Solving One-Variable Inequalities

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Overview

One-variable inequalities can be solved by changing an inequality to an equation, then solving the equation. The solution to the equation is called a "boundary point" and is plotted on a number line. This point separates the number line into two regions. For situations involving $\geq$ or $\leq$, the boundary point is included in the solution and is represented with a solid circle. For situations involving strictly $>$ or $<$, the boundary point is excluded from the solution and is represented with an open circle.

Next, to determine the solution region on the number line, choose a test point from within each region separated by the boundary point. Substitute each value into the inequality and check whether the resulting inequality is true or false. If the inequality is true, then every number in that region is a solution to the inequality. Note that there are an infinite number of solutions to an inequality. If it is false, then no number in that region is a solution to the inequality.

For additional information, see the Methods & Meanings boxes in Lessons 9.1.1 and 9.1.3.

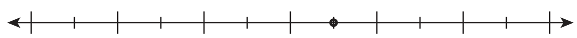
Examples

1. Solve the inequality $3x - (x + 2) \geq 0$.

Change the inequality to an equation and solve.

$$\begin{aligned} 3x - (x + 2) &= 0 \\ 3x - x - 2 &= 0 \\ 2x - 2 &= 0 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

Place the boundary point on the number line as a solid circle. The inequality symbol, $\geq$, indicates that $x = 1$ is included in the solution.



Choose two test points, one from each side of the boundary point, to substitute into the original inequality. In this example, the test points $x = 0$ and $x = 3$ are used.

$x = 0$

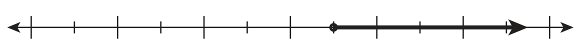
$$\begin{aligned} 3x - (x + 2) &\geq 0 \\ 3(0) - (0 + 2) &\geq 0 \\ 0 - 2 &\geq 0 \\ -2 &\geq 0 \end{aligned}$$

false

$x = 3$

$$\begin{aligned} 3x - (x + 2) &\geq 0 \\ 3(3) - (3 + 2) &\geq 0 \\ 9 - 5 &\geq 0 \\ 4 &\geq 0 \end{aligned}$$

true



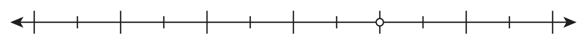
The solution is $x \geq 1$.

2. Solve the inequality $-x + 6 > x + 2$.

Change the inequality to an equation and solve.

$$\begin{aligned} -x + 6 &= x + 2 \\ -2x + 6 &= 2 \\ -2x &= -4 \\ x &= 2 \end{aligned}$$

Place the boundary point on the number line as an open circle. The inequality symbol, >, indicates that $x = 2$ is *not* included in the solution.



Choose two test points, one from each side of the boundary point, to substitute into the original inequality. In this example, the test points $x = 0$ and $x = 4$ are used.

$x = \boxed{0}$

$$\begin{aligned} -x + 6 &> x + 2 \\ -0 + 6 &> 0 + 2 \\ 6 &> 2 \end{aligned}$$

true

$x = \boxed{4}$

$$\begin{aligned} -x + 6 &> x + 2 \\ -4 + 6 &> 4 + 2 \\ 2 &> 6 \end{aligned}$$

false

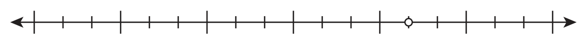
The solution is $x < 2$.

3. Solve the inequality $x < \frac{1}{4}x + 1$.

Change the inequality to an equation and solve.

$$x = \frac{1}{4}x + 1$$
$$\frac{3}{4}x = 1$$
$$x = \frac{4}{3}$$

Place the boundary point on the number line as an open circle. The inequality symbol, <, indicates that $x = \frac{4}{3}$ is not included in the solution.



Choose two test points, one from each side of the boundary point, to substitute into the original inequality. In this example, the test points $x = 0$ and $x = 2$ are used.

$x = 0$	$x = 2$
$x < \frac{1}{4}x + 1$	$x < \frac{1}{4}x + 1$
$0 < \frac{1}{4}(0) + 1$	$2 < \frac{1}{4}(2) + 1$
$0 < 1$	$2 < 1\frac{1}{2}$
true	false

The solution is $x < \frac{4}{3}$.

Practice

Solve each inequality.

1. $4x - 1 \geq 7$
2. $2(x - 5) \leq 8$
3. $3 - 2x < x + 6$
4. $\frac{1}{2}x > 5$
5. $3(x + 4) > 12$
6. $2x - 7 \leq 5 - 4x$
7. $3x + 2 < 11$
8. $4(x - 6) \geq 20$
9. $\frac{1}{4}x < 2$
10. $12 - 3x > 2x + 1$
11. $\frac{1}{7}x - 5 \leq -3$
12. $3(5 - x) \geq 7x - 1$
13. $3x - (2x + 2) \leq 7$
14. $x + \frac{2}{5} < \frac{2}{3}x$
15. $\frac{1}{2}x - \frac{2}{3} \geq 2x + 1$

Answers

1. $x \geq 2$
2. $x \leq 9$
3. $x > -1$
4. $x > 10$
5. $x > 0$
6. $x \leq 2$
7. $x < 3$
8. $x \geq 11$
9. $x < 8$
10. $x < \frac{11}{5}$
11. $x \leq 14$
12. $x \leq 1.6$
13. $x \leq 9$
14. $x < -\frac{6}{5}$
15. $x \leq -\frac{10}{9}$

Solving Absolute Value Equations and Inequalities

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Overview

Absolute value is the numerical value of a number without its sign. The absolute value of a number represents the distance on a number line between that number and 0.

To solve an equation with absolute value, use the Looking Inside strategy introduced in Chapter 3 to write two separate equations, as the quantity inside the absolute value can be either positive or negative. Then solve each equation.

One method to solve absolute value inequalities is to rewrite the inequality as an equation, then solve the equation and place the solution(s) on a number line. The solutions are called boundary points, and they divide the number line into regions. Test points are used to determine whether a region is part of the solution to the inequality.

Additional information on solving absolute value equations can be found in the Lesson 9.2.2 Methods & Meanings box. For more on solving inequalities, reference the Lesson 9.1.3 Methods & Meanings box.

Examples

1. Solve the absolute value equation.

$$|2x + 3| = 11$$

Since $|2x + 3| = 11$, the "Looking Inside" method can be used to write two equations using the expression $2x + 3$ and the opposite of the expression $-(2x + 3)$. With these two expressions, write new equations and solve as shown.

$$|2x + 3| = 11$$

$$\begin{array}{rcl} 2x + 3 = 11 & \quad & -(2x + 3) = 11 \\ -3 \quad -3 & & 2x + 3 = -11 \\ 2x = 8 & & -3 \quad -3 \\ x = 4 & & 2x = -14 \\ & & x = -7 \end{array}$$

Check the solutions by substituting the values into the original equation.

$$\begin{array}{rcl} x = 4 & \quad & x = -7 \\ |2x + 3| = 11 & & |2x + 3| = 11 \\ |2(4) + 3| = 11 & & |2(-7) + 3| = 11 \\ |11| = 11 & & |-11| = 11 \\ 11 = 11 & & 11 = 11 \\ \text{true} & & \text{true} \end{array}$$

2. Solve the absolute value inequality and represent the solution on a number line.

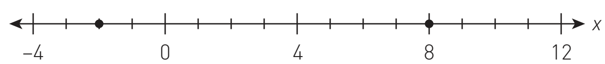
$$|x - 3| \leq 5$$

Write the equation that corresponds to the inequality. Since $|x - 3| = 5$, the "Looking Inside" method can be used to write two equations using the expression $(x - 3)$ and the opposite of the expression $-(x - 3)$. With these two expressions, write new equations and solve as shown.

$$|x - 3| = 5$$

$$\begin{array}{rcl} x - 3 = 5 & \qquad & -(x - 3) = 5 \\ +3\ \ +3 & & x - 3 = -5 \\ x = 8 & & +3\ \ \ +3 \\ & & x = -2 \end{array}$$

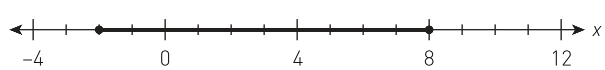
Place these points on a number line as closed circles since the inequality symbol, $\leq$, indicates the boundary points are included in the solution.



Then, choose a point from each of the three regions created. Test the points in the original inequality. For this example, the x-values of −3, 0, and 10 are used.

$$\begin{array}{ccc} x = -3 & x = 0 & x = 10 \\ |x - 3| \leq 5 & |x - 3| \leq 5 & |x - 3| \leq 5 \\ |-3 - 3| \leq 5 & |0 - 3| \leq 5 & |10 - 3| \leq 5 \\ |-6| \leq 5 & |-3| \leq 5 & |7| \leq 5 \\ 6 \leq 5 & 3 \leq 5 & 7 \leq 5 \\ \text{false} & \text{true} & \text{false} \end{array}$$

Since one value yields a true inequality statement, there is one solution region for the absolute value inequality. All the points in this region are solutions to the inequality.



The solution is all numbers greater than or equal to −2 and less than or equal to 8, written as $-2 \leq x \leq 8$.

3. Solve the absolute value inequality and represent the solution on a number line.

$$3|2x + 1| + 1 > 28$$

Write the equation that corresponds to the inequality, then rewrite it so the absolute value expression is by itself on one side of the equation.

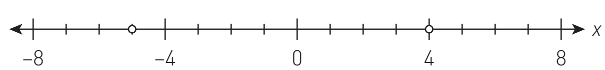
$$\begin{aligned} 3|2x + 1| + 1 &= 28 \\ 3|2x + 1| &= 27 \\ |2x + 1| &= 9 \end{aligned}$$

Since $|2x + 1| = 9$, the "Looking Inside" method can be used to write two equations using the expression $(2x + 1)$ and the opposite expression $-(2x + 1)$. With these two expressions, write new equations and solve as shown.

$$|2x + 1| = 9$$

$$\begin{aligned} 2x + 1 &= 9 \\ -1 \quad & -1 \\ 2x &= 8 \\ x &= 4 \end{aligned} \qquad \begin{aligned} -(2x + 1) &= 9 \\ 2x + 1 &= -9 \\ -1 \quad & -1 \\ 2x &= -10 \\ x &= -5 \end{aligned}$$

Place these points on a number line as open circles since the inequality symbol, >, indicates the boundary points are not included in the solution.



Then, choose a point from each of the three regions created. Test the points in the original inequality. For this example, the x-values of −6, 0, and 5 are used.

$x = -6$

$$\begin{aligned} 3|2x + 1| + 1 &> 28 \\ 3|2(-6) + 1| + 1 &> 28 \\ 3|-11| + 1 &> 28 \\ 33 + 1 &> 28 \\ 34 &> 28 \end{aligned}$$

true

$x = 0$

$$\begin{aligned} 3|2x + 1| + 1 &> 28 \\ 3|2(0) + 1| + 1 &> 28 \\ 3|1| + 1 &> 28 \\ 3 + 1 &> 28 \\ 4 &> 28 \end{aligned}$$

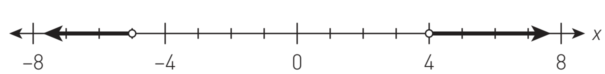
false

$x = 5$

$$\begin{aligned} 3|2x + 1| + 1 &> 28 \\ 3|2(5) + 1| + 1 &> 28 \\ 3|11| + 1 &> 28 \\ 33 + 1 &> 28 \\ 34 &> 28 \end{aligned}$$

true

Since two values yielded true inequality statements, there are two solution regions for the absolute value inequality. All the points in these regions are solutions to the inequality.



The solution is all numbers less than −5 or greater than 4, written as $y < -5$ or $y > 4$.

Practice

Solve each absolute value equation.

1. $|x-2|=5$

2. $|3x+2|=11$

3. $|5-x|=9$

4. $|3-2x|=7$

5. $|2x+3|=-7$

6. $|4x+1|=10$

Solve each absolute value inequality and represent the solution on a number line.

7. $|x+4|\geq 7$

8. $|x|-5\leq 8$

9. $|x-5|\leq 8$

10. $|4x-2|>8$

11. $|3x|\leq 12$

12. $|1-3x|\leq 13$

13. $|2x-3|>15$

14. $|5x|>-15$

15. $-2|x-3|+6<-4$

16. $|4-x|\leq 7$

17. $|x-4|\leq 0$

18. $|2x+1|-2<-3$

Answers

1. $x=7$ or -3

2. $x=3$ or $-\frac{13}{3}$

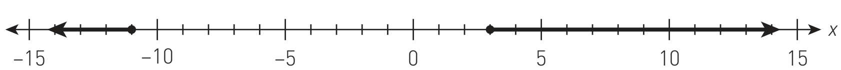
3. $x=-4$ or 14

4. $x=-2$ or 5

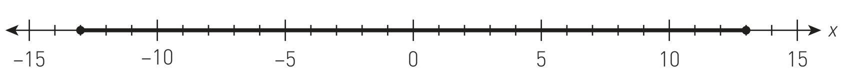
5. no solution

6. $x=\frac{9}{4}$ or $-\frac{11}{4}$

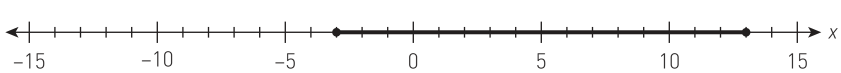
7. $x\geq 3$ or $x\leq -11$



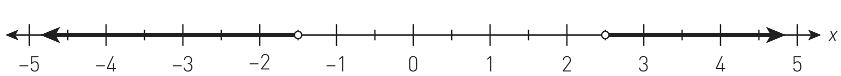
8. $-13\leq x\leq 13$



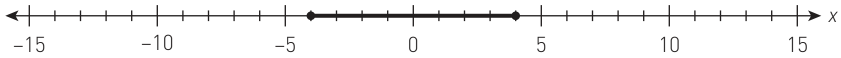
9. $-3\leq x\leq 13$



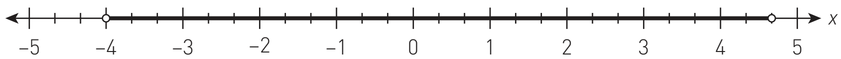
10. $x<-\frac{3}{2}$ or $x>\frac{5}{2}$



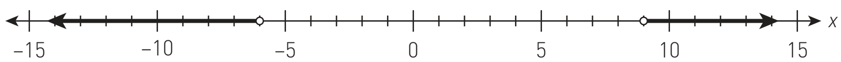
11. $-4 \le x \le 4$



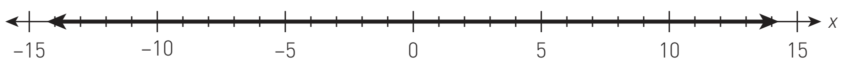
12. $-4 < x < \frac{14}{3}$



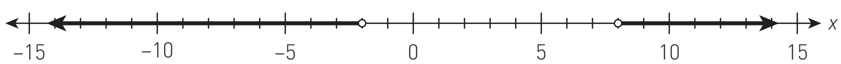
13. $x < -6$ or $x > 9$



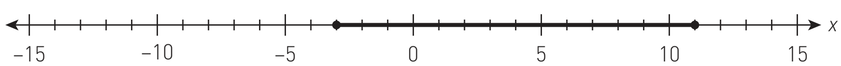
14. all real numbers



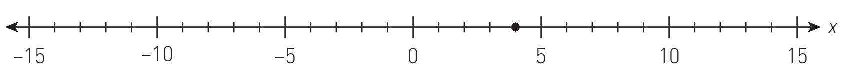
15. $x > 8$ or $x < -2$



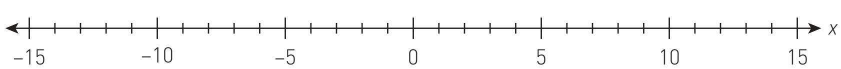
16. $-3 \le d \le 11$



17. $x = 4$



18. no solution



Graphing Two-Variable Inequalities

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Overview

Two-variable inequalities are graphed on a coordinate plane. To graph the solution region to a two-variable inequality, first graph the corresponding equation. The graph of the equation is the boundary line (or curve), since all points that make the inequality true lie on only one side of the line. If the inequality symbol is either $\le$ or $\ge$, then the boundary line is included in the solution to the inequality, and it is drawn as a solid line. If the inequality symbol is either $<$ or $>$, then the boundary line is *not* included in the solution to the inequality, and it is drawn as a dashed line.

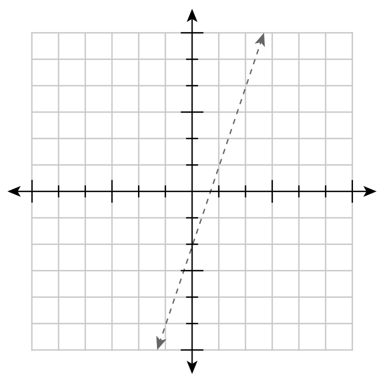
Next, as when solving one-variable inequalities, test points are used to determine which side of the boundary is the solution region. The point (0, 0) is often a convenient point to use, but any point can be used as the test point as long as it is not on the boundary. If the inequality is true for the test point, then shade the graph on this side of the boundary. If the inequality is false for the test point, then shade the opposite side of the boundary. The solution region represents all values that make the inequality true. There are an infinite number of solution points.

For additional information, see the Methods & Meanings box in Lesson 9.3.1.

Examples

1. Graph the solutions to the inequality $y > 3x - 2$.

First, graph the line $y = 3x - 2$ using a dashed line since the inequality symbol, >, indicates that the boundary line is not part of the solution. For example, the point (0, –2) is on the boundary line, and it is not a solution to the inequality. When the point is substituted into the original inequality, it results in a false inequality statement: $-2 > 3(0) - 2$.



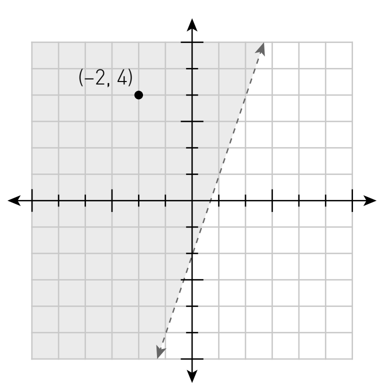
Next, test a point that is not on the boundary line by substituting the point into the original inequality. For this example, the point (–2, 4) is used.

$(-2, 4)$

$$y > 3x - 2$$
$$4 > 3(-2) - 2$$
$$4 > -8$$

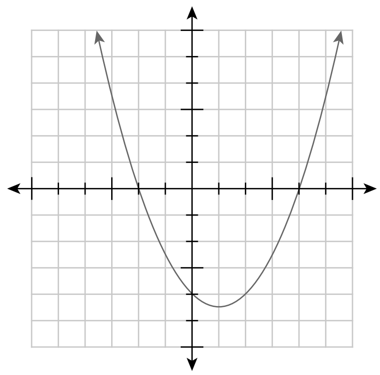
true

This is a true statement. Since the inequality is true for the test point, shade the region containing the point (–2, 4). The shaded region represents all possible solutions to the inequality.



2. Graph the solutions to the inequality $y \le x^2 - 2x - 8$.

Graph the equation $y = x^2 - 2x - 8$ using a solid line since the inequality symbol, $\le$, indicates that the boundary is part of the solution.



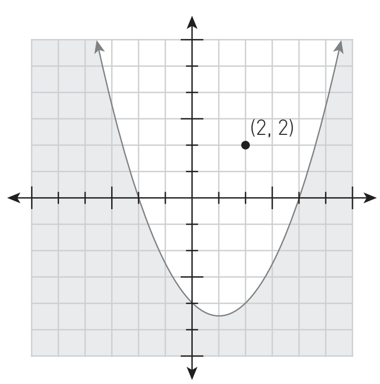
Next, test a point that is not on the boundary by substituting the point into the original inequality. For example, use the point (2, 2).

$$(2, 2)$$

$$y \le x^2 - 2x - 8$$
$$2 \le (2)^2 - 2(2) - 8$$
$$2 \le -8$$

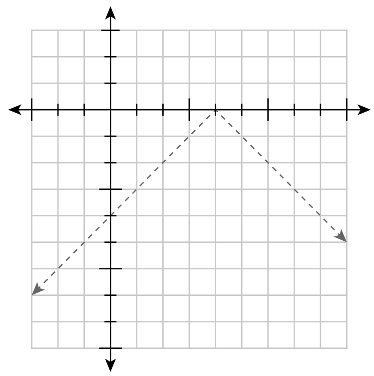
false

The inequality is false for the test point, which lies above the boundary. Therefore, the solution region is the area below the boundary. Shade below the boundary.



3. Graph the solutions to the inequality $y < -|x - 4|$.

Graph the equation $y < -|x - 4|$ using a dashed line since the inequality symbol, <, indicates that the boundary is not part of the solution.



Next, test a point that is not on the boundary by substituting the point into the original inequality. For example, use the point (0, 0).

(0, 0)

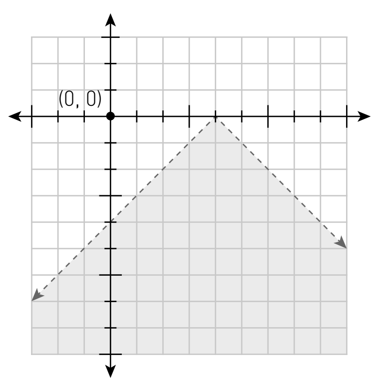
$y < -|x - 4|$

$0 < -|0 - 4|$

$0 \leq -4$

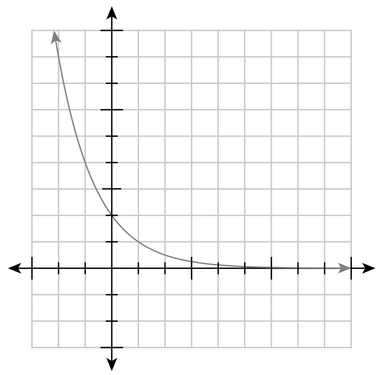
false

The inequality is false for the test point, which lies above the boundary. Therefore, the solution region is the area below the boundary. Shade below the boundary.



4. Graph the solutions to the inequality $y \geq 2(0.5)^x$.

Graph the equation $y = 2(0.5)^x$ using a solid line since the inequality symbol, $\geq$, indicates that the boundary is part of the solution.



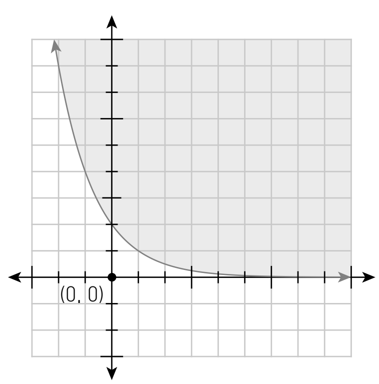
Next, test a point that is not on the boundary by substituting the point into the original inequality. For example, use the point (0, 0).

(0, 0)

$$y \geq 2(0.5)^x$$
$$0 \geq 2(0.5)^0$$
$$0 \geq 2(1)$$
$$0 \geq 2$$

false

The inequality is false for the test point, which lies below the boundary. Therefore, the solution region is the area above the boundary. Shade above the boundary.



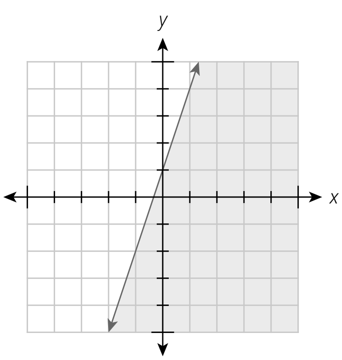
Practice

Graph the solutions to each of the following inequalities on a separate set of axes.

1. $y \le 3x + 1$
2. $y \ge -2x + 3$
3. $y > 4x - 2$
4. $y < -3x - 5$
5. $y \le 3$
6. $y > 0.2(5)^x$
7. $y > \frac{2}{3}x + 8$
8. $y < -\frac{3}{5}x - 7$
9. $3x + 2y \ge 7$
10. $-4x + 2y < 3$
11. $y \ge x^2 - 3$
12. $y \le 0.4(2)^x$
13. $y < 4 - x^2$
14. $y \le x + 2$
15. $y \ge -|x| + 3$

Answers

1.



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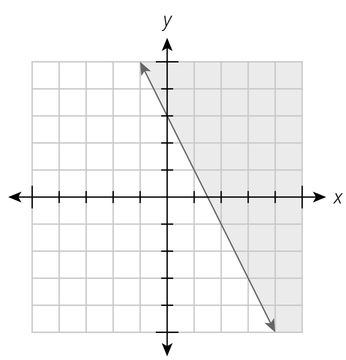
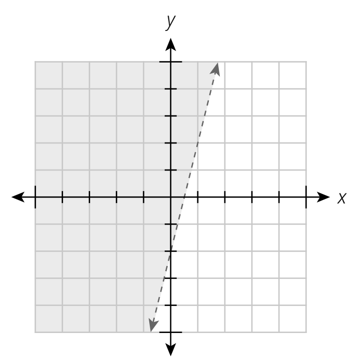
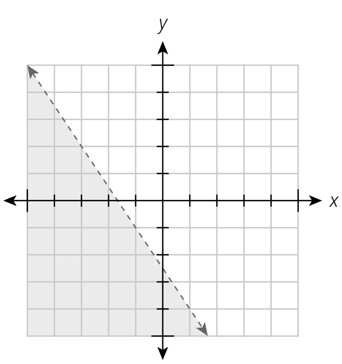


3.



4.



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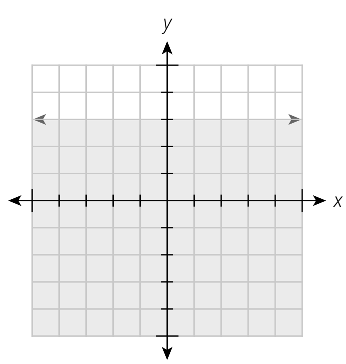

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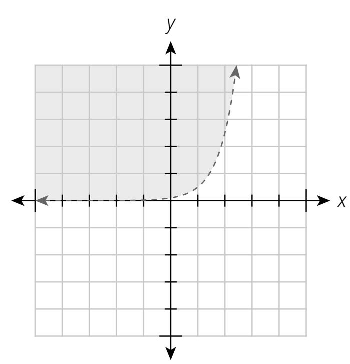

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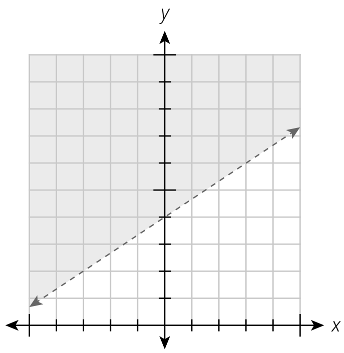

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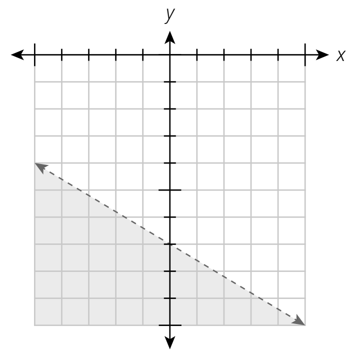

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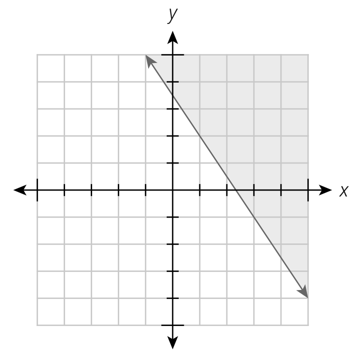

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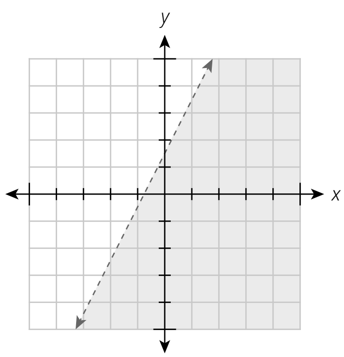

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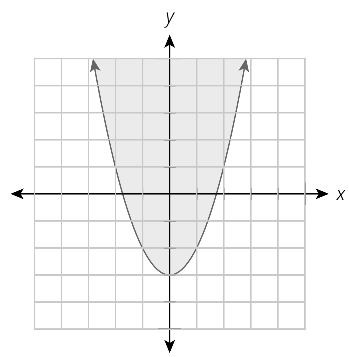

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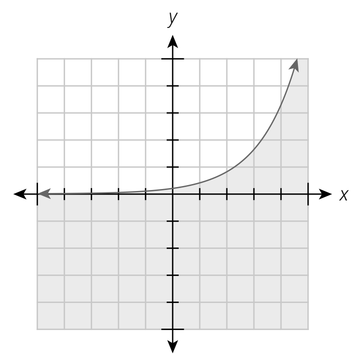

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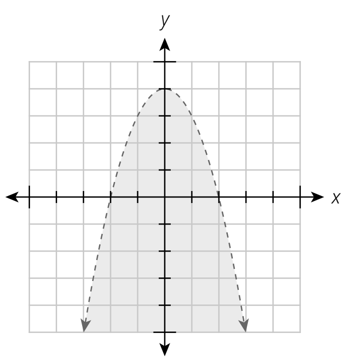

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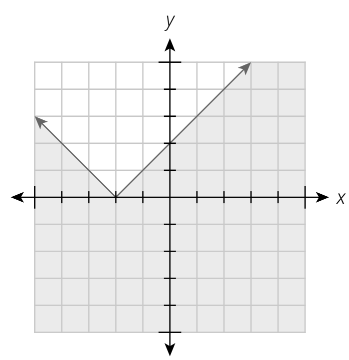

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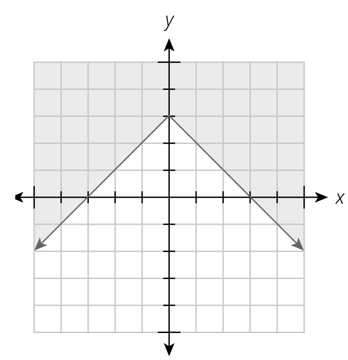


Systems of Inequalities

GUIDED SKILL SUPPORT

Overview

A system of inequalities is similar to a system of equations. They are graphed on a coordinate plane with two lines. In a system of inequalities, these lines are referred to as boundary lines, and the solution includes an infinite number of solutions, or points. A system of equations has none no, one, or infinitely many solutions.

Systems of inequalities can be graphed in a similar way to two-variable inequalities. First, graph the line of the corresponding equation for each inequality, then use a test point from each region formed to identify and shade the solution region. The overlap of the shaded regions from both inequalities represents the solution to the system of inequalities.

Additional information can be found in the Lesson 9.4.2 Reflection Journal.

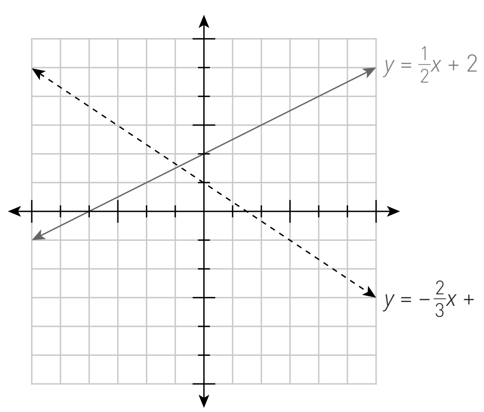
Examples

1. Graph the system of inequalities.

$$y \le \frac{1}{2}x + 2$$

$$y > -\frac{2}{3}x + 1$$

Graph the lines $y = \frac{1}{2}x + 2$ and $y = -\frac{2}{3}x + 1$. The first line is solid and the second is dashed.



Example continues on next page.

1. Example continued from previous page.

Choose test points from each of the four regions created by the two lines. In this example, the points (–5, 0), (–4, 5), (3, 0), and (0, 0) are used. Substitute each point into both of the original inequalities. In order for a point to be a solution, both inequalities must result in a true statement.

(–5, 0)

$y \le \frac{1}{2}x + 2$

$0 \le \frac{1}{2}(-5) + 2$

$0 \le -\frac{1}{2}$

false

$y > -\frac{2}{3}x + 1$

$0 > -\frac{2}{3}(-5) + 1$

$0 > \frac{13}{3}$

false

(–4, 5)

$y \le \frac{1}{2}x + 2$

$5 \le \frac{1}{2}(-4) + 2$

$5 \le 0$

false

$y > -\frac{2}{3}x + 1$

$5 > -\frac{2}{3}(-4) + 1$

$5 > \frac{11}{3}$

true

(3, 0)

$y \le \frac{1}{2}x + 2$

$0 \le \frac{1}{2}(3) + 2$

$0 \le \frac{7}{2}$

true

$y > -\frac{2}{3}x + 1$

$0 > -\frac{2}{3}(3) + 1$

$0 > -1$

true

(0, 0)

$y \le \frac{1}{2}x + 2$

$0 \le \frac{1}{2}(0) + 2$

$0 \le 2$

true

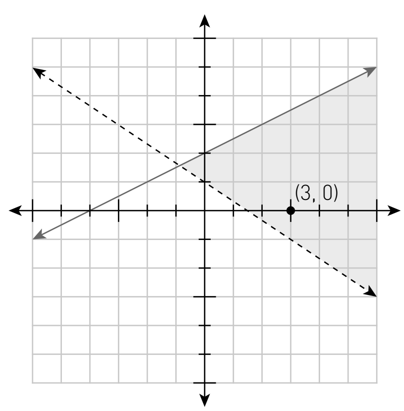
$y > -\frac{2}{3}x + 1$

$0 > -\frac{2}{3}(0) + 1$

$0 > 1$

false

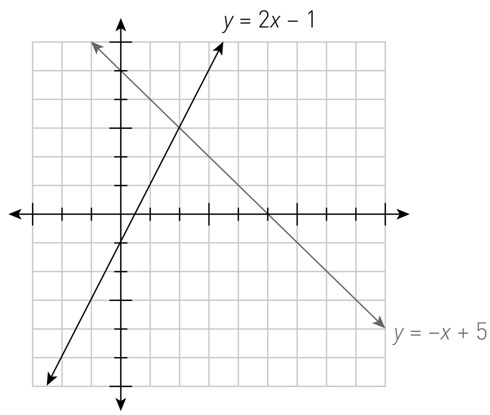
The point (3, 0) gave a true statement for both inequalities. Therefore, the region that includes this point is the solution.



2. Graph the system of inequalities.

$$y \le -x + 5$$
$$y > 2x - 1$$

Graph the lines $y = -x + 5$ and $y = 2x - 1$ with solid lines.



Choose test points from each of the four regions created by the two lines. In this example, the points (0, 0), (0, 6), (6, 0), and (0, −2) are used. Substitute each point into both of the original inequalities. In order for a point to be a solution, both inequalities must result in a true statement.

(0, 0)

$y \le -x + 5$
$0 \le -(0) + 5$
$0 \le 5$
true

$y \ge 2x - 1$
$0 \ge 2(0) - 1$
$0 \ge -1$
true

(0, 6)

$y \le -x + 5$
$6 \le -(0) + 5$
$6 \le 5$
false

$y \ge 2x - 1$
$6 \ge 2(0) - 1$
$6 \ge -1$
true

(6, 0)

$y \le -x + 5$
$0 \le -(6) + 5$
$0 \le -1$
false

$y \ge 2x - 1$
$0 \ge 2(6) - 1$
$0 \ge 11$
false

(0, −2)

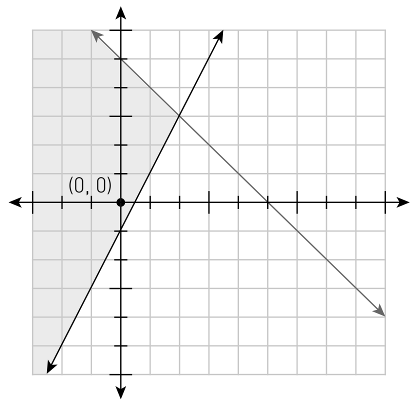
$y \le -x + 5$
$-2 \le -(0) + 5$
$-2 \le 5$
true

$y \ge 2x - 1$
$-2 \ge 2(0) - 1$
$-2 \ge -1$
false

Example continues on next page.

2. Example continued from previous page.

The point (0, 0) gave a true statement for both inequalities. Therefore, the region that includes this point is the solution.



Practice

Graph the solutions to each of the following pairs of inequalities on the same set of axes.

1. $y > 3x - 4$
$y \le -2x + 5$

2. $y \ge -3x - 6$
$y > 4x - 4$

3. $y < -\frac{3}{5}x + 4$
$y < \frac{1}{3}x + 3$

4. $y < -\frac{3}{7}x - 1$
$y > \frac{4}{5}x + 1$

5. $y < 3$
$y > \frac{1}{2}x + 2$

6. $x \le 3$
$y < \frac{1}{3}x - 4$

7. $y \le 2x + 1$
$y \ge 3x - 4$

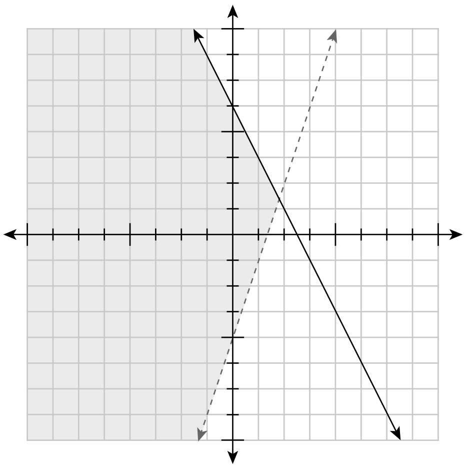
8. $y < -\frac{1}{2}x + 5$
$y \ge x + 1$

9. $y < -x + 6$
$y \le \frac{1}{4}x - 2$

10. $y > -x + 4$
$y < x - 1$

Answers

1.



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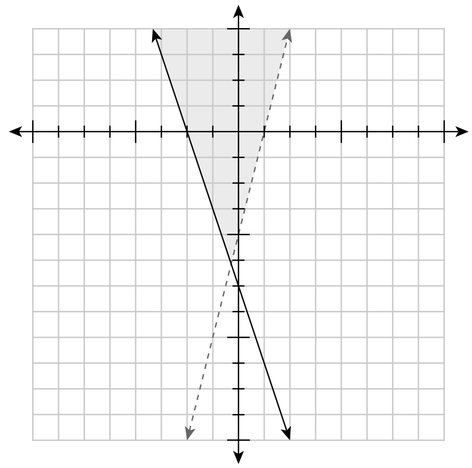

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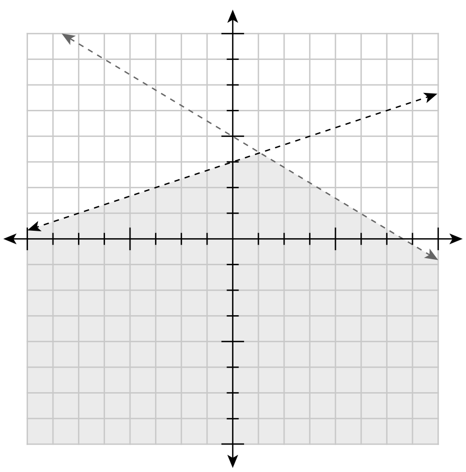

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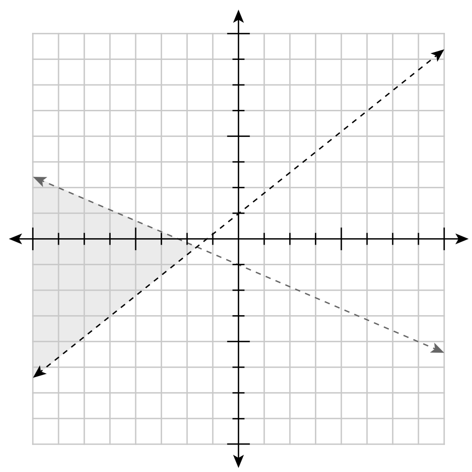

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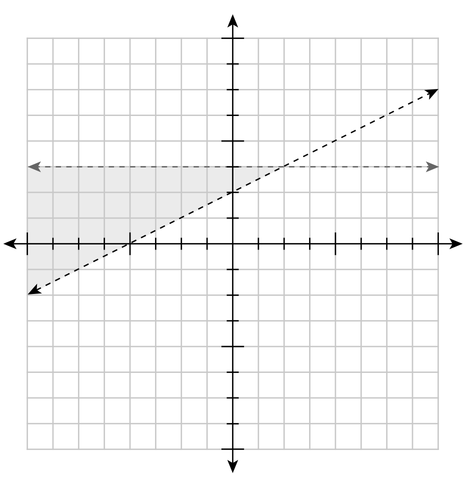

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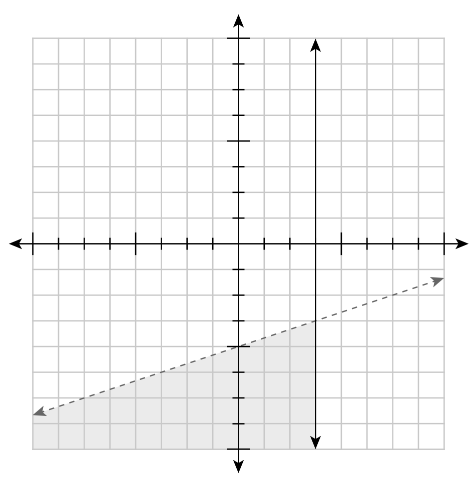

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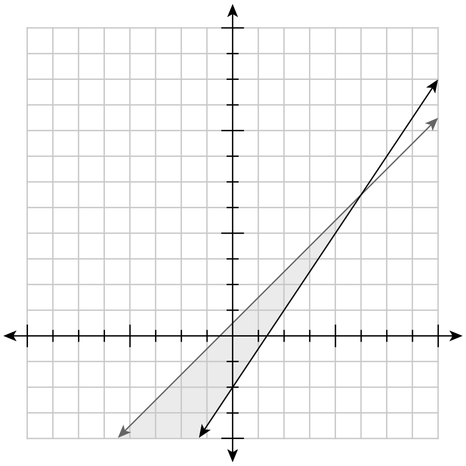

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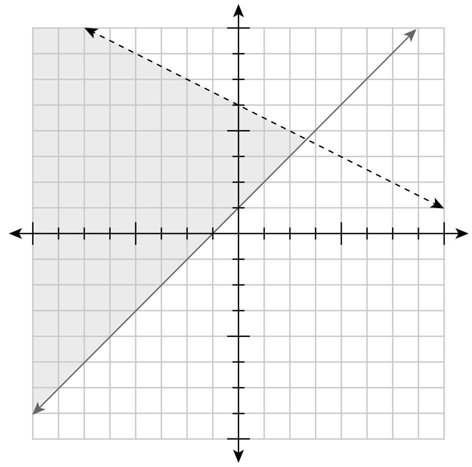

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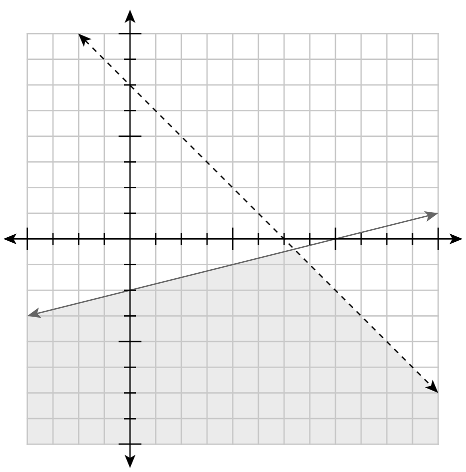

10.