Association in a Two-Way Table

GUIDED SKILL SUPPORT

Overview

Data based on measurements such as height, speed, and temperature is **numerical**. Data can also be counts or percentages in categories such as gender, religion, or blood type. This kind of non-numerical data is called **categorical**. Categorical data cannot be graphed on a scatter plot, but there are other techniques to look for a relationship (an association) between categorical variables, including Venn diagrams, bar charts, and two-way tables.

A two-way table is similar to a generic rectangle. To make sense of the inner cells, you must carefully make sense of the row and column headings. The inner cells are a count of the intersection of their row and column. For example, respondents to a survey identified whether they have siblings and whether they have pets. The results are shown.

	Have Siblings	No Siblings
Have Pets	People who have pets and and have siblings: 78	People who have pets and do not have siblings: 57
No Pets	People who do not have pets and have siblings: 32	People who do not have pets and do not have siblings: 15

To determine if there is a potential association between the data, convert the table to a **conditional relative frequency table**. Calculate the total of each row and column. Then, use those totals to calculate the percentage for each cell.

	Have Siblings	No Siblings	Total
Have Pets	$\frac{78}{110} = 71\%$	$\frac{57}{72} = 79\%$	People with pets: 135
No Pets	$\frac{32}{110} = 29\%$	$\frac{15}{72} = 21\%$	People without pets: 47
Total	People with siblings: 110	People without siblings: 72	

Note that labeling the rows with the column headings and vice versa does not change the outcome of the two-way table analysis. In the example provided, the headings “Have Siblings” and “Have Pets” could be swapped. If those are swapped, “No Siblings” and “No Pets” would also have to be swapped.

Additional information about two-way tables can be found in the Lesson 10.1.2 Methods & Meanings box.

Examples

1. Analyze the fictional data collected. Is there an association between a voter's political party and whether the voter supported the ballot proposition?

	Republican	Democrat
Supported the Proposition	234	286
Did Not Support the Proposition	162	198
Undecided	54	66

Determine the row and column totals.

	Republican	Democrat	Total
Supported the Proposition	234	286	520
Did Not Support the Proposition	162	198	360
Undecided	54	66	120
Total	450	550	1,000

Calculate the percents, relative frequencies, down each column. For example, the percentage of Republicans who supported the proposition is $\frac{234}{450}$ = 52%. Use the percentages to make a two-way relative frequency table.

	Republican	Democrat
Supported the Proposition	$\frac{234}{450}$ = 52%	$\frac{286}{550}$ = 52%
Did Not Support the Proposition	$\frac{162}{450}$ = 36%	$\frac{198}{550}$ = 36%
Undecided	$\frac{54}{450}$ = 12%	$\frac{66}{550}$ = 12%
Total	100%	100%

Compare the percentages across the row "Supported the proposition." 52% of voters supported the ballot proposition regardless of whether they were a Democrat or Republican. This indicates that there is no association between political party and whether the voter supported the ballot proposition.

2. A survey of 155 recent high school graduates found that 130 had driver's licenses and 58 had jobs. According to the survey, 21 graduates had neither a driver's license nor a job. Is there an association between having a driver's license and having a job among recent graduates?

Make a two-way table to display the counts (the number of graduates) in each category.

Determine the known information.

	Job	No Job	Total
License			130
No License		21	
Total	58		155

Use the information to fill in the remaining counts. For example, calculate 155 – 58 to know the total number of graduates without a job is 97. Continue using addition or subtraction to calculate the remaining unknown values.

	Job	No Job	Total
License	54	76	130
No License	4	21	25
Total	58	97	155

Convert the counts to percentages (relative frequencies). For example, $\frac{54}{58}$ = 93% of graduates who have jobs also have a driver's license.

	Job	No Job
License	93%	78%
No License	7%	22%
Total	100%	100%

Compare the percentages across the rows. In this case, the percentages across rows are different, indicating an association between having a job and a driver's license. A higher percentage of graduates with jobs have licenses (93%) than those without jobs (78%).

Practice

1. In a survey of 200 pet owners, 76 do not own a cat, and 63 do not own a dog. According to the survey, 82 own both a cat and a dog. Make a two-way table displaying the counts for each category. Is there an association between cat and dog ownership?

2. In a softball game, Brandon batted 32 times when Parker was pitching. Brandon got a base hit 8 times. Parker used his fastball pitch 12 times total, but Brandon got a base hit only three times when Parker used his fastball. Is there an association between Parker's pitching style and Brandon's chance of getting a base hit when batting in softball games? Make a two-way table and decide whether there is an association between pitching style and getting a base hit.

Answers

1.

	Dog	No Dog	Total
Cat	82	42	124
No Cat	55	21	76
Total	137	63	200

	Dog	No Dog
Cat	60%	67%
No Cat	40%	33%
Total	100%	100%

There appears to be a weak association between cat and dog ownership. The probability of owning a cat is slightly lower if you own a dog. Only 60% of dog owners have a cat, while 67% of non-dog owners have a cat. Pet owners are more likely to own a cat if they are not a dog owner. There is a weak association.

An alternate set up for the tables is shown. The conclusions are the same.

	Cat	No Cat	Total
Dog	82	55	137
No Dog	42	21	63
Total	124	76	200

	Cat	No Cat
Dog	66%	72%
No Dog	34%	28%
Total	100%	100%

2.

		Parker		
		Fastball	Other Pitch	Total
Brandon	Base Hit	3	5	8
	No Base Hit	9	15	24
	Total	12	20	32

		Parker	
		Fastball	Other Pitch
Brandon	Base Hit	25%	25%
	No Base Hit	75%	75%
	Total	100%	100%

There is no association between Parker's pitch and Brandon's ability to get a base hit. The probability that Brandon gets a base hit is 25%, regardless of whether Parker uses his fastball pitch or not. An alternate set up for the tables is also shown.

		Brandon		
		Base Hit	No Base Hit	Total
Parker	Fastball	3	9	12
	Other Pitch	5	15	20
	Total	8	24	32

		Brandon	
		Base Hit	No Base Hit
Parker	Fastball	37.5%	37.5%
	Other Pitch	62.5%	62.5%
	Total	100%	100%

Solving by Rewriting: Fraction Busters

GUIDED SKILL SUPPORT

Overview

Equations with fractions and/or decimals can be rewritten as equivalent equations without fractions and/or decimals and then solved by **Undoing**. Undoing is the process of using inverse operations to isolate the variable.

Equations containing decimals can be rewritten by multiplying by a multiple of 10 that converts all of the decimals to integers. For example, to rewrite the equation $3.2x - 12 = 0.04x + 15$ so it only has integer coefficients, multiply both sides of the equation by 100. **Rewriting** is also an effective strategy when equations contain fractions. One Rewriting method that eliminates fractions from an equation is called **Fraction Busters**. In this method, all terms on each side of an equation are multiplied by a common denominator. Any common denominator works, including the product of all of the denominators. Check the solution by verifying that it makes the equation true.

Additional information about equivalent equations can be found in the Lessons 10.2.1 and 10.2.2 Methods & Meanings boxes.

Examples

For problems 1 through 4, solve each equation.

1. $0.12x + 7.5 = 0.2x + 3$

$0.12x + 7.5 = 0.2x + 3$	
$100(0.12x + 7.5) = (0.2x + 3)100$	Multiply both sides of the equation by 100.
$12x + 750 = 20x + 300$ $-12x \qquad -12x$	Subtract $12x$ from both sides of the equation.
$750 = 8x + 300$ $-300 \qquad -300$	Subtract 300 from both sides of the equation.
$\frac{450}{8} = \frac{8x}{8}$	Divide both sides of the equation by 8.
$x = 56\frac{1}{4}$	

2. $25x^2 + 125x + 150 = 0$

$\frac{25x^2 + 125x + 150}{25} = \frac{0}{25}$	Divide both sides by the common factor 25.
$x^2 + 5x + 6 = 0$	
$(x + 2)(x + 3) = 0$	Rewrite the trinomial as a product of two factors.
$x + 2 = 0$ $\quad$ $x + 3 = 0$	
$x = -2$ $\quad$ $x = -3$	Solve using the Zero Product Property.

3. $\frac{x}{2} + \frac{x}{6} = 7$

$\frac{x}{2} + \frac{x}{6} = 7$	
$(6)\left(\frac{x}{2} + \frac{x}{6}\right) = (7)(6)$	Multiply by the Fraction Buster 6 to eliminate the fractions.
$3x + x = 42$	
$\frac{4x}{4} = \frac{42}{4}$	Combine like terms. Divide both sides by 4.
$x = \frac{21}{2}$ or 10.5	

4. $\frac{5}{2x} + \frac{1}{6} = 8$

$\frac{5}{2x} + \frac{1}{6} = 8$	
$(6x)\left(\frac{5}{2x} + \frac{1}{6}\right) = 8(6x)$	Multiply by the Fraction Buster $6x$ to eliminate the fractions.
$15 + x = 48x$ $\quad - x \quad - x$	Subtract x from both sides of the equation.
$\frac{15}{47} = \frac{47x}{47}$	Divide both sides of the equation by 47.
$x = \frac{15}{47}$ or ≈ 0.32	

Practice

Solve each equation. Use Rewriting as the first step to make each equation easier to solve.

1. $\frac{x}{3} + \frac{x}{2} = 5$

2. $3{,}000x + 2{,}000 = -1{,}000$

3. $0.02y - 1.5 = 17$

4. $\frac{x}{2} + \frac{x}{3} - \frac{x}{4} = 12$

5. $50x^2 - 200 = 0$

6. $\frac{x}{9} + \frac{2x}{5} = 3$

7. $\frac{3x}{10} + \frac{x}{10} = \frac{15}{10}$

8. $\frac{3}{2x} + \frac{5}{x} = \frac{13}{6}$

9. $x^2 - 2.5x + 1 = 0$

10. $\frac{2}{3x} - \frac{1}{x} = \frac{1}{36}$

11. $0.002x = 5$

12. $10 + \frac{5}{x} + \frac{3}{3x} = 11$

13. $0.3(x + 7) = 0.2(x - 2)$

14. $x + \frac{x}{2} + \frac{3x}{5} = 21$

15. $32 \cdot 3x - 32 \cdot 1 = 32 \cdot 8$

16. $5 + \frac{2}{x} + \frac{5}{4x} = \frac{73}{12}$

17. $\frac{17}{2x + 1} = \frac{17}{5}$

18. $2 + \frac{6}{x} + \frac{6}{3x} = 3$

19. $2.5x^2 + 3x + 0.5 = 0$

20. $\frac{x}{x - 2} = \frac{7}{x - 2}$

Answers

Note: excluded values are not provided.

1. $x = 6$

2. $x = -1$

3. $y = 925$

4. $x = \frac{144}{7}$ or ≈ 20.57

5. $x = \pm 2$

6. $x = \frac{135}{23}$ or ≈ 5.87

7. $x = \frac{15}{4}$ or 3.75

8. $x = 3$

9. $x = \frac{1}{2}$ or $x = 2$

10. $x = -12$

11. $x = 2{,}500$

12. $x = 6$

13. $x = -25$

14. $x = 10$

15. $x = 3$

16. $x = 3$

17. $x = 2$

18. $x = 8$

19. $x = -\frac{1}{5}$ or $x = -1$

20. $x = 7$

Multiple Methods for Solving Equations

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Overview

Equations may be solved in a variety of ways including **Rewriting**, **Looking Inside**, and **Undoing**. Rewriting is used to analyze an equation from another perspective. Looking Inside involves focusing on part of the equation to make more sense of the whole equation. Undoing uses inverse operations to isolate the variable.

Remember that it is necessary to find all of the solutions. For example, problems with squares may have two solutions, one solution, or no solution.

$x^2 = 144$	$(x - 3)^2 = 0$	$(2x + 1)^2 = -7$
$x = \pm 12$	$x = 3$	no solution

Additional information about solving equations can be found in the Lessons 10.2.3 and 10.2.6 Methods & Meanings boxes.

Examples

For problems 1 through 4, solve each of the equations for x.

1. $\frac{x}{2}+\frac{x}{5}=3$

Use **Rewriting** because the equation contains more than one fraction.

$\frac{x}{2}+\frac{x}{5}=3$	
$10\left(\frac{x}{2}+\frac{x}{5}\right)=3\cdot 10$	Multiply by a common denominator.
$5x+2x=30$	Rewrite using Fraction Busters.
$7x=30$	Combine like terms.
$x=\frac{30}{7}$ or ≈ 4.29	Divide both sides by 7.

The solution can be checked by substituting the value of x into the original equation.

$$x=\boxed{\frac{30}{7}}$$

$$\frac{\boxed{x}}{2}+\frac{\boxed{x}}{5}=3$$

$$\frac{\frac{30}{7}}{2}+\frac{\frac{30}{7}}{5}=3$$

$$70\left(\frac{30}{14}+\frac{30}{35}\right)=3\cdot 70$$

$$150+60=210$$

$$210=210$$

true

2. $\sqrt{x} + 1 = 6$

Use **Looking Inside**, because the equation involves taking the square root of a variable.

$\sqrt{x} + 1 = 6$	
$? + 1 = 6$	Use Looking Inside to make sense of the radical.
$\sqrt{x} = 5$	Think, "What plus 1 equals 6?"
$x = 25$	Think, "The square root of what number is 5?"

Check that the solution makes the original equation true.

$$x = \boxed{25}$$

$$\sqrt{\boxed{x}} + 1 = 6$$
$$\sqrt{25} + 1 = 6$$
$$5 + 1 = 6$$
$$6 = 6$$

true

3. $\frac{x+1}{2} = -6$

Use **Looking Inside**, because you can use number reasoning to make sense of the fraction.

$\frac{x+1}{2} = -6$	
$\frac{?}{2} = -6$	Think, "What divided by 2 equals −6?"
$x + 1 = -12$ $-1 \quad -1$	Rewrite the equation: the numerator equals −12.
$x = -13$	Subtract 1 from both sides.

Check the solution.

$$x = \boxed{-13}$$

$$\frac{\boxed{x} + 1}{2} = -6$$
$$\frac{(-13) + 1}{2} = -6$$
$$\frac{-12}{2} = -6$$
$$-6 = -6$$

true

4. $|2x+3| = 11$

Use **Looking Inside**, because the absolute value of two quantities could make this equation true. Solve two equations to determine all possible solutions.

$$|2x+3| = 11$$

$2x + 3 = 11$	$\dfrac{-(2x+3)}{-1} = \dfrac{11}{-1}$	Rewrite it as two equations.
	$2x + 3 = -11$	Divide by -1.
$2x = 8$	$2x = -14$	Undo addition or subtraction.
$x = 4$	$x = -7$	Undo multiplying by the coefficient.

Check the solutions.

$x = 4$	$x = -7$
$\lvert 2x + 3\rvert = 11$	$\lvert 2x + 3\rvert = 11$
$\lvert 2(4) + 3\rvert = 11$	$\lvert 2(-7) + 3\rvert = 11$
$\lvert 11\rvert = 11$	$\lvert -11\rvert = 11$
$11 = 11$ true	$11 = 11$ true

Practice

Solve each equation. Identify all possible solutions.

1. $\frac{3x+1}{2} = 5$

2. $4(x-1) + 3 = 15$

3. $\sqrt{2x+5} = 10$

4. $10 - (x+7) = 5$

5. $3(2x-7) = -21$

6. $8 + \left(\frac{y}{2}\right) = 10$

7. $\sqrt{x} - 3 = 7$

8. $(x+1)^2 = 81$

9. $\frac{x}{2} - \frac{x}{5} = 3$

10. $|x-2| = 5$

11. $2\sqrt{x-3} = 8$

12. $4(x-1) = 16$

13. $|2x+1| = 9$

14. $\frac{y+7}{3} = 10$

15. $\frac{m}{3} - \frac{2m}{5} = \frac{1}{5}$

16. $x^2 + 5 = 4$

17. $\frac{3y-1}{3} = 10$

18. $2(3x-1) + 7 = -13$

19. $(y-1)^2 = 9$

20. $20 - (3x) = 10$

Answers

1.	$x = 3$	**2.**	$x = 4$	**3.**	$x = \frac{95}{2}$ or 47.5
4.	$x = -2$	**5.**	$x = 0$	**6.**	$y = 4$
7.	$x = 100$	**8.**	$x = 8$ or -10	**9.**	$x = 10$
10.	$x = 7$ or -3	**11.**	$x = 19$	**12.**	$x = 5$
13.	$x = 4$ or -5	**14.**	$y = 23$	**15.**	$m = -3$
16.	no solution	**17.**	$y = \frac{31}{3}$ or $10.\overline{3}$	**18.**	$x = -3$
19.	$y = 4$ or -2	**20.**	$x = \frac{10}{3}$ or $3\frac{1}{3}$		

Intercepts and Intersections

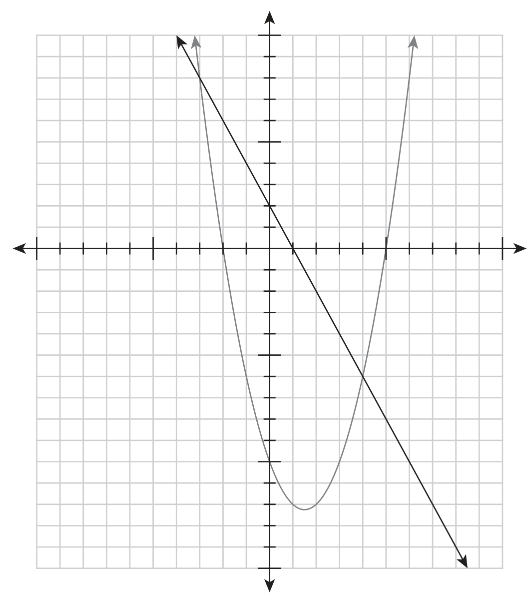
GUIDED SKILL SUPPORT

Overview

Intercepts and **intersections** both represent a point at which two paths cross, but an intercept specifies where a curve or line crosses an *axis*, whereas an intersection refers to any point where the graphs of two equations cross. When analyzing a graph, the points of intercepts or intersections may be estimated. When accuracy is needed, algebra can be used to determine points of intercepts and intersections.

Examples

1. The graphs of the parabola $y = x^2 - 3x - 10$ and the line $y = -2x + 2$ are shown. What are the points of intersection and intercepts?



Scan the x- and y-axes to identify potential **intercepts**.

- The parabola has two x-intercepts $(-2, 0)$ and $(5, 0)$ and one y-intercept $(0, -10)$.
- The line has x-intercept $(1, 0)$ and y-intercept $(0, 2)$.

Scan the graph to determine whether the line and the parabola ever cross and identify potential points of **intersection**. There are two points of intersection: $(-3, 8)$ and $(4, -6)$.

These points can be verified using algebra.

- To determine the intercepts, consider what they represent. The form for x-intercepts is $(x, 0)$, meaning $y = 0$. Similarly, the form for y-intercepts is $(0, y)$, meaning $x = 0$.
- To determine the points of intersection, solve the system of equations.

Example continues on next page.

1. Example continued from previous page.

INTERCEPTS OF A PARABOLA

To determine the x-intercept(s) of the parabola, substitute 0 for y and solve for x by factoring and using the Zero Product Property.

$y = 0$

$$y = x^2 - 3x - 10$$
$$0 = x^2 - 3x - 10$$
$$0 = (x - 5)(x + 2)$$

$0 = (x - 5)$ $\quad$ $0 = (x + 2)$

$5 = x$ $\quad$ $-2 = x$

The x-intercepts are (5, 0) and (−2, 0).

To determine the y-intercepts of the parabola, substitute 0 for x and solve for y.

$x = 0$

$$y = x^2 - 3x - 10$$
$$y = (0)^2 - 3(0) - 10$$
$$y = -10$$

The y-intercept is (0, −10).

INTERCEPTS OF A LINE

To determine the x- and y-intercepts of the line, substitute 0 for x then solve for y, then substitute 0 for y then solve for x.

$x = 0$

$$y = -2x + 2$$
$$y = -2(0) + 2$$
$$y = 2$$

The y-intercept is (0, 2).

$y = 0$

$$y = -2x + 2$$
$$0 = -2x + 2$$
$$1 = x$$

The x-intercept is (1, 0).

Example continues on next page.

1. Example continued from previous page.

INTERSECTION

To determine the point(s) of intersection of the system of equations, write an equation using the Equal Values Method or substitution.

$$x^2 - 3x - 10 = -2x + 2$$

Make one side equal to zero, factor, and use the Zero Product Property or the Quadratic Formula to solve for x.

$$x^2 - x - 12 = 0$$
$$(x - 4)(x + 3) = 0$$

$$x - 4 = 0 \qquad\qquad x + 3 = 0$$
$$x = 4 \qquad\qquad x = -3$$

Substituting $x = 4$ into either of the original equations yields $y = -6$, so $(4, -6)$ is a point of intersection.

$x = 4$

$y = x^2 - 3x - 10$
$y = (4)^2 - 3(4) - 10$
$y = 16 - 12 - 10$
$y = -6$

$x = 4$

$y = -2x + 2$
$y = -2(4) + 2$
$y = -8 + 2$
$y = -6$

Substituting $x = -3$ into either equation yields $y = 8$, so $(-3, 8)$ is a point of intersection.

$x = -3$

$y = x^2 - 3x - 10$
$y = (-3)^2 - 3(-3) - 10$
$y = 9 + 9 - 10$
$y = 8$

$x = -3$

$y = -2x + 2$
$y = -2(-3) + 2$
$y = 6 + 2$
$y = 8$

2. Consider the lines $y = \frac{1}{3}x + 8$ and $y = \frac{1}{2}x - 3$. What are the intercepts and points of intersection? Solve without graphing.

INTERCEPTS

To determine the intercepts, substitute 0 for x and solve for y then substitute 0 for y and solve for x.

Solve for the y-intercept of $y = \frac{1}{3}x + 8$.

$$x = 0$$

$$y = \frac{1}{3}x + 8$$

$$y = \frac{1}{3}(0) + 8$$

$$y = 8$$

The y-intercept is (0, 8).

Solve for the x-intercept of $y = \frac{1}{3}x + 8$.

$$y = 0$$

$$y = \frac{1}{3}x + 8$$

$$0 = \frac{1}{3}x + 8$$

$$-24 = x$$

The x-intercept is (–24, 0).

Solve for the y-intercept of $y = \frac{1}{2}x - 3$.

$$x = 0$$

$$y = \frac{1}{2}x - 3$$

$$y = \frac{1}{2}(0) - 3$$

$$y = -3$$

The y-intercept is (0, –3).

Solve for the x-intercept of $y = \frac{1}{2}x - 3$.

$$y = 0$$

$$y = \frac{1}{2}x - 3$$

$$0 = \frac{1}{2}x - 3$$

$$6 = x$$

The x-intercept is (6, 0).

Example continues on next page.

2. Example continued from previous page.

INTERSECTION

To determine the points of intersection, use the Equal Values Method and solve for x. Then substitute the value of x and solve for y.

In this case, the Equal Values Method can be used because both equations are equal to y; $y=\frac{1}{3}x+8$ and $y=\frac{1}{2}x-3$.

$$y=\frac{1}{3}x+8 \qquad y=\frac{1}{2}x-3$$

$$\frac{1}{3}x+8=\frac{1}{2}x-3$$
$$(6)\left(\frac{1}{3}x+8\right)=\left(\frac{1}{2}x-3\right)(6)$$
$$2x+48=3x-18$$
$$66=x$$

Substitute 66 for x into either original equation to determine $y=30$.

$x=66$	$x=66$
$y=\frac{1}{3}x+8$	$y=\frac{1}{2}x-3$
$y=\frac{1}{3}(66)+8$	$y=\frac{1}{2}(66)-3$
$y=22+8$	$y=33-3$
$y=30$	$y=30$

The point of intersection is (66, 30).

Practice

For each equation, identify the x- and y-intercepts.

1. $y=3x-2$

2. $y=\frac{1}{4}x-2$

3. $3x+2y=12$

4. $-x+2y=15$

5. $2y=15-3x$

6. $y=x^2+5x+6$

7. $y=x^2-3x-10$

8. $y=4x^2-11x-3$

9. $y=x^2+3x-2$

For each system of equations, identify the points of intersection.

10. $y=5x+1$, $y=-3x-15$

11. $y=x+7$, $y=4x-5$

12. $x=7+3y$, $x=4y+5$

13. $x=-\frac{1}{2}y+4y$, $8x+3y=31$

14. $4x-3y=-10$, $x=\frac{1}{4}y-1$

15. $2x-y=6$, $4x-y=12$

16. $y=x^2-3x-8$, $y=2$

17. $y=x^2-7$, $y=8+2x$

18. $y=x^2+2x+8$, $y=-4x-1$

Answers

1.	$(0.\overline{6}, 0), (0, -2)$	**2.**	(8, 0), (0, −2)	**3.**	(4, 0), (0, 6)
4.	(−15, 0), (0, 5)	**5.**	(5, 0), (0, 7.5)	**6.**	(−2, 0), (−3, 0), (0, 6)
7.	(5, 0), (−2, 0), (0, −10)	**8.**	(−0.25, 0), (3, 0), (0, −3)	**9.**	(0.56, 0), (−3.56, 0), (0, −2)
10.	(−2, −9)	**11.**	(4, 11)	**12.**	(13, 2)
13.	(3.5, 1)	**14.**	(−0.25, 3)	**15.**	(3, 0)
16.	(5, 2), (−2, 2)	**17.**	(5, 18), (−3, 2)	**18.**	(−3, 11)

Solving Quadratic and Absolute Value Inequalities

GUIDED SKILL SUPPORT

Overview

Solving an absolute value or quadratic inequality is very similar to solving a linear inequality in one variable, except that there are often three or more solution regions on the number line instead of just two. One method for solving absolute value and quadratic inequalities is to change the inequality to an equation, solve it, and then graph the solution(s) on a number line. The solution(s), called "**boundary point(s)**," divide the number line into regions. Check any point within each region in the *original* inequality. If the inequality is true, then all the points in that region are solutions. If the inequality is false, then none of the points in that region are solutions. The inequality sign determines whether a boundary point is included. The inequality symbols $\geq$ or $\leq$ indicate that the boundary point is included in the solution, and the symbols $>$ or $<$ indicate that the point is not part of the solution.

Additional information about inequality symbols and solving linear inequalities can be found in the Lessons 9.2.1 and 9.3.2 Methods & Meanings boxes.

Examples

1. Solve $|x - 3| \leq 5$

$$|x - 3| \leq 5$$

$$|x - 3| = 5$$

Change the inequality into an equation.

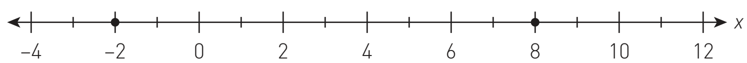
$$x - 3 = 5 \qquad\qquad -(x - 3) = 5$$

$$x = 8 \qquad\qquad x - 3 = -5$$

$$x = -2$$

Rewrite it as two equations.

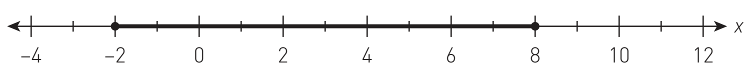
The solutions represent boundary points. Graph the boundary points on a number line.



Choose a point in each region to test (three total). Test the points in the original inequality.

$x = -3$	$x = 0$	$x = 9$
$\lvert x - 3\rvert \leq 5$	$\lvert x - 3\rvert \leq 5$	$\lvert x - 3\rvert \leq 5$
$\lvert -3 - 3\rvert \leq 5$	$\lvert 0 - 3\rvert \leq 5$	$\lvert 9 - 3\rvert \leq 5$
$\lvert -6\rvert \leq 5$	$\lvert -3\rvert \leq 5$	$\lvert 6\rvert \leq 5$
$6 \leq 5$	$3 \leq 5$	$6 \leq 5$
false	true	false

The solution is all numbers between and including the boundary points, $-2 \leq x \leq 8$.



2. Solve $|2y + 1| > 9$

$$|2y + 1| > 9$$

$$|2y + 1| = 9$$

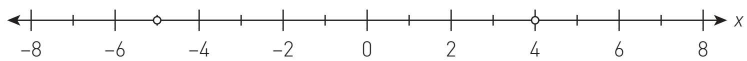
Change the inequality into an equation.

$2y + 1 = 9$
$2y = 8$
$y = 4$

$-(2y + 1) = 9$
$2y + 1 = -9$
$2y = -10$
$y = -5$

Rewrite it as two equations.

Graph the boundary points on a number line.



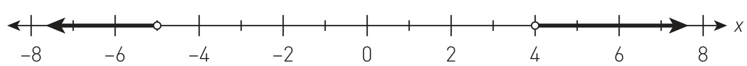
Choose a point in each region to test (three total). Test the points in the original inequality.

$y = -10$
$|2y + 1| > 9$
$|2(-10) + 1| > 9$
$|-20 + 1| > 9$
$|-19| > 9$
$-19 > 9$
true

$y = 0$
$|2y + 1| > 9$
$|2(0) + 1| > 9$
$|0 + 1| > 9$
$|1| > 9$
$1 > 9$
false

$y = 10$
$|2y + 1| > 9$
$|2(10) + 1| > 9$
$|20 + 1| > 9$
$|21| > 9$
$21 > 9$
true

The solution is all numbers less than −5 or greater than 4, $y < -5$ or $y > 4$.



3. Solve $x^2 - 3x - 18 < 0$

Determine the boundary points.

$$x^2 - 3x - 18 < 0$$

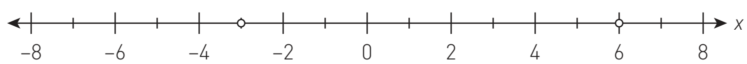
$$x^2 - 3x - 18 = 0$$ Change the inequality into an equation.

$$(x + 3)(x - 6) = 0$$ Factor the trinomial.

$$(x + 3) = 0 \qquad (x - 6) = 0$$
$$x = -3 \qquad x = 6$$

Rewrite it as two equations.

Graph the boundary points on a number line.



Choose a point in each region to test. Test the points in the original inequality.

$x = -10$

$$\begin{aligned} x^2 - 3x - 18 &< 0 \\ (-10)^2 - 3(-10) - 18 &< 0 \\ 100 + 30 - 18 &< 0 \\ 112 &< 0 \end{aligned}$$

false

$x = 0$

$$\begin{aligned} x^2 - 3x - 18 &< 0 \\ (0)^2 - 3(0) - 18 &< 0 \\ 0 - 0 - 18 &< 0 \\ -18 &< 0 \end{aligned}$$

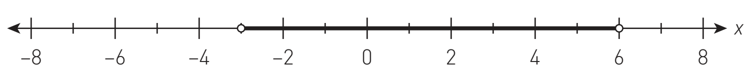
true

$x = 10$

$$\begin{aligned} x^2 - 3x - 18 &< 0 \\ (10)^2 - 3(10) - 18 &< 0 \\ 100 - 30 - 18 &< 0 \\ 52 &< 0 \end{aligned}$$

false

The solution is all numbers between the boundary points, $-3 < x < 6$.



4. Solve $m^2 - 3 \geq 1$

Determine the boundary points.

$$m^2 - 3 \geq 1$$

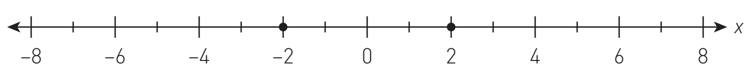
$$\downarrow$$

$$m^2 - 3 = 1$$ Change the inequality into an equation.

$$m^2 = 4$$

$$m = \pm 2$$

Graph the boundary points on a number line.



Choose a point in each region to test. Test the points in the original inequality.

$$m = -4$$

$$\downarrow$$

$$m^2 - 3 \geq 1$$

$$(-4)^2 - 3 \geq 1$$

$$16 - 3 \geq 1$$

$$13 \geq 1$$

true

$$m = 0$$

$$\downarrow$$

$$m^2 - 3 \geq 1$$

$$(0)^2 - 3 \geq 1$$

$$0 - 3 \geq 1$$

$$-3 \geq 1$$

false

$$m = 6$$

$$\downarrow$$

$$m^2 - 3 \geq 1$$

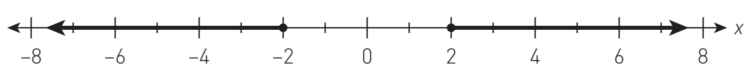
$$(6)^2 - 3 \geq 1$$

$$36 - 3 \geq 1$$

$$33 \geq 1$$

true

The solution is any number less than or equal to –2 or greater than or equal to 2, $m \leq -2$ or $m \geq 2$.



5. Solve $-|3x + 4| < -12 + 20$

Determine the boundary points.

$$-|3x + 4| < -12 + 20$$

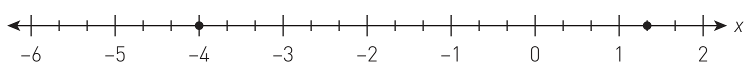
$$-|3x + 4| = -12 + 20$$
$$-|3x + 4| = 8$$

Change the inequality into an equation.

$$-(3x + 4) = 8 \qquad -(-(3x + 4)) = 8$$
$$-3x - 4 = 8 \qquad 3x + 4 = 8$$
$$-3x = 12 \qquad 3x = 4$$
$$x = -4 \qquad x = \frac{4}{3}$$

Rewrite it as two equations.

Graph the boundary points on a number line.



Choose a point in each region to test. Test the points in the original inequality.

$x = -10$

$$-|3x + 4| < -12 + 20$$
$$-|3(-10) + 4| < -12 + 20$$
$$-|-30 + 4| < 8$$
$$-|-26| < 8$$
$$-26 < 8$$

true

$x = 0$

$$-|3x + 4| < -12 + 20$$
$$-|3(0) + 4| < -12 + 20$$
$$-|0 + 4| < 8$$
$$-|4| < 8$$
$$-4 < 8$$

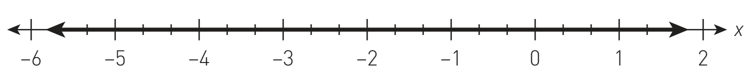
true

$x = 10$

$$-|3x + 4| < -12 + 20$$
$$-|3(10) + 4| < -12 + 20$$
$$-|30 + 4| < 8$$
$$-|34| < 8$$
$$-34 < 8$$

true

In this case, all of the regions make the inequality true, so x = all real numbers.



Practice

Solve each inequality.

1. $\lvert x + 4\rvert \geq 7$	**2.** $x^2 + 6x + 8 < 0$	**3.** $y^2 - 5y > 0$
4. $\lvert x\rvert - 5 \leq 8$	**5.** $\lvert x - 5\rvert \leq 8$	**6.** $y^2 - 5y < 0$
7. $\lvert 4r - 2\rvert > 8$	**8.** $x^2 - 3x - 4 < 0$	**9.** $\lvert 3x\rvert \leq 12$
10. $-x^2 - 9x - 14 < 0$	**11.** $\lvert 1 - 3x\rvert \leq 13$	**12.** $y^2 \leq 16$
13. $\lvert 2x - 3\rvert > 15$	**14.** $3x^2 + 7x - 6 \geq 0$	**15.** $\lvert 5x\rvert > -15$
16. $-2\lvert x - 3\rvert + 6 < -4$	**17.** $x^2 + 4x - 12 < 0$	**18.** $y^2 + 6y + 9 > 0$
19. $\lvert 4 - d\rvert \leq 7$	**20.** $x(7x - 26) \leq 8$	

Answers

1. $x \geq 3$ or $x \leq -11$	**2.** $-4 < x < -2$	**3.** $y < 0$ or $y > 5$
4. $-13 \leq x \leq 13$	**5.** $-3 \leq x \leq 13$	**6.** $0 < y < 5$
7. $r < \frac{-3}{2}$ or $r > \frac{5}{2}$	**8.** $-1 < x < 4$	**9.** $-4 \leq x \leq 4$
10. $x < -7$ or $x > -2$	**11.** $-4 < x < \frac{14}{3}$	**12.** $-4 \leq y \leq 4$
13. $x < -6$ or $x > 9$	**14.** $x \leq -3$ or $x \geq \frac{2}{3}$	**15.** all numbers
16. $x > 8$ or $x < -2$	**17.** $-6 < x < 2$	**18.** all numbers except $y = -3$
19. $-3 \leq d \leq 11$	**20.** $\frac{-2}{7} \leq x \leq 4$	