same length, the polygon is a rhombus.

2. On a set of coordinate axes, plot the points $A(-4, 1)$, $B(1, 3)$, $C(8, -1)$, and $D(4, -3)$, and connect them in the order given. Is this quadrilateral a parallelogram? Justify your answer.

After plotting the points, the quadrilateral appears to be a parallelogram. This can be proven by showing that the opposite sides are parallel. Lines are parallel if they have the same slope. Use slope triangles to determine the slope of each side.



slope of $\overline{BC} = \frac{-4}{7} = -\frac{4}{7}$

slope of $\overline{AD} = \frac{-4}{8} = -\frac{1}{2}$

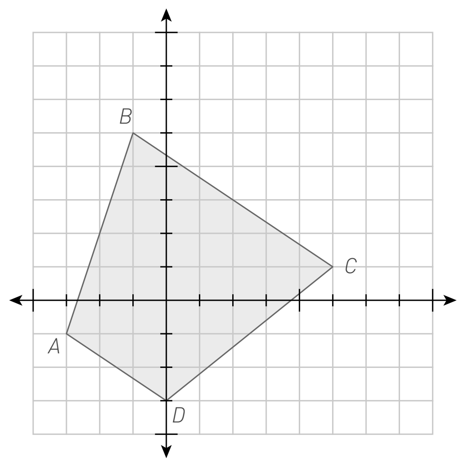
slope of $\overline{BA} = \frac{2}{5}$

slope of $\overline{DC} = \frac{2}{4} = \frac{1}{2}$

Although the values for the slopes of the opposite sides are close, they are not equal. Therefore, this quadrilateral is not a parallelogram.

3. Consider quadrilateral *ABCD* with vertices at *A*(−3, −1), *B*(−1, 5), *C*(5, 1), *D*(0, −3).

a. Plot the points on a coordinate grid and connect them in the given order.



b. Decide whether the quadrilateral is a special kind. Justify your answer.

The quadrilateral appears to be a trapezoid. This can be justified if it has a pair of parallel sides. To determine whether opposite sides of the quadrilateral are parallel, use slope triangles to calculate the slopes for each side. Sides *AB* and *CD* are opposite sides, as are sides *AD* and *BC*.

$$\text{slope of } \overline{AB} = \frac{6}{2} = 3 \qquad \text{slope of } \overline{CD} = \frac{4}{5}$$

Sides *AB* and *CD* are not parallel because their slopes are different.

$$\text{slope of } \overline{AD} = -\frac{2}{3} \qquad \text{slope of } \overline{BC} = -\frac{4}{6} = -\frac{2}{3}$$

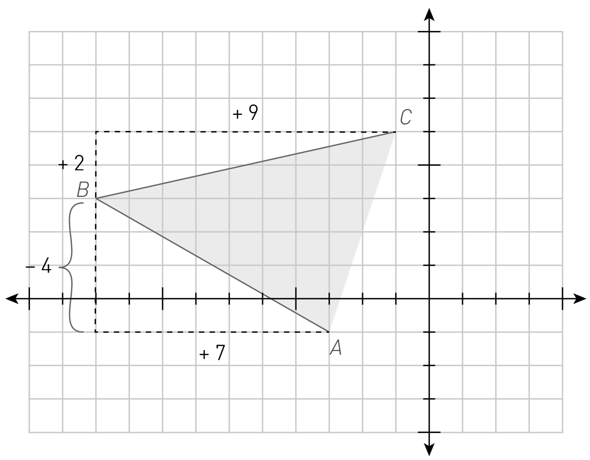
Sides *AD* and *BC* are parallel because their slopes are the same.

Since the quadrilateral has one pair of parallel sides, it is a trapezoid.

4. Consider quadrilateral *ABCD* with vertices at *A*(–3, –1), *B*(–10, 3), and *C*(–1, 5). Connect the given points in order. Do not connect point *C* to point *A*. What is the coordinate of point *D* that would make the quadrilateral a parallelogram?

A parallelogram has opposite sides that are parallel to each other. The location of point *D* must ensure that $\overline{AD}$ and $\overline{BC}$, as well as $\overline{AB}$ and $\overline{CD}$, are parallel.

Draw a slope triangle between points *A* and *B* and another between points *B* and *C*. Calculate the slopes of these sides.

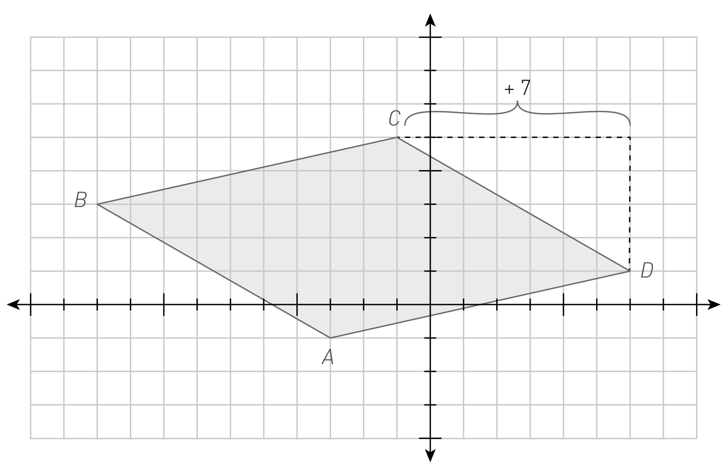


slope of $\overline{AB} = \frac{-4}{7} = -\frac{4}{7}$

slope of $\overline{BC} = \frac{2}{9}$

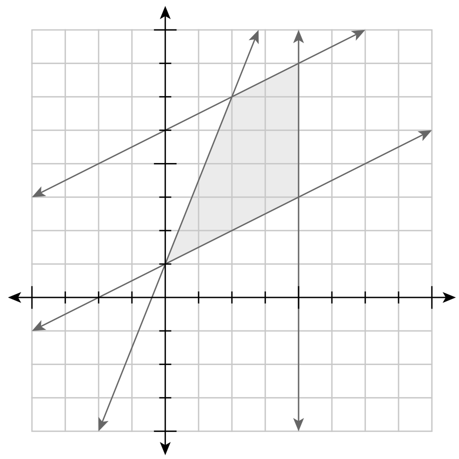
For opposite sides to be parallel, the slope of side *CD* needs to be the same as the slope of side *AB*. The slope of side *AD* must be the same as that of side *BC*.

A slope triangle can be made starting at either point *A* or point *C*. If starting at point *C*, draw a slope triangle with a height of 4 and a width of 7. The far right vertex of the triangle is *D*(6, 1).



5. Graph the following lines on the same set of axes. These lines enclose a shape. What is the name of that shape? Justify your answer.

$y = \frac{1}{2}x + 1$ $\quad y = \frac{1}{2}x + 5$ $\quad y = \frac{5}{2}x + 1$ $\quad x = 4$



Since the sides of the quadrilateral formed by the lines $y = \frac{1}{2}x + 1$ and $y = \frac{1}{2}x + 5$ have the same slope of $\frac{1}{2}$, the sides are parallel. The other pair of opposite sides have different slopes, so they are not parallel. A quadrilateral with one pair of opposite sides parallel is a trapezoid.

Practice

1. If *ABCD* is a rectangle, and the points *A*(1, 2), *B*(5, 2), and *C*(5, 5) are given, what are the coordinates of *D*?

2. Graph the following lines on the same set of axes. These lines enclose a quadrilateral. What is the name of the quadrilateral? Justify your answer.

$y = -2x + 7$ $\quad y = 3$ $\quad y = 2x + 1$ $\quad y = -2$

3. The three points *S*(1, 4), *A*(−1, −1), and *M*(2, −1) are vertices of a parallelogram. Connect the points in the order given, except *M* to *S*. What are the coordinates of the fourth vertex, *E*?

4. Graph the following lines on the same set of axes. These lines enclose a shape. What is the name of that shape? Justify your answer.

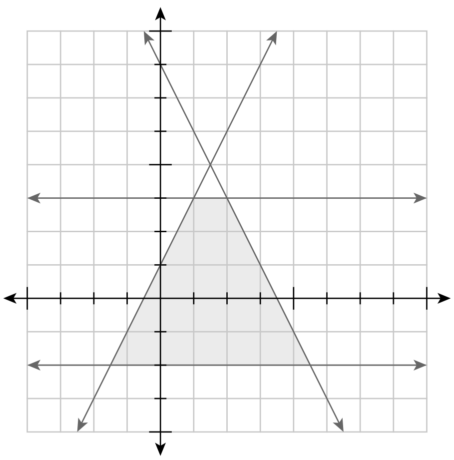
$y = \frac{3}{5}x + 7$ $\quad y = 0.6x$ $\quad y = -\frac{10}{6}x - 1$ $\quad y = -\frac{5}{3}x + 9$

5. If *W*(−4, −5), *X*(1, 0), *Y*(−1, 2), and *Z*(−6, −3) form a polygon, what shape is *WXYZ*? Justify your answer.

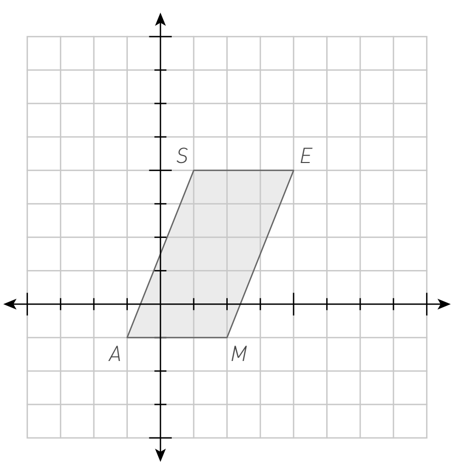
Answers

1. (1, 5)

2. Trapezoid; the quadrilateral may appear isosceles, but the nonparallel opposite sides do not have opposite slopes.



3. $E(4, 4)$



4. Since the slopes of the opposite sides are equal, this is a parallelogram. Additionally, since the slopes of intersecting lines are negative reciprocals of each other, they are perpendicular. This means the angles are all right angles, so the figure is a rectangle.

5. The slope of $\overline{WX} = 1$, the slope of $\overline{XY} = -1$, the slope of $\overline{YZ} = 1$, and the slope of $\overline{ZW} = -1$. This shows that opposite sides are parallel and adjacent sides are perpendicular. Also, all of the angles are 90°. $WXYZ$ is a rectangle.