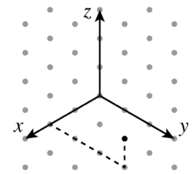
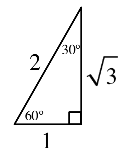
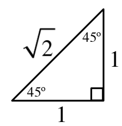
Glossary



3-D coordinate system In three dimensions, the z-axis is perpendicular to the x-y plane at $(0, 0)$. Points in 3-dimensions are represented with coordinates (x, y, z). The first octant where x, y, and z are positive is shown at right with the point $(2, 3, 1)$. (p. 580)

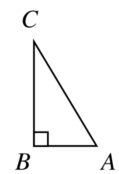


30º-60º-90º triangle A special right triangle with acute angle measures of 30º and 60º. The side lengths are always in the ratio of $1:\sqrt{3}:2$.



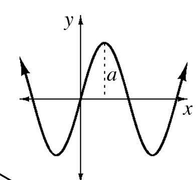
45º-45º-90º triangle A special right triangle with acute angle measures of 45º. The side lengths are always in the ratio of $1:1:\sqrt{2}$.

absolute value The absolute value of a number is the distance of the number from zero. Since the absolute value represents a distance without regard to direction, it is always a positive real number. For example, $|-7| = 7$, and $|7| = 7$. (p. 128)

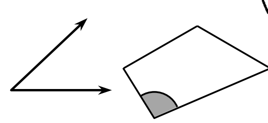


acute angle An angle with a measure greater than 0º and less than 90º. Also see *obtuse angle*.

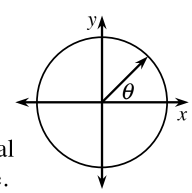
adjacent leg In a right triangle, the leg adjacent to an acute angle is the side of the angle that is not the hypotenuse. For example, in ΔABC shown at right, $\overline{AB}$ is the leg adjacent to $\angle A$. (p. 55)



amplitude (of a periodic function) The amplitude of a periodic graph is one-half the distance between the highest and lowest points. In the graph at right, a is the amplitude. (p. 477)

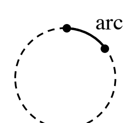


angle In general, an angle is formed by two rays joined at a common endpoint. Angles in geometric figures are usually formed by two segments with a common endpoint (such as the angle shaded in the figure at right).

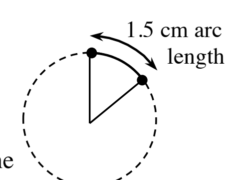


angle of rotation An angle with its vertex at the origin, formed by counterclockwise rotation from the positive x-axis, is called an angle of rotation in standard position. For an angle θ, in standard position, the positive x-axis is the initial ray and the terminal ray may point in any direction. The measure of such an angle may have any real value. (p. 472)

approximate answer An answer that is not precisely accurate and not exact. For example, if the length of a side of a triangle is $\sqrt{10}$, the decimal approximation of the answer to the question "*How long is that side of the triangle?*" would be 3.162, while the exact answer would be $\sqrt{10}$. Also see *exact answer*.

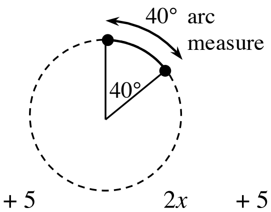


arc A connected part of a circle. Because a circle does not contain its interior, an arc is more like a portion of a bicycle tire than a slice of pizza.

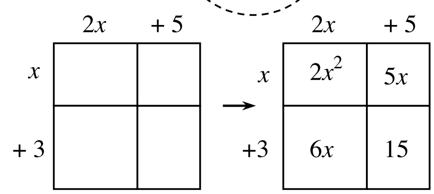


arc length The length of an arc (in inches, centimeters, feet, etc.) is the distance from one of the arc's endpoints to the arc's other endpoint, measured around the circle. Note that arc length is different from arc measure. Arcs of two different circles may have the same arc measure (like 90°) but different arc lengths if one circle has a larger radius than the other. Also see *arc measure*.

arc measure The measure of an arc's central angle. Note that arc measure is often measured in degrees but can also be measured in radians. It is different from arc length. Also see *arc length*.



area model A type of diagram used to visualize multiplying expressions without algebra tiles. Each expression to be multiplied forms a side length of the rectangle, and the product is the sum of the areas of the sections of the rectangle. For example, the area model at right can be used to multiply $(2x + 5)$ by $(x + 3)$.



$(2x + 5)(x + 3) = 2x^2 + 11x + 15$

area as a product area as a sum

area model (for probability) See *probability area model*.

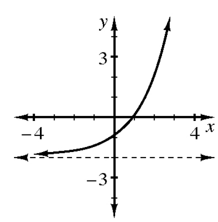
argument Used with sigma notation for sequences to describe the n^{th} term. In the expression $\sum_{k=1}^{10}(2k+1)$, the expression $2k + 1$ is the argument. (p. 508)

arithmetic sequence In an arithmetic sequence the difference between sequential terms is constant. Each term of an arithmetic sequence can be generated by adding the common difference to the previous term. For example, in the sequence 4, 7, 10, 13, …, the common difference is 3.

arithmetic series An arithmetic series is the sum of the terms of an arithmetic sequence. Given the arithmetic sequence 3, 7, 11, …, 43, the corresponding arithmetic series is 3 + 7 + 11 + … + 43. Also see *arithmetic sequence*. (p. 504)

association A relationship between two (or more) variables. An association between numerical variables can be displayed on a scatterplot, and described by its form, direction, strength, and outliers. Possible association between two categorical variables can be studied in a relative frequency table. If one variable increases as the other increases, the direction is said to be a positive association. If one variable increases as the other decreases, there is said to be a negative association. If there is no apparent pattern in the scatterplot, then the variables have no association. Also see *scatterplot* and *direction (of an association)*.

asymptote In this course, a line that a graph of a curve approaches but never meets. An asymptote is often represented by a dashed line on a graph. For example, the graph at right has a horizontal asymptote at $y = –2$. A more complete definition of an asymptote requires some ideas from calculus. (p. 20)



average See *mean*.

average rate of change The average rate of change of a function over an interval is the change in the value of the dependent quantity divided by the change in the independent quantity. It is the change in the y-value divided by the change in the x-value for two distinct points on the graph, which is the slope of the line through these two distinct points. (p. 87)

axes In a coordinate plane, the two perpendicular number lines that intersect at the origin (0, 0). The x-axis is horizontal and the y-axis is vertical. In three dimensions a third number line, the z-axis, is perpendicular to the x-y plane at the origin (0, 0, 0). See *coordinate system* and *3-D coordinate system* for an illustration.

base (of a shape) (1) Triangle: any side of a triangle to which a height is drawn. There are three possible bases in each triangle; (2) Trapezoid: the two parallel sides; (3) Parallelogram (and rectangle, rhombus, and square): any side to which a height is drawn (there are four possible bases in each parallelogram).

base (of an exponent) When working with an exponential expression in the form b^a, b is called the base. For example, 2 is the base in 2^5. (5 is the exponent, and 32 is the value.) Also see *exponent*. (p. 31)

bias A systematic inaccuracy in data due to a process that favors certain outcomes. (p. 158)

biased wording Words used in a survey question that intentionally or unintentionally influence results. (p. 159)

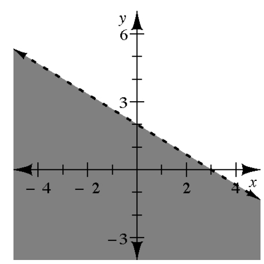
bin width An interval, or the width of a bar, on a histogram. Also see *histogram*.

binomial An expression that is the sum or difference of exactly two terms, each of which is a monomial. For example, $-2x + 3y^2$ is a binomial.

binomial expansion The polynomial that results when a power of a binomial is multiplied out. For example, the third power of $(ax + b)$ or $(ax + b)^3 = a^3x^3 + 3a^2bx^2 + 3ab^2x + b^3$. (p. 542)

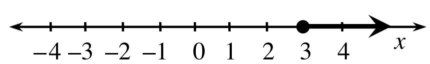
bound The highest or lowest value that a statistical prediction is within. Also see *margin of error* and *boundary line*. (p. 277)

boundary line or curve (1) A line or curve on a two-dimensional graph that divides the graph into two regions. A boundary line or curve is used when graphing inequalities with two variables. For example, the inequality $y < -\frac{2}{3}x + 2$ is graphed at right. The dashed boundary line has the equation $y = -\frac{2}{3}x + 2$. A boundary line is also sometimes called a "dividing line". (2) A line drawn parallel to and above or below the least squares regression line at a distance equivalent to the largest residual. The line determines the upper or lower limit on the values that a prediction is likely to be. (p. 140)

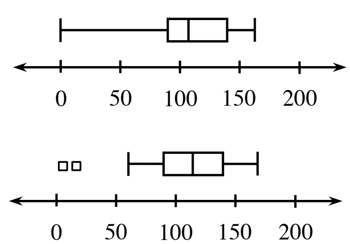


$y < -\frac{2}{3}x + 2$

boundary point The endpoint of a ray or segment on a number line where an inequality is true. For strict inequalities (that is, inequalities involving < or >), the point is not part of the solution. We determine boundary points by solving the equation associated with our inequality. For example, the solution to the equation $2x + 5 = 11$ is $x = 3$, so the inequality $2x + 5 \geq 11$ has a boundary point at 3. The solution to that inequality is illustrated on the number line above. A boundary point is also sometimes called a "dividing point". (p. 137)



boxplot A graphical way of displaying the five-number summary of a distribution of data: minimum, first quartile, median, third quartile, and maximum. A **modified boxplot** displays outliers separately from the boxplot. Also see *five-number summary*. (p. 43)

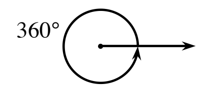


Cartesian Plane See *coordinate system*.

census The process of measuring every member of the population. (p. 161)

center (of a data distribution) A number that represents the middle of a data set, or that represents a "typical" value of the set. Two ways to measure the center of a data set are the mean and the median. When dealing with measures of center, it is often useful to consider the distribution of the data. For symmetric distributions with no outliers, either the mean or the median can represent the middle, or "typical" value, of the data well. However, in the presence of outliers or non-symmetrical data distributions, the median may be a better measure. Also see *mean* and *median*. (p. 43)

circular angle An angle with a measure of 360°.



closed question A question that limits respondents to some set number of responses from which to choose. (p. 160)

closed set A set of numbers is said to be closed under an operation if the result of applying the operation to any two numbers in the set produces a number in the set. For example, the whole numbers are a closed set under addition because if you add any two whole numbers, the result is always a whole number. However, the whole numbers are not closed under division: if you divide any two whole numbers, you do not always get a whole number. (p. 25)

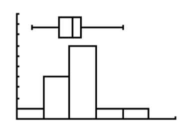
cluster (of data) A group of data points that occur together.

coefficient When variable(s) are multiplied by a number, the number is called a coefficient of that term. The numbers that are multiplied by the variables in the terms of a polynomial are called the coefficients of the polynomial. For example, 3 is the coefficient of $3x^2$. (p. 26)

cofunction A trigonometric function whose value for the complement of an angle is equal to the value of a given trigonometric function for the angle itself. For example, the cofunction of sine is cosine and the cofunction of tangent is cotangent. (p. 639)

combination A combination is the number of ways we can select items from a larger set without regard to order. For instance, choosing a committee of 3 students from a group of 5 volunteers is a combination because the order in which committee members are selected does not matter. We write ${}_nC_r$ to represent the number of combinations of n things taken r at a time (or n choose r). For instance, the number of ways to select a committee of 3 students from a group of 5 is ${}_5C_3$. You can use Pascal's Triangle to find ${}_5C_3$ or use permutations and divide by the number of arrangements $\frac{{}_5P_3}{{}_3P_3} = \frac{5!}{3!2!}$. Formulas for combinations include ${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}$. Also see *permutation*. (p. 493)

combination boxplot and histogram A way to visually represent a distribution of data. The boxplot is drawn with the same x-axis as the histogram. (p. 43)



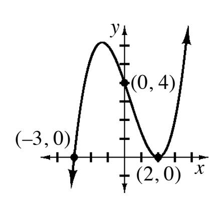
common difference The difference between consecutive terms of an arithmetic sequence or the *generator* of the sequence. When the common difference is positive, the sequence increases; when it is negative, the sequence decreases. In the sequence 3, 7, 11, …, 43, the common difference is 4.

common logarithm The logarithm with base 10. If no base is given, the logarithm is understood to be base 10. (p. 253)

common multiplier of a sequence Another name for *common ratio*. (p. 538)

common ratio Common ratio is another name for the *multiplier* or *generator* of a geometric sequence. It is the number to multiply one term by to get the next one. In the sequence 96, 48, 24, …, the common ratio is $\frac{1}{2}$. (p. 538)

complete graph A complete graph is one that includes everything that is important about the graph (such as intercepts and other key points, asymptotes, or limitations on the domain or range), and that makes the rest of the graph predictable based on what is shown. Points are plotted accurately. Both axes are labeled with the variable name and with the units of measurement. The scale on each axis should be marked in equal intervals and be appropriate to highlight key features. Connect continuous data but do not connect discrete data. Use arrowheads if the line or curve extends beyond the edge of the grid. For example, a complete graph of the equation $y = \frac{1}{3}(x-2)^2(x+3)$ is shown at right. (p. 7)



completing the square A standard procedure for rewriting a quadratic equation from standard form to perfect square form (or graphing form), or for rewriting the equation of a circle from general form to graphing form. One method for completing the square to rewrite $y = x^2 - 6x + 4$ in graphing form follows: (p. 93)

$$\begin{aligned} y &= x^2 - 6x + 4 \\ y - 4 &= x^2 - 6x \\ y - 4 + 9 &= x^2 - 6x + 9 \\ y + 5 &= (x-3)^2 \\ y &= (x-3)^2 - 5 \end{aligned}$$

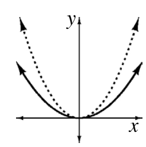
complex conjugate The complex number $a + bi$ has a complex conjugate $a - bi$. Similarly, the conjugate of $c - di$ is $c + di$. What is noteworthy about complex conjugates is that both their product $(a + bi)(a - bi) = a^2 - b^2i^2 = a^2 + b^2$ and their sum $(a + bi) + (a - bi) = 2a$ are real numbers. If a complex number is a zero (or root) of a polynomial function with real coefficients, then its complex conjugate is also a zero (or root). (p. 390)

complex number A number written in the form $a + bi$ where a and b are real numbers. Each complex number has a real part, a, and an imaginary part, bi. Note that real numbers are also complex numbers where $b = 0$, and imaginary numbers are complex numbers where $a = 0$. (p. 72)

composite function A function that is created as the result of using the outputs of one function as the inputs of another can be expressed as a composite function. For example, the function $h(x) = |\log(x)|$ can be expressed as the composite function $f(g(x))$ where $f(x) = |x|$ and $g(x) = \log(x)$. (p. 244)

compound interest Interest that is paid on both the principal and the previously accrued interest. (p. 565)

compress A term used informally to describe the relationship of a graph to its parent graph when the graph increases or decreases more slowly than the parent. For example, the solid parabola shown at right is a compressed version of its parent, shown as a dashed curve. (p. 53)



conjugate Every complex number $a + bi$ has a conjugate $a - bi$, and both the sum and product of the conjugates are real. Similarly, an irrational number that can be written $a+\sqrt{b}$, where a and b are rational, has a conjugate, $a-\sqrt{b}$, and the product and sum of these conjugates are rational. Also see *complex conjugate*. (p. 390)

constant function A constant function is a function where every input value gives the same output value. For example, the function $f(x) = 5$ is a constant function because the output value is 5 for every value of x.

constant term A number that is not multiplied by a variable. In the expression $2x + 3(5 - 2x) + 8$, the number 8 is a constant term. The number 3 is not a constant term, because it is multiplied by a variable inside the parentheses. (p. 26)

constraint A limitation or restriction placed on a function or situation, either by the context of the situation or by the nature of the function. (p. 210)

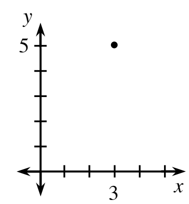
continuous For this course, when the points on a graph are connected and it makes sense to connect them, we say the graph is continuous. A continuous graph will have no holes or breaks in it. This term will be more completely defined in a later course.

continuously compounded interest For this course, continuously compounded interest can be thought of as interest that is computed and added to the balance of an account every instant. The formula for continuous compounded interest is $A = Pe^{rt}$, where A is the total amount at any time, P is the principal, r is the interest rate, and t is time. (p. 565)

control limit The upper and lower limits of a critical measurement that determines acceptable quality in a manufacturing process. See *statistical process control*. (p. 297)

convenience sample A subgroup of the population for which it was easy to collect data. A convenience sample is not a random sample. (p. 170)

coordinate system A system of graphing ordered pairs of numbers on a coordinate plane. An ordered pair represents a point, with the first number giving the horizontal position relative to the x-axis and the second number giving the vertical position relative to the y-axis. For example, the diagram at right shows the point (3, 5) graphed on a coordinate plane.

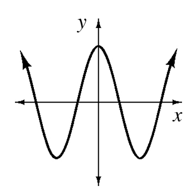


coordinates The numbers in an ordered pair (a, b) or triple (a, b, c) used to locate a point in the plane or in space in relation to a set of coordinate axes.

correlation coefficient, *r* A measure of how much or how little data is scattered around the least squares regression line. It is a measure of the strength of an association that has already been determined to be linear. The correlation coefficient takes on values between –1 and 1. The closer to 1 or –1 the correlation coefficient is, the less scattered the data is around the *least squares regression line* (LSRL).

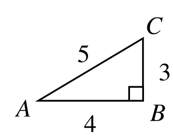
cosecant The cosecant is the reciprocal of the sine given by $\frac{1}{\sin(x)}$. Also see *reciprocal trigonometric functions*. (p. 622)

cosine function The general equation for the cosine function is $y = a\cos(b(x - h)) + k$. This function has amplitude a, period $\frac{2\pi}{b}$, horizontal shift h, and vertical shift k. (p. 445)

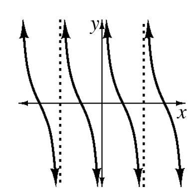


cosine inverse ($\cos^{-1}(x)$) See *inverse cosine*.

cosine ratio In a right triangle, the cosine ratio of an acute angle, A, is $\cos(A) = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$. In the triangle at right, $\cos(A) = \frac{AB}{AC} = \frac{4}{5}$. On a unit circle, the cosine of angle θ, denoted $\cos(\theta)$, is the x-coordinate of the point on the unit circle reached by a rotation angle of θ radians in standard position. Also see *sine ratio, tangent ratio,* and *inverse cosine*. (p. 55)



cotangent The cotangent is the reciprocal of the tangent given by $\frac{1}{\tan(x)}$. The graph of the cotangent function is at right. Also see *reciprocal trigonometric functions*. (p. 623)

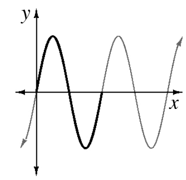


cross-section The intersection of a three-dimensional solid and a plane. The result is a two-dimensional diagram that represents the flat surface of a slice of a solid. The cross-sections of an infinite double cone are called conic sections. (p. 207)

cube A polyhedron, all of whose faces are squares.

cubic A cubic polynomial is a polynomial of the form $y = ax^3 + bx^2 + cx + d$. "Cube" refers to the third (3) power.

cube root The cube root of a number is the value that, when cubed (taken to the third power), gives the number. That is, the cube root of x is the value a such that $a^3 = x$. For example, the cube root of 8 is 2 because $8 = 2 \cdot 2 \cdot 2 = 2^3$. This is written $\sqrt[3]{8} = 2$.



cycle One cycle of a graph of a trigonometric function is a connected piece of the graph of the function that is one complete repetition of the function. (p. 477)

data distribution The statistical distribution of a variable is a description (usually a list, table, or graph) of the number of times each possible outcome occurs. A distribution of data is often summarized using its center, shape, spread, and outliers, and can be displayed with a combination boxplot and histogram. (p. 43)

decision chart A method for counting the number of outcomes (the size of the sample space) for a sequence of probabilistic situations, often in which the order of the items matters. A tree diagram could be used to count the number of outcomes, but if the number of outcomes is large, a decision chart is more useful. For example, how many three-letter arrangements could be made by lining up any three blocks chosen from a set of 26 alphabet blocks, if the first letter must be a vowel and the blocks are not reused? There are three decisions (three blocks to be chosen), with 5 choices for the first letter (a vowel), 25 for the second, and 24 for the third. According to the Fundamental Counting Principle, the total number of possibilities is:

$$\frac{5}{\text{1st decision}} \cdot \frac{25}{\text{2nd decision}} \cdot \frac{24}{\text{3rd decision}} = 3000$$

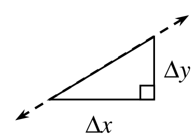
This decision chart is a short way to represent a tree with 5 branches for the first alphabet block, followed by 25 branches for each of those branches; each of those 125 branches would then have 24 branches representing the possibilities for the third alphabet block. Also see *Fundamental Counting Principle*.

decreasing function A function is decreasing if the y-value decreases as the x-value increases.

degree One degree is an angle measure that is $\frac{1}{360}$ of a full circle or $\frac{\pi}{180}$ radians. (p. 451)

degree of a polynomial The degree of a polynomial (in one variable) is the highest power of the variable. The degree of a polynomial function also indicates the maximum number of factors of the polynomial and provides information for predicting the number of "turns" the graph can take. The degree of a monomial is the sum of the exponents of its variables. For example, the degree of $3x^2y^5z$ is 8. For a polynomial, the degree is the degree of the monomial term with the highest degree. Example: for the polynomial $2x^2y^2 - 4x^4z^6 + x^7z$, the degree is 10. (p. 26)

delta (Δ) A Greek letter that is often used to represent a difference, or change, in values. Its uses include Δx and Δy, which represent the lengths of the horizontal and vertical legs of a slope triangle, respectively.



density The quantity of something per unit measure, especially length, area, or volume. For example, the density of birds on a power line may be 7 birds/meter, the population density of Singapore is 7953 people per square kilometer, or the mass density of iron is 7.874 $\frac{g}{cm^3}$. (p. 191)

dependent variable When one quantity's value depends on one or more others, it is called the dependent variable. For example, we might relate the speed of a car to the amount of force you apply to the gas pedal. Here, the speed of the car is the dependent variable; it depends on how hard you push the pedal. The dependent variable appears as the output value in an $x \rightarrow y$ table, and is usually placed relative to the vertical axis of a graph. We often use the letter y and the vertical y-axis for the dependent variable. When working with functions or relationships, the dependent variable represents the output value. In Statistics, the dependent variable is often called the response variable. Also see *independent variable*. (p. 14)

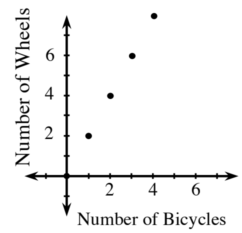
difference of cubes A polynomial of the form $x^3 - y^3$. It can be factored as follows: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$. For example, the difference of cubes $8x^3 - 27$ can be factored as $(2x - 3)(4x^2 + 6x + 9)$. (p. 419)

difference of squares A polynomial that can be factored as the product of the sum and difference of two terms. The general pattern is $x^2 - y^2 = (x + y)(x - y)$. For example, the difference of squares $4x^2 - 9$ can be factored as $(2x + 3)(2x - 3)$. (p. 419)

dimensional analysis The analysis of units of measurement as a check on the plausibility of solutions and mathematical models. Dimensional analysis is also used to convert from one unit of measure to another. For example, make Giant Ones out of the units to convert inches per hour to inches per minute.

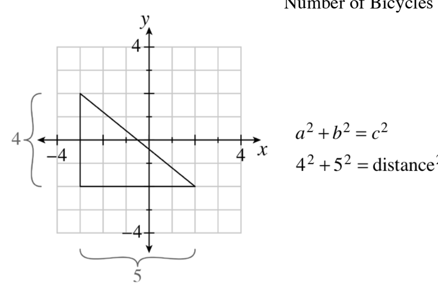
$$\frac{96 \text{ inches}}{1 \text{ hour}} \cdot \boxed{\frac{1 \text{ hour}}{60 \text{ minutes}}} = \frac{96}{60} \cdot \frac{\text{inches}}{\text{minute}} \cdot \boxed{\frac{1}{1} \cdot \frac{\text{hour}}{\text{hour}}} = \frac{1.6 \text{ inches}}{1 \text{ minute}}$$

direction (of an association) If one variable in a relationship increases as the other variable increases, the direction of the association is said to be positive. If one variable decreases as the other variable increases, there is said to be a negative association. If there is no apparent pattern in the scatterplot, then the variables have no association. When describing a linear association, you can use the slope to describe the direction of the association.

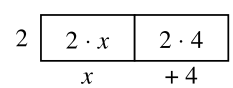


discrete graph A graph that consists entirely of separated points is called a discrete graph. For example, the graph shown at right is discrete. Also see *continuous*.

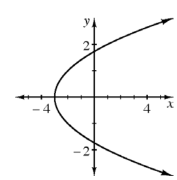
distance formula An application of the Pythagorean Theorem to calculate the distance between two points in a plane. The distance between any two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. In the example at right, the distance is $\sqrt{4^2 + 5^2}$.



Distributive Property We use the Distributive Property to write a product of expressions as a sum of terms. The Distributive Property states that for any numbers or expressions a, b, and c, $a(b + c) = ab + ac$. For example, $2(x + 4) = 2 \cdot x + 2 \cdot 4 = 2x + 8$. We can demonstrate this with an area model.

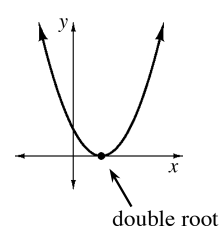


domain The set of all input values for a relationship or function. For example, the domain of the relationship graphed at right is $x \geq -3$. For variables, the domain is the set of numbers the variable may represent. (p. 6)



double-peaked See *shape (of a data distribution)*.

double root A root of a function that occurs exactly twice. If an expression of the form $(x - a)^2$ is a factor of a polynomial, then the polynomial has a double root at $x = a$. The graph of the polynomial does not pass through the x-axis at $x = a$, but is tangent to the axis at $x = a$. (p. 400)



e The number e is the base of natural logarithms and is one of the most important numbers in mathematics, especially for calculus. It is an irrational number, with $e \approx 2.71828\ldots$ To define it properly requires a good understanding of limits, a topic that is explored deeply in calculus. (p. 549)

Elimination Method A method for solving a system of equations. The key step in using the Elimination Method is to add both sides of two equations to eliminate one of the variables. For example, the two equations in the system at right can be added together to get the simplified result $7x = 14$. We can solve this equation for x, then substitute the x-value back into either of the original equations to determine the value of y.

$$\begin{aligned} 5x + 2y &= 10 \\ 2x - 2y &= 4 \end{aligned}$$

Equal Values Method A method for solving a system of equations. To use the Equal Values Method, take two expressions that are each equal to the same variable and set those expressions equal to each other. For example, in the system of equations at right, $-2x + 5$ and $x - 1$ each equal y. So we write $-2x + 5 = x - 1$, and then solve that equation for x. Once we have x, we substitute that value back into either of the original equations to determine the value of y.

$$\begin{aligned} y &= -2x + 5 \\ y &= x - 1 \end{aligned}$$

equation A mathematical sentence in which two expressions appear on either side of an "equals" sign (=), stating that the two expressions are equivalent. For example, the equation $7x + 4.2 = -8$ states that the expression $7x + 4.2$ has the value -8. In this course, an equation is often used to represent a relationship between two quantities. For example, an equation for determining the area y of a tile pattern with figure number x might be written $y = 4x - 3$.

equivalent Two expressions are equivalent if they have the same value. For example, $2 + 3$ is equivalent to $1 + 4$. Two equations are equivalent if they have all the same solutions. For example, $y = 3x$ is equivalent to $2y = 6x$.

evaluate To evaluate an expression, substitute the value(s) given for the variable(s) and perform the operations according to the Order of Operations. For example, evaluating $2x + y - 10$ for $x = 4$ and $y = 3$ gives the value 1.

even function A function where $f(-x) = f(x)$ for all values of x defined for the function. The graph of the function will be symmetric about the y-axis. (p. 83)

event One or more results of an experiment or a probabilistic situation.

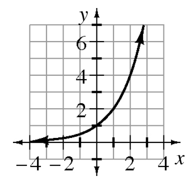
exact answer An answer that is precisely accurate and not approximate. For example, if the length of a side of a triangle is exactly $\sqrt{10}$, the exact answer to the question *"How long is that side of the triangle?"* would be $\sqrt{10}$, while 3.162 would be a decimal approximation of the answer. Also see *approximate answer*.

excluded value A value that is excluded from the domain of a function (or expression), possibly because the function is undefined for that value. (p. 573)

experiment A process that applies a treatment (change) to a group of subjects to determine if there is a measurable difference from another group. If an experiment properly implements randomization, controls (balances) the effects of lurking variables between the groups, and is replicated with many subjects, cause and effect can often be determined. (p. 174)

experimental probability The probability based on data collected in experiments. The experimental probability of an event is defined to be $\frac{\text{number of successful outcomes in the experiment}}{\text{total number of outcomes in the experiment}}$.

exponent In an expression of the form b^a, a is called the exponent. For example, in the expression 2^5, 5 is called the exponent. (2 is the base, and 32 is the value.) The exponent indicates how many times to use the base as a multiplier. For example, in 2^5, 2 is used 5 times: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$. For exponents of zero, the rule is that for any number $x \neq 0$, $x^0 = 1$. For negative exponents, the rule is that for any number $x \neq 0$, $x^{-n} = \frac{1}{x^n}$, and $\frac{1}{x^{-n}} = x^n$. Also see *laws of exponents*. (p. 31)



exponential function An exponential function in this course has an equation of the form $y = ab^x + k$, where a is the initial value, b is positive and is the multiplier, and $y = k$ is the equation of the horizontal asymptote. An example of an exponential function is graphed at right. (p. 60)

expression An expression is a combination of individual terms separated by plus or minus signs. Numerical expressions combine numbers and operation symbols; algebraic (variable) expressions include variables. For example, $4 + (5 - 3)$ is a numerical expression. In an algebraic expression, if each of the following terms, $6xy^2$, 24, and $\frac{y-3}{4+x}$, are combined, the result may be $6xy^2 + 24 - \frac{y-3}{4+x}$. An expression does not have an "equals" sign.

extraneous solution Sometimes in the process of solving equations, multiplying or squaring expressions involving a variable will lead to a numerical result that does not make the original equation true. This false result is called an extraneous solution. For example, in the process of solving the equation $\sqrt{x+3} = 9 - x$, both sides of the equation are squared to get $x + 3 = x^2 - 18x + 81$, which has solutions $x = 6$ and $x = 13$. $x = 6$ is a solution of the original equation, but 13 is extraneous, because $\sqrt{13+3} \neq 9 - 13$. (p. 114)

$\boldsymbol{f^{-1}(x)}$ Read this as "f inverse of x", the inverse function for $f(x)$. (p. 238)

factor (1) In arithmetic: When two or more integers are multiplied, each of the integers is a factor of the product. For example, 4 is a factor of 24 because $4 \cdot 6 = 24$. (2) In algebra: When two or more algebraic expressions are multiplied together, each of the expressions is a factor of the product. For example, x^2 is a factor of $-17x^2y^3$ because $(x^2)(-17y^3) = -17x^2y^3$. (3) To factor an expression is to write it as a product. For example, the factored form of $x^2 - 3x - 18$ is $(x - 6)(x + 3)$. (4) A factor is part of a product. A polynomial expression $p(x)$ is a factor of another polynomial expression $P(x)$ when there is a polynomial $q(x)$ such that $p(x)q(x) = P(x)$. In the equation $3x^2 - 9x + 6 = 3(x - 2)(x - 1)$, the expressions $(x - 2)$, $(x - 1)$ and 3 are factors. (p. 405)

Factor Theorem If a polynomial has a root $x = c$ (i.e., $P(c) = 0$), then $(x - c)$ is a factor of P. For example, given $P(x) = x^3 - 3x^2 + 4$, $P(-1) = 0$, therefore $(x + 1)$ is a factor of $P(x)$. (p. 408)

factorial A shorthand notation for the product of a list of consecutive positive integers from the given number to 1: $n! = n(n-1)(n-2)(n-3) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$. For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

family of functions A group of functions that have at least one common characteristic, usually the shape and the form of the equation. For example, the quadratic family of functions has graphs that are parabolas, and equations of the form $y = ax^2 + bx + c$. Examples of other families are linear functions, exponential functions, and absolute value functions. (p. 89)

first quartile (Q1) The median of the lower half of a set of data that has been written in numerical order. (p. 36)

five-number summary A way of summarizing the center and spread of a single-variable distribution of data. The five-number summary includes the minimum, first quartile, median, third quartile, and maximum of the set of data. Also see *boxplot*. (p. 193)

form (of an association) The form of an association can be linear or nonlinear. The form can contain a cluster of data. A residual plot can help determine if a particular form is appropriate for modeling the relationship.

fraction The quotient of two quantities in the form $\frac{a}{b}$ where b is not equal to 0.

fractional exponents Raising a number to a fractional exponent indicates a power as well as a root. $x^{a/b} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$

frequency table A two-way table that contains entries that are counts (as opposed to fractions or percents). Also see *two-way table*. (p. 182)

function A relationship in which for each input value there is one and only one output value. For example, the relationship $f(x) = x + 4$ is a function; for each input value (x) there is exactly one output value. In terms of ordered pairs (x, y), no two ordered pairs of a function have the same first member (x). Also see *function notation*. (p. 6)

function notation When an equation expressing a function is written using function notation, the function is given a name, most commonly "f", "g", or "h". The notation $f(x)$ represents the output of a function, named f, when x is the input. It is pronounced "f of x". For example, $g(2)$, pronounced "g of 2", represents the output of the function g when $x = 2$. If $g(x) = x^2 + 3$, then $g(2) = 7$. (p. 6)

Fundamental Counting Principle A method for counting the number of outcomes (the size of the sample space) of a probabilistic situation, often in which the order of the items matters. If event {A} has m outcomes, and event {B} has n outcomes after event {A} has occurred, then event {A} followed by event {B} has $m \cdot n$ outcomes. For a sequence of events, a tree diagram could be used to count the number of outcomes, but if the number of outcomes is large, a decision chart is more useful. Also see *decision chart*. (p. 535)

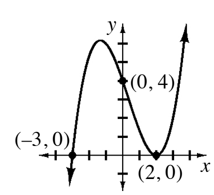
Fundamental Theorem of Algebra The Fundamental Theorem of Algebra states that every one-variable polynomial has at least one complex root (remember that every real number is also a complex number). This theorem can be used to show that every polynomial of degree n has n complex roots. (p. 400)

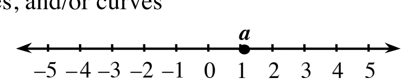
geometric sequence A geometric sequence is a sequence that is generated by a multiplier. This means that each term of a geometric sequence can be found by multiplying the previous term by a constant. For example: 5, 15, 45… is the beginning of a geometric sequence with generator (or *common ratio*) 3. In general, a geometric sequence can be represented $a, ar, ar^2, \ldots ar^{n-1}$. (p. 538)

geometric series The sum of a geometric sequence is a geometric series, for example: $5 + 15 + 45 + \ldots$ The sum of the first n terms of a geometric sequence, $a + ar + ar^2 + \ldots + ar^{n-1}$ is given by the formula at right. (p. 538)

$$S = \frac{a(r^n - 1)}{r - 1}$$

graph A graph represents numerical information spatially. The numbers may come from a table, situation (pattern), or equation/inequality. The graph of an equation is the set of points representing the coordinates that make the equation true. The direction to "graph an equation" or "draw a graph" means use graph paper, scale your axes appropriately, label key points, and plot points accurately. This is different from *sketching* a graph. The equation $y = \frac{1}{3}(x-2)^2(x+3)$ is graphed at right. Most of the graphs in this course show points, lines, and/or curves on a two-dimensional coordinate system or on a single axis called a number line. Also see *complete graph*. (p. 10)



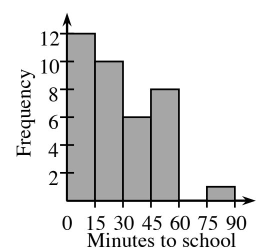


graphing form A form of the equation of a function or relationship that clearly shows key information about the graph. For example, the graphing form for the general equation of a quadratic function is $y = a(x - h)^2 + k$. The vertex (h, k), the orientation (whether a is positive or negative), and the amount of stretch or compression based on $|a| > 1$ or $|a| < 1$ appear in the equation. Also see *locator point*. (p. 89)

greatest common factor (GCF) (1) For integers, the greatest positive integer that is a common factor of two or more integers. For example, the greatest common factor of 28 and 42 is 14. (2) For two or more algebraic monomials, the product of the greatest common integer factor of the coefficients of the monomials and the variable(s) in each algebraic term with the smallest degree of that variable in every term. For example, the greatest common factor of $12x^3y^2$ and $8xy^4$ is $4xy^2$. (3) For a polynomial, the greatest common monomial factor of its terms. For example, the greatest common factor of $16x^4 + 8x^3 + 12x$ is $4x$.

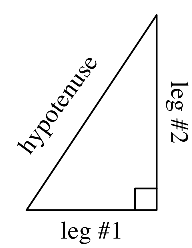
(h, k) In this course h and k are used as parameters in general equations for families of functions $f(x) = af(x - h) + k$, and families of relationships to represent the horizontal and vertical shifts of the parent graph. The point (h, k) is the locator point. (p. 89)

height (1) Triangle: the length of a segment that connects a vertex of the triangle to a line containing the opposite base (side) and is perpendicular to that line; (2) Trapezoid: the length of any segment that connects a point on one base of the trapezoid to the line containing the opposite base and is perpendicular to that line; (3) Parallelogram (includes rectangle, rhombus, and square): the length of any segment that connects a point on one base of the parallelogram to the line containing the opposite base and is perpendicular to that line; (4) Pyramid and cone: the length of the segment that connects the apex to a point in the plane containing the figure's base and is perpendicular to that plane; (5) Prism or cylinder: the length of a segment that connects one base of the figure to the plane containing the other base and is perpendicular to that plane. Some texts make a distinction between height and altitude, where the altitude is the segment described in the definition above and the height is its length.



histogram A way of displaying single-variable data that is like a bar graph in that the height of the bars is proportional to the number of elements. However, a histogram is for numerical data. Each bin (bar) of a histogram represents the number of data elements in a range of values, such as the number of people who take from 15 minutes up to, but not including, 30 minutes to get to school. Each bin should be the same width and the bins should touch each other. (p. 43)

hypothesis test See *statistical hypothesis test*.



hypotenuse The side of a right triangle opposite the right angle. Note that legs of a right triangle are always shorter than its hypotenuse. Also see *legs*. (p. 55)

identity (trigonometric) Equations that are true for all values for which the functions are defined. For example, $\sin^2(x) + \cos^2(x) = 1$, which is true for all values of x, or $\cot(x) = \frac{1}{\tan(x)}$, which is true whenever the tangent and cotangent are defined. (p. 639)

imaginary numbers The set of numbers created by multiplying the imaginary number i by every possible real number. The imaginary number i is defined as the square root of -1, or $i = \sqrt{-1}$. Therefore $i^2 = -1$. They are not positive, negative, or zero. These numbers are solutions of equations of the form x^2 = (a negative number). In general, imaginary numbers follow the rules of real number arithmetic (e.g. $i + i = 2i$). (p. 72)

increasing function A function is increasing if the y-value increases as the x-value increases.

independent events If the outcome of a probabilistic event does not affect the probability of another event, the events are independent. For example, assume you plan to roll a normal six-sided die twice and want to know the probability of rolling a 1 twice. The result of the first roll does not affect the probability of rolling a 1 on the second roll; the events are independent. If two events {A} and {B} are independent, then P(A given B) = P(A). The converse is also true. Alternatively, by rewriting the Multiplication Rule, if two events are independent, then P(A and B) = $P(\text{A}) \cdot P(\text{B})$. Again, the converse is also true.

independent variable When one quantity's value changes in a way that does not depend on the value of another quantity, the value that changes independently is represented with the independent variable. For example, we might relate the speed of a car to the amount of force you apply to the gas pedal. Here, the amount of force applied may be whatever the driver chooses, so it represents the independent variable. The independent variable appears as the input value in an $x \rightarrow y$ table, and is usually placed relative to the horizontal axis of a graph. We often use the letter x and the horizontal x-axis for the independent variable. When working with functions or relationships, the independent variable represents the input value. In Statistics, the independent variable is often called the explanatory variable. Also see *dependent variable*. (p. 14)

index (plural indices) In summation notation, the indices are the numbers below and above the sigma that indicate which term to start with and which to end with. For a series, they show the first and last replacement values for n. When the symbol above sigma is ∞ the series continues without ending. Example: $\sum_{n=1}^{8} 5n-7 = -2+3+8+\ldots+33$ (p. 508)

inequality An inequality consists of two expressions on either side of an inequality symbol. For example, the inequality $7x + 4.2 < -8$ states that the expression $7x + 4.2$ has a value less than -8. (p. 137)

inference A statistical prediction.

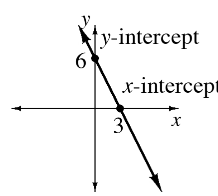
infinite geometric series An infinite geometric series is a geometric series which never ends. The sum of such a series with an initial value a and common ratio r, with $-1 < r < 1$, is given by the formula at right. (p. 538)

$$S = \frac{a}{1-r}$$

infinity The concept that something is without limit or unending. The symbol for infinity is ∞. Note that infinity is an idea, not a number, even though it is often treated as a number. (p. 10)

integer Any whole number or the opposite of a whole number: $\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$

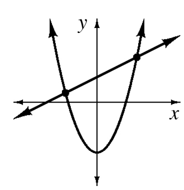
intercepts Points where a graph crosses the axes. x-intercepts are points at which the graph crosses the x-axis and y-intercepts are points at which the graph crosses the y-axis. On the graph at right, the x-intercept is $(3, 0)$ and the y-intercept is $(0, 6)$.



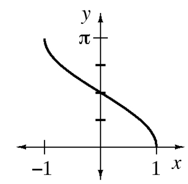
interest An amount paid which is a percentage of the principal. For example, a savings account may offer 4% annual interest, which means it will pay \$4.00 in interest for a principal of \$100 kept in the account for one year. (p. 565)

interquartile range (IQR) A way to measure the spread of data. It is calculated by subtracting the first quartile from the third quartile. (p. 36)

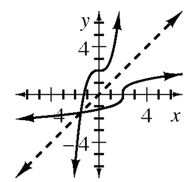
intersection A point of intersection is a point that the graphs of two or more equations have in common. Graphs may intersect in one, two, several, many, or no points. The coordinates of each point of intersection satisfy the equations of both (or all) graphs. (p. 29)



inverse cosine ($\cos^{-1}(x)$) Read as the inverse of cosine x, $\cos^{-1}(x)$ is the measure of the angle with cosine x. The inverse of the cosine function is the function that "undoes" cosine. When the ratio between the lengths of two sides of a right triangle is known, $\sin^{-1}(x)$, $\cos^{-1}(x)$, or $\tan^{-1}(x)$, can be used to determine the measure of one of the triangle's acute angles. We can also write $y = \arccos(x)$. Note that the notation refers to the inverse of the cosine function, *not* $\frac{1}{\cos(x)}$. Because $y = \cos^{-1}(x)$ is equivalent to $x = \cos(y)$ and there are infinitely many angles y such that $\cos(y) = x$, the inverse *function* is restricted to select the *principal* value of y such that $0 \le y \le \pi$. Also see *inverse sine* and *inverse tangent*. (p. 624)

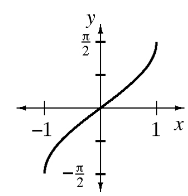


inverse function A function that "undoes" what the original function does. The inverse of a function performs in reverse order the inverse operation for each operation of the function. The graph of an inverse function is a reflection of the original function across the line $y = x$. For example, $y = x^3 + 2$ is equivalent to $x = \sqrt[3]{y-2}$; its inverse function is written $y = \sqrt[3]{x-2}$. Sometimes the domain of a function must be restricted to ensure that its inverse is also a function. (p. 226)

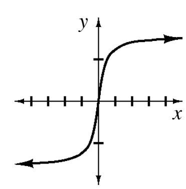


inverse operations Subtraction is the inverse operation for addition and vice versa, division for multiplication, square root for squaring, and more generally taking the n^{th} root for raising to the n^{th} power.

inverse sine ($\sin^{-1}(x)$) Read as the inverse of sine x, $\sin^{-1}(x)$ is the measure of the angle with sine x. The inverse of the sine function is the function that "undoes" sine. When the ratio between the lengths of two sides of a right triangle is known, $\sin^{-1}(x)$, $\cos^{-1}(x)$, or $\tan^{-1}(x)$, can be used to determine the measure of one of the triangle's acute angles. We can also write $y = \arcsin(x)$. Note that the notation refers to the inverse of the sine function, *not* $\frac{1}{\sin(x)}$. Because $y = \sin^{-1}(x)$ is equivalent to $x = \sin(y)$ and there are infinitely many angles y such that $\sin(y) = x$, the inverse *function* is restricted to select the *principal* value of y such that $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. The graph of the inverse function, $y = \sin^{-1}(x)$ with $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, is at right. Also see *inverse cosine* and *inverse tangent*. (p. 624)

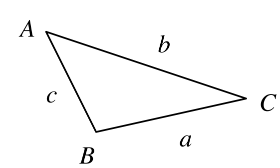


inverse tangent ($\tan^{-1}(x)$) Read as the inverse of tangent x, $\tan^{-1}(x)$ is the measure of the angle that has tangent x. The inverse of the tangent function is the function that "undoes" tangent. When the ratio between the lengths of two sides of a right triangle is known, $\sin^{-1}(x)$, $\cos^{-1}(x)$, or $\tan^{-1}(x)$ can be used to determine the measure of one of the triangle's acute angles. We can also write $y = \arctan(x)$. Note that the notation refers to the inverse of the tangent function, *not* $\frac{1}{\tan(x)}$. Because $y = \tan^{-1}(x)$ is equivalent to $x = \tan(y)$ and there are infinitely many angles y such that $\tan(y) = x$, the inverse *function* is restricted to select the *principal* value of y such that $-\frac{\pi}{2} < y < \frac{\pi}{2}$. The graph of the inverse tangent function is at right. Also see *inverse cosine* and *inverse sine*. (p. 624)



irrational numbers The set of numbers that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \ne 0$. For example, π and $\sqrt{2}$ are irrational numbers.

isosceles triangle A triangle with at least two sides of equal length.



Law of Cosines For any ΔABC with sides a, b, and c opposite $\angle A$, $\angle B$, and $\angle C$ respectively, it is always true that $c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$, $b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$, and $a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$. (p. 349)

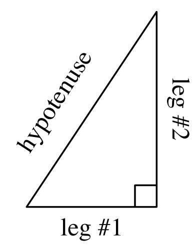
Law of Large Numbers A theorem stating that as the number of trials of a random process increases (for example, as the number of trials in a simulation increases), the results will approach the theoretical value more closely. (p. 269)

Law of Sines For any ΔABC with sides a, b, and c opposite $\angle A$, $\angle B$, and $\angle C$ respectively, it is always true that $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$. (p. 345)

laws of exponents (p. 31)

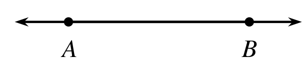
Law	Examples	
$x^m x^n = x^{m+n}$ for all x	$x^3 x^4 = x^{3+4} = x^7$	$2^5 \cdot 2^{-1} = 2^4$
$\frac{x^m}{x^n} = x^{m-n}$ for $x \neq 0$	$x^{10} \div x^4 = x^{10-4} = x^6$	$\frac{5^4}{5^7} = 5^{-3}$
$(x^m)^n = x^{mn}$ for all x	$(x^4)^3 = x^{4\cdot 3} = x^{12}$	$(10^5)^6 = 10^{30}$
$x^0 = 1$ for $x \neq 0$	$\frac{y^2}{y^2} = y^0 = 1$	$9^0 = 1$
$x^{-1} = \frac{1}{x}$ for $x \neq 0$	$\frac{1}{x^2} = \left(\frac{1}{x}\right)^2 = (x^{-1})^2 = x^{-2}$	$3^{-1} = \frac{1}{3}$
$x^{m/n} = \sqrt[n]{x^m}$ for $x \geq 0$	$\sqrt{k} = k^{1/2}$	$y^{2/3} = \sqrt[3]{y^2}$

least squares regression line (LSRL) A unique best-fit line that is found by making the squares of the residuals as small as possible.

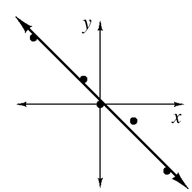


legs The two sides of a right triangle that form the right angle. Note that the legs of a right triangle are always shorter than its hypotenuse. Also see *hypotenuse*.

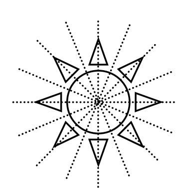
line Graphed, a line is made up of an infinite number of points, is one-dimensional, and extends without end in two directions. It is made up of points and has no thickness. A line can be named with a letter (such as l), but also can be labeled using two points on the line, such as $\overleftrightarrow{AB}$ above. A line is a mathematically undefined term in geometry. Also see *line segment*. In two dimensions, a line is the graph of an equation in standard form $ax + by = c$. Also see $y = mx + b$.

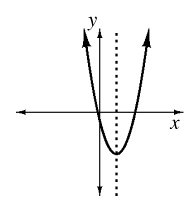


line of best fit A line that represents, in general, data on a scatterplot. The line of best fit is a model of numerical two-variable data that helps describe the data. It is also used to make predictions for other data. Also see *least squares regression line*.

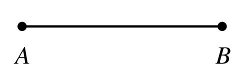


line of symmetry A line that divides a shape into two congruent parts that are reflections of each other across the line. If you fold a shape over its line of symmetry, the shapes on both sides of the line will match each other perfectly. Some shapes have more than one line of symmetry, such as the example at right. Other shapes may only have one line of symmetry or no lines of symmetry.





line segment The portion of a line between two points. A line segment is named using its endpoints. For example, the line segment at right may be named either $\overline{AB}$ or $\overline{BA}$.



linear equation An equation with at least one variable of degree one and no variables of degree greater than one. The graph of a linear equation of two variables is a line in the plane. $ax + by = c$ is the standard form of a linear equation. Also see $y = mx + b$. (p. 78)

linear factor A factor of the form $(ax + b)$.

linear function A polynomial function of degree one or zero, with general equation $f(x) = a(x - h) + k$. The graph of a linear function is a line. Also see *linear equation*.

linear programming A method for solving a problem with several conditions or constraints that can be represented as linear equations or inequalities. (p. 139)

locator point A locator point is a point which gives the position of a graph with respect to the axes. For a parabola, the vertex is a locator point. (p. 89)

log base 10 $\log_{10}(m) = n$ if $10^n = m$. See *common logarithm*. (p. 253)

logarithm The logarithm of a given number, x, with respect to a base, b, is the exponent to which b must be raised to give x. For example, since we know that $32 = 2^5$, it follows that 5 is the logarithm, base 2, of 32, or $\log_2(32) = 5$. (p. 255)

logarithmic functions Inverse exponential functions. The base of the logarithm is the same base as that of the exponential function. For instance, $y = \log_2(x)$ can be read as "y is the exponent needed for base 2 to get x", and is equivalent to $x = 2^y$. The short version is stated "log, base 2, of x", and written $\log_2(x)$. (p. 249)

Looking Inside "Looking Inside" is a method of solving one-variable equations containing parentheses or an absolute value symbol. To use "looking inside", we first determine what the value of the entire expression inside the parentheses (or absolute value symbol) must be. We then use that fact to solve for the value of the variable. For example, to use "looking inside" to solve the equation $4(x + 2) = 36$, we first determine that $x + 2$ must equal 9. We then solve the equation $x + 2 = 9$ to get $x = 7$. (p. 108)

lower boundary line A line drawn parallel to and below the least squares regression line at a distance equivalent to the largest residual. The lower boundary line determines the lower bound on the values that a prediction is likely to be. (p. 277)

lower control limit (LCL) See *control limit*. (p. 297)

lurking variable A hidden variable that was not part of the statistical study under investigation. Sometimes a lurking variable explains the true cause of an association between two other variables that are linked. (p. 174)

m When the equation of a line is expressed in $y = mx + b$ form, the parameter m gives the slope of the line. For example, the slope of the line $y = -\frac{1}{3}x + 7$ is $-\frac{1}{3}$.

margin of error Half of the spread between the upper and lower bounds of a statistical prediction. For example, if the predicted weight is likely to be between a lower bound of 12 kg and an upper bound of 18 kg, the margin of error is 3 kg, and the prediction of the weight can be written as 15 ± 3 kg. (p. 298)

maximum value The largest value in the range of a function. For example, the y-coordinate of the vertex of a parabola that opens downward.

mean The arithmetic mean, or average, of several numbers is one way of defining the "center", "middle", or "typical value" of a set of numbers. The mean represents the center of a set of data well if the distribution of the data is symmetrical and has no outliers. To calculate the mean of a numerical data set, add the values together and then divide by the number of values in the set. For example, the mean of the numbers 1, 5, and 6 is $(1 + 5 + 6) \div 3 = 4$.

measure of central tendency Mean and median are measures of central tendency, representing the "center" or "middle" or "typical value" of a set of data. See *center (of a data distribution)*.

median (of a data set) The middle number of a set of data which has been written in numerical order. If there is no distinct middle, then the mean of the two middle numbers is the median. The median is generally more representative of the “middle” or of a “typical value” than the mean if there are outliers or if the distribution of the data in not symmetric.

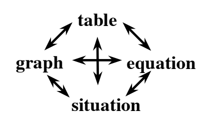
midline The horizontal axis of the graph of a trigonometric function. The midline is halfway between the maximum and minimum values of the function. (p. 477)

minimum value The smallest value in the range of a function. For example, the y-coordinate of the vertex of a parabola that opens upward.

model A model is a mathematical way of describing a real situation. It can be as simple as adding numbers as a way to calculate how much someone has spent, but most models involve approximations and estimations. The weather predictions that you see on the news are based on complex models that rely on calculations done by very large computer programs. Also see *regression*.

monomial An expression with only one term. It can be a number, a variable, or the product of a number and one or more variables. For example, 7, $3x$, $-4ab$, and $3x^2y$ are each monomials.

multiple representations web An organizational tool we use to keep track of connections between the four representations of relationships between quantities emphasized in this course. In this course, we emphasize four different ways of representing a numerical relationship: with a graph, table, situation (pattern), or equation or inequality.



multiplier In a geometric sequence, the number multiplied to each term to get the next term is called the multiplier, the *common* ratio, or *generator*. The multiplier is also the number you can multiply by in order to increase or decrease an amount by a given percentage in one step. For example, to increase a number by 4%, the multiplier is 1.04. We would multiply the number by 1.04. The multiplier for decreasing by 4% is 0.96. (p. 60)

natural logarithm A logarithm base e. Abbreviated “ln”. (p. 550)

natural numbers The natural numbers are the positive integers, $\{1, 2, 3, \ldots\}$. They are also called the counting numbers.

negative exponent The value of a number raised to a negative exponent is the reciprocal of the number raised to the absolute value of the exponent. That is, $x^{-a} = \frac{1}{x^a}$ for $x \neq 0$.

negative number A negative number is a number less than zero.

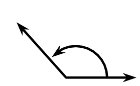
normal distribution A distribution of data that can be modeled with the normal probability density function. See *normal probability density function*. (p. 187)

normal probability density function A very specific function that is used to model single-peaked, symmetric, bell-shaped data. (p. 195)

observational study A study where data is collected by observing an existing situation without the researcher (or observer) imposing any type of change or treatment. (p. 174)

observed value An actual measurement, as opposed to a prediction made from a model.

obtuse angle Any angle that measures between (but not including) 90º and 180º. Also see *acute angle*.

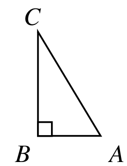


odd function A function where $f(-x) = -f(x)$ for all values of x defined for the function. The graph of the function will have 180º rotational symmetry about the origin. (p. 83)

open question In a survey, a question that allows respondents to offer any answer they like. (p. 160)

one-dimensional Not having any width or depth. Lines and curves are one-dimensional.

opposite (1) In algebra: Two numbers are opposites if they are the same distance from zero, but one is positive and one is negative. For example, 5 and –5 are opposites. The opposite of a number is sometimes called its additive inverse, indicating that the sum of a number and its opposite is zero. (2) In geometry: In a figure, opposite means "across from". For example, in a triangle, an opposite side is the side that is across from a particular angle and is not a side that makes the angle. For example, the side $\overline{AB}$ in ΔABC at right is opposite $\angle C$, while $\overline{AC}$ is opposite the right angle. (p. 55)



Order of Operations The specific order in which certain operations are to be carried out to evaluate or simplify expressions. The order is: parentheses (or other grouping symbols), exponents (powers or roots), multiplication and division (from left to right), and addition and subtraction (from left to right).

ordered pair A pair of numbers written (x, y) used to represent the coordinates of a point in an xy-plane where x represents the horizontal distance from 0 and y is the vertical distance from 0. The input and output values of a function or relationship can be represented as ordered pairs where x is the input and y is the output. (p. 590)

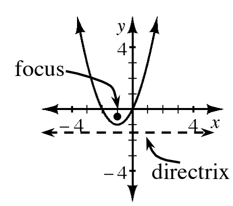
ordered triple Three real numbers written in order (x, y, z) represent a point in space or replacement values for a situation involving three variables. See *3-D coordinate system*. (p. 590)

origin The point on a coordinate plane where the x- and y-axes intersect is called the origin. This point has coordinates (0, 0). The point assigned to zero on a number line is also called the origin.

outcome The result of a probabilistic situation. The set of all possible outcomes of a probabilistic situation is called the sample space. Also see *event*.

outlier A number in a set of data that is far away from the bulk of the data. (p. 43)

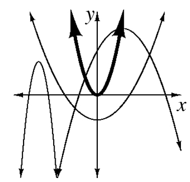
parabola A parabola is a conic section created by slicing a cone with a plane parallel to a lateral edge of the cone. Parabolas are most commonly seen in algebra courses as the graphs of quadratic functions. Two commonly used forms of quadratic functions are standard form $y = ax^2 + bx + c$ and graphing form $y = a(x - h)^2 + k$, with the vertex of the parabola at (h, k).



A parabola is also described as the set of all points that are equidistant from a single point (the focus) and a line (the directrix). A general equation for parabolas that are not functions, "sleeping" parabolas, is $x - h = a(y - k)^2$. (p. 69)

parameter In a general equations where x and y represent the inputs and outputs of the function, variables such as a, b, c, m, h, and k are often referred to as parameters, and they are often replaced with specific values. For example, in the equation $y = a(x - h)^2 + k$, which represents all parabolas that are functions, the a, h, and k are (variable) parameters that give the shape and location, while x and y are the independent and dependent variables. (p. 19)

parent function The simplest version of a family of functions. For instance, $y = x^2$ is considered the parent function for all quadratics that are functions. (p. 19)



parent graph The simplest version of a family of graphs. For instance, the graph of $y = x^2$ is considered the parent graph for parabolas that are functions. (p. 63)

Pascal's Triangle The array of numbers at right. The triangular pattern continues downward. This array shows all the values of $_nC_r$ where n is the row number when the vertex is $_0C_0$, and r is the number of places to the right in row n (when the counting begins with 0). For instance, $_5C_2$ is equal to 10. (p. 541)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

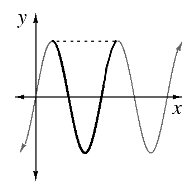
percentile A percentile ranking indicates the percentage of scores that are below the score in question. For example, if you scored at the 90th percentile on a test, your score was higher than the scores of 90% of the other test takers. (p. 192)

perfect square For numbers, a whole number that can be written as the second power of another whole number. For example, 1, 4, 9, 16, and 25 are perfect squares. See also *perfect square form* and *perfect square trinomials*.

perfect square form A quadratic equation in the form $(ax + b)^2 = c$, where a is nonzero, is said to be in perfect square form. For example, $(2x + 3)^2 = 20$ is a quadratic equation in perfect square form.

perfect square trinomials Trinomials of the form $a^2x^2 + 2abx + b^2$, where a and b are nonzero real numbers, are known as perfect square trinomials and factor as $(ax + b)^2$. For example, the perfect square trinomial $9x^2 - 24x + 16$ can be factored as $(3x - 4)^2$.

period The horizontal length of one complete cycle of a graph, as shown by the dashed line in the graph at right. (p. 477)

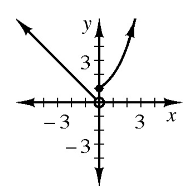


periodic function A function which has a repetitive section or cycle such as the sine, cosine, and tangent functions. In a periodic function, the pattern continues forever both to the left and to the right. (p. 477)

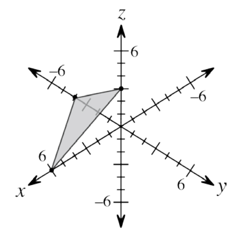
permutation A permutation is an arrangement in which the order of selection matters. For example, a batting line-up is a permutation because it is an ordered list of players. If each of five letters, A, B, C, D, E is printed on a card, the number of three letter sequences you can make by selecting three of the five cards is a permutation. Permutations can be represented with tree diagrams, decision charts, and their value calculated by using the formula for $_nP_r = \frac{n!}{(n-r)!}$. In the example given above, $_5P_3 = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 5 \cdot 4 \cdot 3 = 60$. Also see *combination*. (p. 493)

perpendicular Two rays, line segments, or lines that meet (intersect) to form a right angle (90°) are called perpendicular. The slopes of lines that are perpendicular are opposite reciprocals. A line and a flat surface can also be perpendicular if the line does not lie on the flat surface, but intersects it and forms a right angle with every line on the flat surface passing through the point of intersection. A small square at the point of intersection of two lines or segments indicates that the lines are perpendicular.

piecewise-defined function A function composed of parts of two or more functions, also called a piecewise function. Each part is a function with a restricted (limited) domain. (p. 87)



placebo A treatment with no effect. A placebo is sometimes used in experiments involving people so that the psychological effects of participating in an experiment are the same in all the treatment groups. (p. 175)



plane A plane is an undefined term in geometry. It is a two-dimensional flat surface that extends without end. It is made up of points and has no thickness. The part of a plane outlined by its *xy*-, *xz*- and *yz*-traces is often used to represent a plane on a 3-dimensional coordinate system. (p. 598)

point-slope form The point-slope form of the equation of a line is $y - k = m(x - h)$ where (h, k) are the coordinates of a point on the line. It is called the point-slope form of a linear equation or function because it shows the slope m and a point (h, k) that is on the graph of the line. For example, given a line that has a slope of $\frac{5}{3}$ and contains the point $(3, -4)$, its equation can be written $y + 4 = \frac{5}{3}(x - 3)$. To write the equation of the line in $y = mx + b$ form, we solve the point-slope form equation for y. Also see $y = mx + b$ or *slope-intercept form*. (p. 78)

polynomial An algebraic expression that involves at most the operations of addition, subtraction, and multiplication. A polynomial in one variable is an expression that can be written as the sum of terms of the form: (any number) $\cdot\, x^{(\text{whole number})}$. These polynomials are usually arranged with the powers of x in order, starting with the highest, left to right. The numbers that multiply the powers of x are called the coefficients of the polynomial. For example, $x^8 - 4x^6 + 6x^2$ is a polynomial. Also see *degree of a polynomial*. (p. 26)

population A collection of objects or group of people about whom information is gathered. (p. 161)

positive number A positive number is a number greater than zero. Zero is neither positive nor negative.

power A number or variable raised to an exponent in the form x^n. See *exponent*.

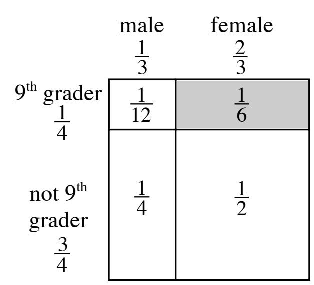
predicted value (of an association) The dependent (y-value) that is predicted for an independent (x-value) by the best-fit model for an association.

principal Initial investment or capital. An initial value.

probabilistic situation A situation in which the outcomes are determined by random processes.

probability The probability that an event A, with a finite number of equally likely outcomes, will occur is the number of outcomes for event A divided by the total number of equally likely outcomes. This can be written as $\frac{\text{number of outcomes for event A}}{\text{total number of possible outcomes}}$. A probability p is a ratio, $0 \le p \le 1$.

probability area model A model that uses the area of rectangles to represent the probabilities of possible outcomes when considering two *independent* events. For example, suppose you are going to randomly select a student from a classroom. If the probability of selecting a female student from the room is $\frac{2}{3}$ and the probability of selecting a 9th grader from the room is $\frac{1}{4}$, the area of the shaded rectangle at right represents the probability that a randomly selected student is a female 9th grader ($\frac{2}{3} \cdot \frac{1}{4} = \frac{1}{6}$). A "generic" area model enables you to calculate the probabilities of outcomes without drawing a diagram to scale. Also see *tree diagram*. (p. 269)

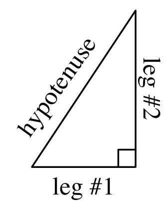


Area model

process control See *statistical process control*.

profit The amount of money after expenses have been accounted for.

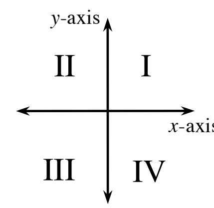
Pythagorean Theorem The statement relating the lengths of the legs of a right triangle to the length of the hypotenuse: $(\text{leg } \#1)^2 + (\text{leg } \#2)^2 = \text{hypotenuse}^2$. The Pythagorean Theorem is powerful because if you know the lengths of any two sides of a right triangle, you can use this relationship to calculate the length of the third side.



Pythagorean Triple Any three positive integers a, b, and c that make the relationship $a^2 + b^2 = c^2$ true. Commonly used Pythagorean Triples include 3, 4, 5 and 5, 12, 13.

quadrants The coordinate plane is divided by its axes into four quadrants. The quadrants are numbered as shown in the first diagram at right. When graphing data that has no negative values, we sometimes use a graph showing only the first quadrant.

four-quadrant graph:



quadratic polynomial A polynomial (in one variable) that can be written in the form $ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. The quadratic term is ax^2, the linear term is bx, and the constant term is c.

quarterly Occurring four times a year.

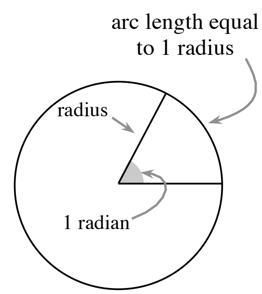
quartile Along with the median, the quartiles divide a set of data into four groups of the same size. Also see *boxplot*. (p. 36)

quotient The result of a division problem is a quotient with a remainder (which could be 0). When a polynomial $P(x)$ is divided by a polynomial $D(x)$, a polynomial $Q(x)$ will be the quotient with a remainder $R(x)$. The product of $Q(x)$ and $D(x)$ plus the remainder $R(x)$ will equal the original polynomial. $P(x) = D(x) \cdot Q(x) + R(x)$. (p. 414)

r See *correlation coefficient*.

$\boldsymbol{R^2}$ See *R*-squared.

radian The ratio of the arc length to the radius for any sector of a circle. A central angle with measure 1 radian intercepts an arc with length equal to one radius of the circle. The ratio will be constant for a given central angle, regardless of the length of the radius of the circle. Along with degrees, radians are a very common unit of measurement for the central angle of a circle. 1 radian = $\frac{180°}{\pi}$. (p. 460)



radical An expression in the form $\sqrt{a}$, where $\sqrt{a}$ is the positive square root of a. For example, $\sqrt{49} = 7$. Also see *square root*.

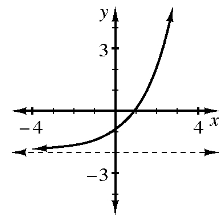
radical form Fractional exponents with a base of x in the form of $x^{m/n}$ can also be written in radical form as $(\sqrt[n]{x})^m$ or $\sqrt[n]{x^m}$. Also called exact form. Also see *approximate answer* and *laws of exponents*.

radicand The expression under a radical sign. For example, in the expression $3 + 2\sqrt{x-7}$, the radicand is $x - 7$.

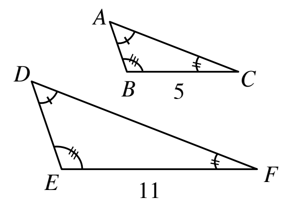
random An outcome is random if it happens by chance; that is, its result cannot be predicted from previous outcomes. A sequence of several random outcomes in a row will have no predictable order or pattern. For example, the outcome of rolling a die is random because the outcome cannot be predicted, and many rolls in a row will have no predictable pattern. (p. 267)

random sample A sample which was chosen as a result of a random process. A random sample can represent the whole population well. (p. 164)

range (1) The set of all output values for a function or relationship. For example, the range of the function graphed at right is $y > -2$. It consists of all the values of the dependent variable, that is, every number that y can represent for the function $f(x) = y$. Also see *domain*. (2) The range of a set of data is a statistic that represents the spread of the data. The range is the difference between the highest and lowest values. (pp. 6, 36)



ratio A ratio compares two quantities by division. A ratio can be written using a colon, but is more often written as a fraction. For example, in the two similar triangles shown at right, a ratio can be used to compare the length of $\overline{BC}$ in ΔABC with the length of $\overline{EF}$ in ΔDEF. This ratio can be written as 5:11 or as the fraction $\frac{5}{11}$.



rational expression An expression in the form of a fraction in which the numerator and denominator are polynomials. For example, $\frac{x-7}{x^2+8x-9}$ is a rational expression. (p. 563)

rational function A function that is the quotient of two polynomials. For example, $f(x) = \frac{5}{x-3}$. (p. 16)

rational number A number that can be written as a fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$. (p. 570)

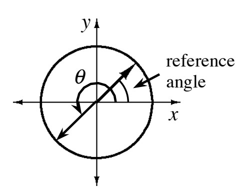
Rational Zeros Theorem If the polynomial $P(x) = (a_n)x^n + (a_{n-1})x^{(n-1)} + \ldots + (a_1)x^1 + a_0$ has integer coefficients, then every rational zero of P is of the form $\frac{p}{q}$ where p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n. The possible rational zeros of $P(x) = 2x^3 + x^2 - 7x + 6$ are of the form $\frac{\text{factor of }6}{\text{factor of }2}$. The factors of 6 are ±1, ±2, ±3, and ±6. The factors of 2 are ±1 and ±2. Thus, the possible rational zeros are $\pm\frac{1}{2}$, $\pm\frac{2}{1}$, $\pm\frac{3}{1}$, $\pm\frac{3}{2}$, and $\pm\frac{6}{1}$. (p. 408)

real numbers Irrational numbers together with rational numbers form the set of the real numbers. For example, the following are all real numbers: 2.78, −13267, 0, $\frac{3}{7}$, π, $\sqrt{2}$. All real numbers are represented on the number line. (p. 72)

reciprocal The reciprocal of a nonzero number is its multiplicative inverse; that is, the reciprocal of x is $\frac{1}{x}$. For a number in the form $\frac{a}{b}$, where a and b are nonzero, the reciprocal is $\frac{b}{a}$. The product of a number and its reciprocal is 1. For example, the reciprocal of 12 is $\frac{1}{12}$, and $12 \cdot \frac{1}{12} = 1$.

reciprocal trigonometric functions The functions $y = \frac{1}{\sin(x)} = \csc(x)$, $y = \frac{1}{\cos(x)} = \sec(x)$, and $y = \frac{1}{\tan(x)} = \cot(x)$. (p. 622)

reference angle For every angle of rotation in standard position, the reference angle is the angle in the first quadrant $0 \leq \theta \leq \frac{\pi}{2}$ whose cosine and sine have the same absolute values as the cosine and sine of the original angle. (p. 455)



regression A method for determining a best-fit line or curve through a set of points on a scatterplot. The most common type of regression is the least squares regression line. Other regressions include: exponential regressions (fitting $y = a \cdot b^x$), quadratic regressions (fitting $y = ax^2 + bx + c$), and power regressions (fitting $y = ax^b$).

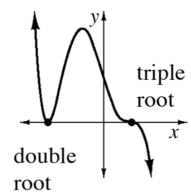
relative frequency A ratio or percent. If 60 people are asked, and 15 people prefer "red", the relative frequency of people preferring red is $\frac{15}{60} = 25\%$. (p. 182)

relative frequency table A two-way table in which the percent of subjects in each combination of categories is displayed.

remainder When dividing polynomials in one variable, the remainder is what is left after the constant term of the quotient has been determined. The degree of the remainder must be less than the degree of the divisor. In the example below, the remainder is $3x - 3$.
$(x^4 + 3x^3 - x^2 + x - 7) \div (x^2 + x + 1) = x^2 + 2x - 4 + \frac{3x-3}{x^2+x+1}$. Also see *quotient*.

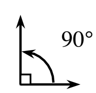
Remainder Theorem For any number c, when a polynomial P is divided by $(x - c)$, the remainder is the value of $P(c)$. For example, if the polynomial $P(x) = x^4 - x^3 + 20x - 48$ is divided by $(x - 5)$, the remainder is $P(5) = 552$. (p. 408)

repeated root A root of a polynomial that occurs more than once. The root r will occur as many times as $(x - r)$ is a factor of the polynomial. See *double root* and *triple root*. (p. 400)



Rewriting To rewrite an equation or expression is to write an equivalent equation or expression. Rewriting could involve using the Distributive Property, following the Order of Operations, using properties of 0 or 1, substitution, inverse operations, Properties of Logarithms, or use of trigonometric identities. We usually rewrite in order to change expressions or equations into more useful forms, or sometimes just simpler forms. In this course, "Rewriting" also refers to a method of solving one-variable equations. In "Rewriting", we use algebraic techniques to write an equation equivalent to the original. We then solve the equation using various solution methods, including perhaps rewriting again. For example, to solve the equation $4(x + 2) = 36$ by "Rewriting", we use the Distributive Property to rewrite the equation as $4x + 8 = 36$. We then solve this equation to get $x = 7$. (p. 108)

right angle An angle that measures 90°. A small square is used to note a right angle, as shown in the example at right.



right triangle A triangle that has one right angle. The side of a right triangle opposite the right angle is called the hypotenuse, and the two sides adjacent to the right angle are called legs. Note that legs of a right triangle are always shorter than its hypotenuse.

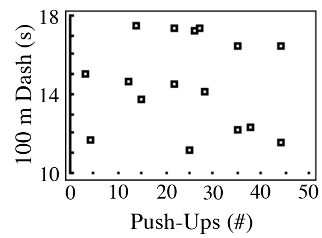
root (of a polynomial function) The roots of a polynomial function $P(x)$ are the values of x such that $P(x) = 0$. For example, the roots of $P(x) = (x - 4)(2x + 3)$ are $x = 4$ and $x = -\frac{3}{2}$, because these are the solutions to the equation $(x - 4)(2x + 3) = 0$. The x-intercepts of a function's graph are the real roots of the function. Roots can be real or complex, but complex roots must be determined algebraically. (p. 400)

***R*-squared** The correlation coefficient squared, and usually expressed as a percent. R^2 is a measure of the strength of a linear relationship. Its interpretation is that $R^2\%$ of the variability in the dependent variable can be explained by a linear relationship with the independent variable. The rest of the variability is explained by other variables that are not part of the study.

sample A subset (group) of a given population with the same characteristics as the whole population. (p. 161)

sample space With probability, all the possible outcomes.

scatterplot A way of displaying two-variable numerical data where two measurements are taken for each subject (like height and forearm length, or surface area of cardboard and volume of cereal held in a cereal box). To create a scatterplot, the two values for each subject are written as coordinate pairs and graphed on a pair of coordinate axes (each axis representing a variable). Also see *association*.



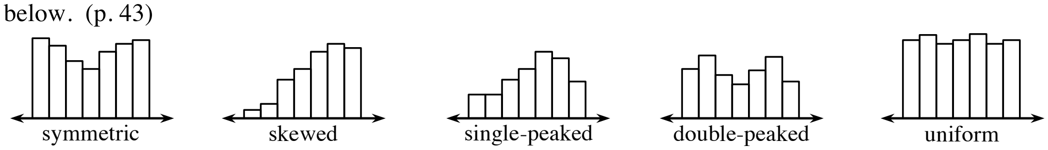
secant The reciprocal of the cosine, given by $\frac{1}{\cos(x)}$.
Also see *reciprocal trigonometric functions*. (p. 623)

sequence A sequence is an ordered set of objects, usually written as a list of numbers. A sequence can be thought of as a function in which the independent variable is the term number and the dependent variable is the term value. (p. 538)

series The sum of the terms of a sequence. (p. 504)

set A collection of data or items. (p. 25)

shape (of a data distribution) Statisticians use the following words to describe the overall shape of a data distribution: symmetric, skewed, single-peaked, double-peaked, and uniform. Examples are shown below. (p. 43)



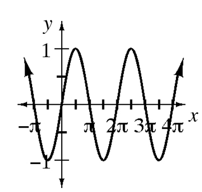
sigma The Greek letter Σ which is used to mean sum. Using Σ provides a way to write a short, compact mathematical representation for a series. See *summation notation*. (p. 508)

simple interest Interest paid on the principal alone.

simplify To simplify an expression is to write a less complicated expression with the same value. A simplified expression has no parentheses and no like terms. For example, the expression $3 - (2x + 7) - 4x$ can be simplified to $-4 - 6x$. When working with algebra tiles, a simplified expression uses the fewest possible tiles to represent the original expression.

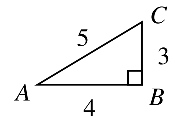
simulation (probability) When conducting an experiment with an event that is unrealistic to perform, a simulation can be used. A simulation is a similar experiment that has the same probabilities as the original experiment. (p. 267)

sine function The general equation for the sine function is $y = a \sin b(x - h) + k$. This function has amplitude a, period $\frac{2\pi}{b}$, horizontal shift h, and vertical shift k. (p. 477)



sine inverse ($\sin^{-1}(x)$) See *inverse sine*.

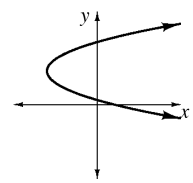
sine ratio In a right triangle, the sine ratio of an acute angle ($\angle A$) is $\sin(A) = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$. In the triangle at right, $\sin(A) = \frac{BC}{AC} = \frac{3}{5}$. On a unit circle, the sine of angle θ, denoted $\sin(\theta)$, is the y-coordinate of the point on the unit circle reached by a rotation angle of θ radians in standard position. Also see *cosine ratio, tangent ratio,* and *inverse sine*. (p. 55)



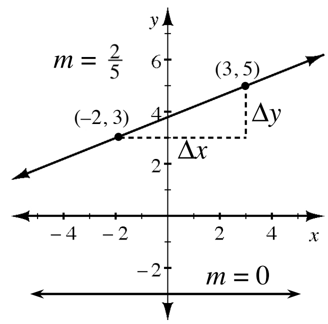
single-peaked See *shape (of a data distribution)*.

skewed (data display) See *shape (of a data distribution)*.

"sleeping" parabola A parabola which opens to the left or right, rather than upward or downward. These parabolas are not the graphs of functions. (p. 81)



slope The ratio of the vertical change to the horizontal change between any two points on a line that describes how steep (or flat) a line is. Slope can be positive, negative, or zero, but a line has only one slope. Slope is the ratio $\frac{\text{vertical change}}{\text{horizontal change}}$ or $\frac{\text{change in } y\text{-value}}{\text{change in } x\text{-value}}$, sometimes written $\frac{\Delta y}{\Delta x}$. For any two points (x_1, y_1) and (x_2, y_2) on a given line, the slope is $\frac{y_2 - y_1}{x_2 - x_1}$. For example, the slope of a line through the points (3, –5) and (–7, 2) is $\frac{2-(-5)}{-7-3} = -\frac{7}{10}$. When the equation of a line is written in $y = mx + b$ form, m is the slope of the line. Some texts refer to slope as the ratio of the "rise over the run". A line has a positive slope if it slopes upward from left to right on a graph, a negative slope if it slopes downward from left to right, zero slope if it is horizontal, and an undefined slope if it is vertical. Slope is interpreted in context as the amount of change in the y-variable for an increase of one unit in the x-variable.



slope angle The acute angle a line forms with the x-axis on a coordinate graph.

slope-intercept form A linear equation written in the form $y = mx + b$ is written in slope-intercept form. In this form, m is the slope of the line and the point $(0, b)$ is the y-intercept.

solid A closed three-dimensional shape and all of its interior points. Examples include regions bounded by pyramids, cylinders, and spheres. (p. 207)

solution The number or numbers that when substituted into an equation or inequality make the equation or inequality true.

space The set of all points in three dimensions.

spread (of a data distribution) A measure of the amount of variability, or how "spread out" a set of data is. Some ways to measure spread are the range, the mean absolute deviation, the standard deviation, and the interquartile range. For symmetric distributions with no outliers, the standard deviation or the interquartile range can represent the spread of the data well. However, in the presence of outliers or non-symmetrical data distributions, the interquartile range may be a better measure. Also see *variability*. (p. 36)

square root The square root of a number is the value that when multiplied by itself gives the number. "Square root" always refers to a positive number. Also see *radical*.

standard deviation A way to measure the spread, or the amount of variability, in a set of data. The standard deviation is the square root of the average of the distances to the mean, after the distances have been made positive by squaring. The standard deviation represents the spread of a set of data well if the distribution of the data is symmetric and has no outliers. If the set of data is a *sample* of the population, instead of the entire population, a slight adjustment is made: for a sample, divide by $n - 1$ (one less than the number of items in the sample) instead of n (the number of items in the population) when calculating the average. (p. 36)

standard form of a polynomial For a polynomial in one variable with an undetermined degree n, the standard form is $P(x) = (a_n)x^n + (a_{n-1})x^{(n-1)} + \ldots + (a_1)x^1 + a_0$. (p. 373)

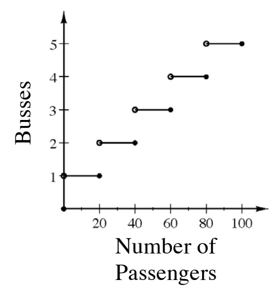
standard position An angle is in standard position if its vertex is located at the origin and one ray is on the positive x-axis. (p. 472)

statistic A numerical fact or calculation that is formed from data. For example, a mean or a correlation coefficient is a statistic. With a capital "S", Statistics is the field of study that is concerned with summarizing data and using probability to make predictions from data with natural variability. (p. 161)

statistical hypothesis test A method of using statistical sampling to determine the probability that a certain claim (hypothesis) about a whole population is true. In a statistical hypothesis test, sample-to-sample variability is analyzed to determine a margin of error for sample statistics. (p. 280)

statistical process control The process of checking whether manufacturing quality remains stable over time. A process is "in control" if critical measurements of quality remain within the upper control limit and lower control limit as expected. (p. 296)

step function A special kind of piecewise function (a function composed of parts of two or more functions). A step function has a graph that is a series of line segments that often looks like a set of steps. (p. 87)

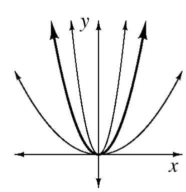


stoplight icon The icon (shown at right) will appear periodically throughout the text. Problems that display this icon contain errors of some type. (p. 21)

streak Consecutive identical outcomes in a probabilistic situation. For example, if a baseball team wins four consecutive games, they have a winning streak of 4.

strength (of an association) A description of how much scatter there is in the data away from the line or curve of best fit. If an association is linear, the correlation coefficient, r, or R-squared can be used to numerically describe and interpret the strength.

stretch factor Used to describe the effect of a in the graphing form of a quadratic, cubic, absolute value, or exponential function. For $a > 1$ or $a < -1$, the outputs increase or decrease faster than the outputs for the parent functions, and the graphs are described as being stretched upwards or downwards in relation to the parent graph. (p. 385)



study See *observational study*.

subjects The people or items being measured in a statistical study.

substitution Replacing a variable or expression with a number, another variable, or another expression. For example, when evaluating the function $f(x) = 5x - 1$ for $x = 3$, substitute 3 for x to get $f(3) = 5(3) - 1 = 14$. (p. 109)

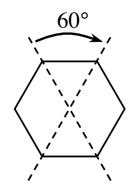
Substitution Method A method for solving a system of equations by replacing one variable with an expression involving the remaining variable(s). For example, in the system of equations at right, the first equation tells you that y is equal to $-3x + 5$. We can substitute $-3x + 5$ in for y in the second equation to get $2(-3x + 5) + 10x = 18$, then solve this equation for x. Once we have x, we substitute that value back into either of the original equations to determine the value of y.

$$y = -3x + 5$$
$$2y + 10x = 18$$

sum of cubes A polynomial of the form $x^3 + y^3$. It can be factored as follows: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$. For example, the sum of cubes $27y^3 + 64$ can be factored as $(3y + 4)(9y^2 - 12y + 16)$. (p. 419)

summation notation A convenient way to represent a series is to use summation notation. The Greek letter sigma, Σ, indicates a sum. For example, $\sum_{n=1}^{4} 3n = 3(1) + 3(2) + 3(3) + 3(4) = 30$. The numbers below and above the sigma are called the indices. The index below, $n = 1$, tells us what value to start with for n. The top index tells us how high the value can go. In the example shown above, n starts at 1 and increases to 4. (p. 508)

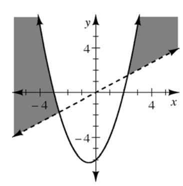
symmetry (1) Rotation symmetry: A shape has rotation symmetry when it can be rotated for less than 360° and it appears not to change. For example, the hexagon at right appears not to change if it is rotated 60° clockwise (or counterclockwise); (2) Reflection symmetry: A shape has reflection symmetry when it appears not to change after being reflected across a line. The regular hexagon at right also has reflection symmetry across any line drawn through opposite vertices, such as those shown in the diagram. (3) There are no polygons with translation symmetry but a line can have translation symmetry as it can be translated on to itself. See *line of symmetry*.



system of equations A system of equations is a set of equations with the same variables that has more than one unknown. Solving a system of equations means determining solutions that make each of the equations in the system true. A solution to a system of equations gives a point of intersection of the graphs of the equations in the system. There may be zero, one, or several solutions to a system of equations. For example, $(1.5, -3)$ is a solution to the system of equations at right; setting $x = 1.5$, $y = -3$ makes both of the equations true. Also, $(1.5, -3)$ is a point of intersection of the graphs of these two equations. (p. 120)

$$y = 2x - 6$$
$$y = -2x$$

system of inequalities A system of inequalities is a set of inequalities with the same variables. Solving a system of inequalities means determining the regions on the coordinate plane whose points represent solutions to each of the inequalities in the system. There may be zero, one, or several such regions for a system of inequalities. For example, the shaded region at right is a graph of the system of inequalities that appears below it. (p. 129)

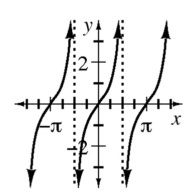


$$y \le x^2 + x - 6$$
$$y > \tfrac{2}{3}x$$

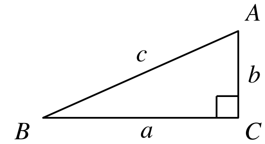
systematic list A list created by following a system (an orderly process).

table The tables used in this course represent numerical information by organizing it into columns and rows. The numbers may come from a graph, situation, or equation.

tangent function The general equation for the tangent function is $y = a\tan b(x - h) + k$. This function has period of $\frac{\pi}{b}$, vertical asymptotes at $\frac{\pi}{2b} + h \pm \frac{n\pi}{b}$ for $n = 1, 2, \ldots$, horizontal shift h, and vertical shift k. (p. 458)



tangent inverse ($\tan^{-1}(x)$) See *inverse tangent*.



tangent ratio In a right triangle (at right) the ratio $\frac{\text{opposite side}}{\text{adjacent side}}$ is known as the tangent of an acute angle. At right, $\tan(B) = \frac{b}{a}$ since the side of length b is opposite angle B and the side length a is adjacent to (or next to) angle B. On a unit circle, the tangent of angle θ, denoted $\tan(\theta)$, is the slope of the line containing the ray which represents a rotation of θ radians in standard position. Also see *sine ratio, cosine ratio,* and *inverse tangent*. (p. 55)

term A single number, variable, or product of numbers and variables. A monomial is a term. Also a component of a sequence. (p. 26)

theorem A conjecture that has been proven to be true. Some examples of theorems are the Pythagorean Theorem and the Triangle Angle Sum Theorem.

theoretical probability A calculated probability based on the possible outcomes when each outcome has the same chance of occurring: $\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$.

theta (θ) A Greek letter that is often used to represent the measure of an angle. Other Greek letters used to represent the measure of an angle include *alpha* (α) and *beta* (β).

third quartile (Q3) The median of the upper half of a set of data which has been written in numerical order. (p. 36)

three-dimensional An object that has height, width, and depth. (p. 579)

transformation (of a function) The conversion of a function to a corresponding function. Transformations often slide, reflect, stretch, and/or compress graphs of functions. For example, the transformation of $f(x) + k$ moves the graph of the function up or down by an amount k. The transformation $f(-x)$ reflects the graph of the function across the y-axis. (p. 76)

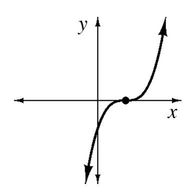
translation Formally, a translation is a function that assigns each point (Q) in the plane a unique point (Q') so that all line segments connecting an original point with its image have equal length and are parallel.

treatment Some type of change imposed on one or more groups in an experiment so that a direct comparison can be made. (p. 174)

tree diagram A model used to organize the possible outcomes and the respective probabilities of two or more events.

trigonometry Literally, the "measure of triangles". In this course, this word is used to refer to the development of triangle tools such as trigonometric ratios (sine, cosine, and tangent) and the Laws of Sines and Cosines.

trinomial A polynomial that is the sum or difference of exactly three terms, each of which is a monomial. For example, $x^2 + 6x + 9$ is a trinomial.



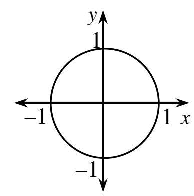
triple root A root of a function that occurs exactly three times. If an expression of the form $(x - a)^3$ is a factor of a polynomial, then the polynomial has a triple root at $x = a$. The graph of the polynomial has an inflection point at $x = a$. (p. 400)

two-dimensional An object having length and width. (p. 579)

two-way table A way to display categorical two-variable data. The categories of one of the variables is the header of the rows, while the other variable is the header of the columns. The entries in the table can be counts (frequencies) or percents (relative frequencies). Also see *relative frequency table*.

Undoing Using inverse operations and reversing the Order of Operations to solve an equation in one variable or to "undo" a function in order to write its inverse. For example, in the equation $4(x + 2) = 36$, the last operation that was applied to the left-hand side was a multiplication by 4. So to use "Undoing", we divide both sides of the equation by 4, giving us $x + 2 = 9$. We then solve the equation $x + 2 = 9$ (perhaps by "Undoing" again and subtracting 2 from both sides) to get $x = 7$. (p. 108)

uniform See *shape (of a data distribution)*.



unit circle A circle with a radius of one unit is called a unit circle. (p. 441)

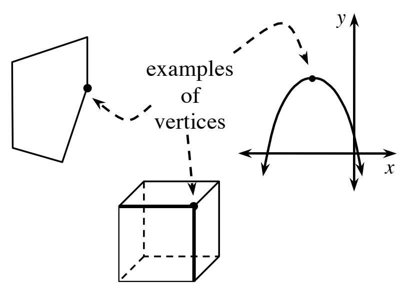
upper boundary line A line drawn parallel to and above the least squares regression line at a distance equivalent to the largest residual. The upper boundary line determines the upper bound on the values that a prediction is likely to be. (p. 277)

upper control limit (UCL) See *control limit*.

variability The inconsistency in measured data. Variability comes from two sources. Measurement error is the inconsistency in measurement of the same element repeated times and/or by different people. Element variability is the natural variation of the elements themselves. For example, even though in general, people with larger shoe sizes are taller, not all people with the same shoe size have the same height. That is element variability. See *spread (of a data distribution)*. (p. 280)

variable A symbol used to represent one or more numbers. In this course, letters of the English or the Greek alphabet are used as variables. For example, in the expression $3x – (8.6xy + z)$, the variables are x, y, and z. (p. 19)

vertex (plural: vertices) (1) For a two-dimensional geometric shape, a vertex is a point where two or more line segments or rays meet to form a "corner", such as in a polygon or angle. (2) For a three-dimensional polyhedron, a vertex is a point where the edges of the solid meet. (3) On a graph, a vertex can be used to describe the highest or lowest point on the graph of a parabola or absolute value function (depending on the graph's orientation).



vertex form The vertex form for the equation of a quadratic function (also called *graphing form*) is written $y = a(x – h)^2 + k$.

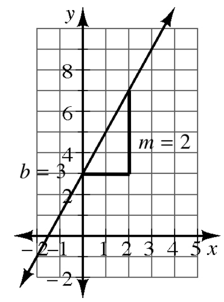
volume A measurement of the size of the three-dimensional region enclosed within an object. It is expressed as the number of $1 \times 1 \times 1$ unit cubes (or parts of cubes) that fit inside a solid.

whole numbers The positive integers and zero, { 0, 1, 2, 3, . . , }.

***x*-bar process control chart** A plot of the mean measurements of samples over time. *x*-bar process control charts are used to monitor whether a manufacturing process is proceeding within limits as expected. (p. 297)

***x*-intercept(s)** The point(s) where a graph intersects the x-axis. A graph may have several x-intercepts, no x-intercepts, or just one. In two dimensions, the coordinates of the x-intercept are $(x, 0)$. In three dimensions they are $(x, 0, 0)$. See *intercept*.

y* = *mx* + *b When two quantities x and y have a linear relationship, that relationship can be represented with an equation in $y = mx + b$ form. The parameter m is the slope, and b is the y-intercept of the graph. For example, the graph at right shows the line represented by the equation $y = 2x + 3$, which has a slope of 2 and a y-intercept of 3. This form of a linear equation is also called the slope-intercept form.



***y*-intercept(s)** The point(s) where a graph intersects the y-axis. A function has at most one y-intercept; a relationship may have several. The y-intercept of a graph is important because it often represents the starting value of a quantity in a real-world situation. In two dimensions, the coordinates of the y-intercept are $(0, y)$. In three dimensions, the coordinates are $(0, y, 0)$. See *intercept*.

Zero Product Property When the product of two or more factors is zero, at least one of the factors must equal zero. That is, if $a \cdot b = 0$, then either $a = 0$ or $b = 0$ (or both). This property is used to solve equations in factored form. For example, given the equation $(x - 4)(x + 5) = 0$, $x - 4 = 0$ or $x + 5 = 0$ (or both). You can see that $x = 4$ and $x = -5$ are solutions and that they are the only possible solutions because there are no other numbers that make either factor zero. The Zero Product Property can be used to solve factorable quadratic equations. (p. 12)

zero factorial Zero factorial is defined to be 1, that is, $0! = 1$.

zero power The result of raising any number (except zero) to the zero power is 1. $x^0 = 1$ for any number $x \neq 0$.

zero of a function See *roots (of a polynomial function)*.